Chapter 5, Trigonometric Identities

ODD Focus Exercise Answers

Section 5.1 Verifying Trigonometric Identities

For #1-12, full answers are not shown. Instead, hints are given as to how to start the proof:

1. On the left side, rearrange the terms to see an important identity.
3. On the right side, convert to sine/cosine. Simplify.
5. **One option:** On the right side, use a Pythagorean identity. Factor and simplify.
Alternative: On the left side, convert to sine/cosine and simplify.
7. On the left side, get common denominators. Simplify.
9. On the left side, distribute and simplify; on scratch paper, simplify the right side to give guidance on how to complete the proof.
11. **One option:** On the left side, Convert to sine/cosine. Simplify the complex fraction.
Alternative: On the left side, split the fraction and simplify.

For #13-17, one option is shown along with the first step. You are encouraged to choose a different value than the one shown.

13. One option: $\theta = \frac{\pi}{2}$: $\sin\left(-\frac{\pi}{2}\right) \stackrel{?}{=} \sin\left(\frac{\pi}{2}\right)$

In general, any value that doesn't make sine 0 is a counterexample.

15. One option: $\theta = \pi$: $(1 + \cos\pi)^2 \stackrel{?}{=} 1 + \cos^2\pi$

In general, any value that doesn't make cosine 0 is a counterexample.

17. One option: $\theta = \frac{\pi}{4}$: $\sin\left(2\cdot\frac{\pi}{4}\right) \stackrel{?}{=} \sin\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{\pi}{4}\right)$

In general, any value that doesn't make sine or cosine 0 is a counterexample.

Section 5.2 Sum and Difference Formulas

1. One option: $A = 45°$ and $B = 135°$: $\cos(45° + 135°) \stackrel{?}{=} \cos(45°) + \cos(135°)$

3. $\frac{\sqrt{6}+\sqrt{2}}{4}$ **5.** $\frac{-\sqrt{6}-\sqrt{2}}{4}$ **7.** $2 - \sqrt{3}$ **9.** $\frac{\sqrt{3}}{2}$

11. $\frac{\sqrt{2}}{2}$ **13.** $-\frac{\sqrt{3}}{3}$ **15.** $-\frac{\sqrt{2}}{2}$ **17.** 1

19. Undefined

Answers for the proofs #21-27, odd, are not shown.

21. Start with $\sin(-x) = \sin(0 - x)$. Then use the difference identity for sine of a difference to simplify the right side.

23. On the left side, carefully use the sum and difference identities for sine; simplify.

25. On the left side, use the difference identity for sine; simplify.

27. On the left side, first convert the expression to sine/cosine and then use the appropriate difference identity; simplify.

Section 5.3 Double Angle Formulas

1. a) $\sin(2A) = \frac{24}{25}$

b) $\cos(2A) = \frac{-7}{25}$

3. a) $\sin(2A) = \frac{3}{5}$

b) $\cos(2A) = \frac{4}{5}$

5. $\tan(2A) = -4\sqrt{5}$

7. $\frac{\sqrt{3}}{2}$

9. $\frac{\sqrt{2}}{2}$

11. $-\frac{1}{2}$

13. $-\frac{1}{4}$

15. $1 - \frac{\sqrt{2}}{2}$

For #17 & 19, full answers are not shown. Instead, hints are given as to how to start the proof:

17. On the left side, multiply out the quantity and simplify.

19. On the left side, use the double angle identity for sine and one of the double angle identifies for cosine; simplify. Note: any one of the three cosine identities for cosine will work (eventually), but one is a better choice than the others.

Section 5.4 Half-Angle Formulas

1. $\frac{\sqrt{2-\sqrt{3}}}{2}$

3. $\frac{\sqrt{2+\sqrt{2}}}{2}$

5. $-2-\sqrt{3}$

7. $\frac{\sqrt{2-\sqrt{2}}}{2}$

9. $\frac{\sqrt{2+\sqrt{2}}}{2}$

11. $1+\sqrt{2}$

13. $\frac{-\sqrt{30}}{6}$

15. $\frac{\sqrt{15}}{5}$

17. $-\sqrt{7}$

19. $\cos\left(\frac{4\pi}{9}\right)$

21. $\sin(31.5°)$

23. $\tan\left(\frac{\pi}{5}\right)$

25. $-\tan(115°)$

27. $\cos^2\left(\frac{A}{2}\right) = \frac{1 + \cos A}{2}$

29. $\frac{1}{2}\sin A$

31. $\sec\left(\frac{A}{2}\right) = \pm\frac{\sqrt{2(1 + \cos A)}}{1 + \cos A}$ *or* $\sec\left(\frac{A}{2}\right) = \pm\frac{\sqrt{2(1 - \cos A)}}{\sin A}$

For #33 & 35, full answers are not shown. Instead, hints are given as to how to start the proof.
Note: the instructions say to *"verify each by using a half-angle identity on the left-side expression."*

33. Of the two options for $\tan\left(\frac{A}{2}\right)$, use the one with a "single-term" denominator.

35. Square each half-angle identity carefully. **Note:** each of these can be done using another technique, but you are to verify each using a half-angle identity.