Chapter 5, Trigonometric Identities

Focus Exercise Answers

Section 5.1 Proving Trigonometric Identities

1. $\sin\theta$

2. $\dfrac{\sin^2\theta}{\cos\theta}$

3. $\cos\theta$

4. $\dfrac{\cos\theta}{\sin^2\theta}$

5. $\dfrac{\cos^2\theta}{\sin\theta}$

6. $\sec\theta$

7. $\dfrac{\cos\theta}{\sin^2\theta}$ or $\csc\theta\cot\theta$

8. $\cos^2\theta$

9. $\csc\theta$

10. $\csc^2\theta$

11. $\dfrac{\sin\theta + 1}{\cos\theta}$

12. $\dfrac{2\cos\theta}{\sin\theta}$ or $2\cot\theta$

13. 0

14. $\dfrac{1 - \sin^2\theta}{\cos\theta}$ or $\cos\theta$

15. LCD $= \cos^2\theta$; $\dfrac{\cos\theta + 1}{\cos^2\theta}$

16. LCD $= \sin\theta$; $\dfrac{1 - \sin^2\theta}{\sin\theta}$ or $\dfrac{\cos^2\theta}{\sin\theta}$

17. LCD $= \sin\theta\cos\theta$; $\dfrac{\sin\theta}{\cos\theta}$

18. LCD $= \sin\theta\cos\theta$; $\dfrac{1}{\sin\theta\cos\theta}$

19. $\sin\theta - \cos^2\theta$

20. $\dfrac{1}{\cos\theta} + 1$ or $\dfrac{1 + \cos\theta}{\cos\theta}$ or $\sec\theta + 1$

21. $\cos^2\theta + 3\cos\theta + 2$

22. $1 + 2\sin\theta\cos\theta$

For #23-30, full answers are not shown. Instead, strategies are given as to how to work the proof:

#23-27: On the left side, write the expression in terms of sine and cosine only; simplify using the trigonometric reciprocal and ratio identities.

#28-29: On the left side, write the expression in terms of sine and cosine, combine the fractions, and use a Pythagorean Identity to arrive at a conclusion.

#30: On the left side, write the expression in terms of sine and cosine and use a Pythagorean Identity to arrive at a conclusion.

Section 5.2 Verifying Trigonometric Identities

For #1-12, full answers are not shown. Instead, hints are given as to how to start the proof:

1. On the left side, rearrange the terms to see an important identity.
2. On the left side, split the fraction.
3. On the right side, convert to sine/cosine. Simplify.
4. On the right side, distribute and simplify. You can convert to sine/cosine before or after distributing.
5. **One option:** On the right side, use a Pythagorean identity. Factor and simplify.
Alternative: On the left side, convert to sine/cosine and simplify.
6. **One option:** On the right side, use a Pythagorean identity.
Alternative: On the right side, multiply by a conjugate and simplify.
7. On the left side, get common denominators. Simplify.
8. On the left side, convert to sine/cosine. Simplify. Eventually, a Pythagorean identity is used.
9. On the left side, distribute and simplify; on scratch paper, simplify the right side to give guidance on how to complete the proof.
10. **One option:** On the left side, multiply by the denominator's conjugate; on scratch paper, simplify the right side to give guidance on how to complete the proof.
Alternative: On the right side, multiply out the quantity; convert to sine/cosine, and factor the numerator and/or denominator.
11. **One option:** On the left side, Convert to sine/cosine. Simplify the complex fraction.
Alternative: On the left side, split the fraction and simplify.
12. **One option:** On the left side, Convert to sine/cosine. Simplify the complex fraction.
Alternative: On the left side, split the fraction and simplify.

For #13-18, one option is shown along with the first step. For some, hints might be given as to alternative answers. You are encouraged to choose a different value than the one shown.

13. One option: $\theta = \frac{\pi}{2}$

$\sin\left(-\frac{\pi}{2}\right) \stackrel{?}{=} \sin\left(\frac{\pi}{2}\right)$

In general, any value that doesn't make sine 0 is a counterexample.

14. One option: $\theta = \frac{\pi}{4}$

$\cos\left(-\frac{\pi}{4}\right) \stackrel{?}{=} -\cos\left(\frac{\pi}{4}\right)$

In general, any value that doesn't make cosine 0 is a counterexample.

15. One option: $x = \pi$

$(1 + \cos\pi)^2 \stackrel{?}{=} 1 + \cos^2\pi$

In general, any value that doesn't make cosine 0 is a counterexample.

16. One option: $x = \frac{\pi}{3}$

$\tan^2\left(\frac{\pi}{3}\right) - 1 \stackrel{?}{=} \sec^2\left(\frac{\pi}{3}\right)$

In general, any value that doesn't make cosine 0 is a counterexample.

17. One option: $x = \frac{\pi}{4}$

$\sin\left(2\cdot\frac{\pi}{4}\right) \stackrel{?}{=} \sin\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{\pi}{4}\right)$

In general, any value that doesn't make sine or cosine 0 is a counterexample.

18. One option: $x = \frac{\pi}{2}$

$\cos^2\left(\frac{\pi}{2}\right) - \sin^2\left(\frac{\pi}{2}\right) \stackrel{?}{=} 1$

In general, any value that doesn't make sine 0 is a counterexample.

Section 5.3 Sum and Difference Formulas

1. One option: $A = 45°$ and $B = 135°$

$\cos(45° + 135°) \stackrel{?}{=} \cos(45°) + \cos(135°)$

2. One option: $A = 45°$ and $B = 135°$

$\sin(45° + 135°) \stackrel{?}{=} \sin(45°) + \sin(135°)$

3. $\frac{\sqrt{6}+\sqrt{2}}{4}$ **4.** $\frac{\sqrt{6}+\sqrt{2}}{4}$ **5.** $\frac{-\sqrt{6}-\sqrt{2}}{4}$ **6.** $\frac{\sqrt{6}-\sqrt{2}}{4}$

7. $2 - \sqrt{3}$ **8.** $-2 - \sqrt{3}$ **9.** $\frac{\sqrt{3}}{2}$ **10.** -1

11. $\frac{\sqrt{2}}{2}$ **12.** $-\frac{\sqrt{2}}{2}$ **13.** $-\frac{\sqrt{3}}{3}$ **14.** $\sqrt{3}$

15. $-\frac{\sqrt{2}}{2}$ **16.** 1 **17.** 1 **18.** $\frac{1}{2}$

19. Undefined **20.** 1

Answers for the proofs #21-28 are not shown, but hints are given.

21. Start with $\sin(-x) = \sin(0 - x)$

Then use the difference identity for sine to simplify the right side.

22. Start with $\tan(-x) = \frac{\sin(-x)}{\cos(-x)}$

Or, start with $\tan(-x) = \tan(0 - x)$

23. On the left side, carefully use the sum and difference identities for sine; simplify.

24. On the left side, carefully use the sum and difference identities for cosine; simplify.

25. On the left side, use the difference identity for sine; simplify.

26. On the left side, use the sum identity for cosine; simplify.

27 & 28. On the left side, first write the expression in terms of sine and cosine; then use the appropriate difference identity. Simplify.

Section 5.4 Double Angle Formulas

1. a) $\sin(2A) = \frac{-4\sqrt{5}}{9}$

b) $\cos(2A) = \frac{1}{9}$

2. a) $\sin(2A) = \frac{24}{25}$

b) $\cos(2A) = \frac{-7}{25}$

3. a) $\sin(2A) = \frac{3}{5}$

b) $\cos(2A) = \frac{4}{5}$

4. a) $\sin(2A) = \frac{3\sqrt{7}}{8}$

b) $\cos(2A) = \frac{-1}{8}$

5. a) $\sin(2A) = \frac{-12}{13}$

b) $\cos(2A) = \frac{5}{13}$

6. a) $\sin(2A) = \frac{40}{41}$

b) $\cos(2A) = \frac{9}{41}$

7. $\tan(2A) = -4\sqrt{5}$

8. $\tan(2A) = \frac{-60}{11}$

9. $\tan(2A) = \frac{-4}{3}$

10. $\tan(2A) = -2\sqrt{2}$

11. $\frac{\sqrt{3}}{2}$

12. $\frac{\sqrt{3}}{2}$

13. $\frac{\sqrt{2}}{2}$

14. $\frac{1}{2}$

15. $-\frac{1}{2}$

16. $-\frac{\sqrt{3}}{3}$

17. -1

18. The author made an error on this exercise. As written, 2sin(63.5°) cos(63.5°) becomes sin(127°), which is not one of our familiar angle measures. Instead, the author meant to write the exercise as 2sin(67.5°) cos(67.5°). When done correctly, this answer is $\frac{\sqrt{2}}{2}$.

19. $-\frac{1}{4}$

20. $-\frac{1}{4}$

21. $\frac{1}{2}$

22. $\frac{-1}{2\sqrt{3}} = \frac{-\sqrt{3}}{6}$

23. $1 - \frac{\sqrt{2}}{2}$

24. $-\frac{\sqrt{3}}{2} + 1$

25. $-\frac{\sqrt{3}}{2} - 1$

26. $-\frac{\sqrt{3}}{2} - 1$

For #27-30, full answers are not shown. Instead, hints are given as to how to start the proof:

27. On the left side, multiply out the quantity and simplify.

28. On the left side, use one of the three double angle identities for cosine and simplify. Note: any one of the three identities will work (eventually), but one identity is a better choice than the other two.

29. On the left side, use the double angle identity for sine and one of the double angle identities for cosine; simplify. Note: any one of the three cosine identities for cosine will work (eventually), but one is a better choice than the others.

30. On the right side, first write the expression in terms of sine and cosine, then use a Pythagorean identity and a double angle identity to simplify.

Section 5.5 Half-Angle Formulas

1. $\frac{\sqrt{2-\sqrt{3}}}{2}$ **2.** $\frac{\sqrt{2+\sqrt{2}}}{2}$ **3.** $\frac{\sqrt{2+\sqrt{2}}}{2}$ **4.** $\frac{\sqrt{2+\sqrt{3}}}{2}$

5. $-2-\sqrt{3}$ **6.** $1+\sqrt{2}$ **7.** $\frac{\sqrt{2-\sqrt{2}}}{2}$ **8.** $-\frac{\sqrt{2-\sqrt{3}}}{2}$

9. $\frac{\sqrt{2+\sqrt{2}}}{2}$ **10.** $\frac{\sqrt{2+\sqrt{3}}}{2}$ **11.** $1+\sqrt{2}$ **12.** $2-\sqrt{3}$

13. $-\frac{\sqrt{30}}{6}$ **14.** $\frac{\sqrt{6}}{3}$ **15.** $\frac{\sqrt{15}}{5}$ **16.** $-\frac{\sqrt{70}}{10}$

17. $-\sqrt{7}$ **18.** $\frac{\sqrt{2}}{2}$ **19.** $\frac{1+\cos(A)}{2}$ **20.** $\frac{1-\cos(A)}{2}$

21. $\frac{\sin(A)}{2}$ **22.** $\frac{1+\cos(A)}{\sin(A)}$

#23-26, hints are given:

23. Use the half angle identity on the left-hand side. Separate the fractions.
24. Use two different half-angle identities on the left-hand side.
25. and 26. Use the expressions found in #19 and #20.