Chapter 5, Trigonometric Identities

Focus Exercise Answers, Odds only

Section 5.1 Proving Trigonometric Identities

- $\cos^2\theta$ 1. 5. $\sin\theta$ 3. $\cos\theta$ $\sin\theta$ $\cos\theta$ $\sin\theta + 1$ or $\csc\theta \cot\theta$ 7. 9. $\csc\theta$ 11. **13.** 0 $\sin^2\theta$ $\cos\theta$ 17. LCD = $\sin\theta\cos\theta$; $\frac{\sin\theta}{\cos\theta}$ **15.** LCD = $\cos^2\theta$; $\frac{\cos\theta + 1}{\cos^2\theta}$ 19. $\sin\theta - \cos^2\theta$ 21. $\cos^2\theta + 3\cos\theta + 2$
- For #23-29, full answers are not shown. Instead, strategies are given as to how to work the proof:
- **#23-27:** On the left side, write the expression in terms of sine and cosine only; simplify using the trigonometric reciprocal and ratio identities.
- **#29:** On the left side, write the expression in terms of sine and cosine, combine the fractions, and use a Pythagorean Identity to arrive at a conclusion.

Section 5.2 Verifying Trigonometric Identities

For #1-11, full answers are not shown. Instead, hints are given as to how to start the proof:

- 1. On the left side, rearrange the terms to see an important identity.
- 3. On the right side, convert to sine/cosine. Simplify.
- 5. One option: On the right side, use a Pythagorean identity. Factor and simplify. Alternative: On the left side, convert to sine/cosine and simplify.
- 7. On the left side, get common denominators. Simplify.
- **9.** On the left side, distribute and simplify; on scratch paper, simplify the right side to give guidance on how to complete the proof.
- **11. One option:** On the left side, Convert to sine/cosine. Simplify the complex fraction. **Alternative:** On the left side, split the fraction and simplify.

For #13-17, one option is shown along with the first step. For some, hints might be given as to alternative answers. You are encouraged to choose a different value than the one shown.

- **13.** One option: $\theta = \frac{\pi}{2}$ $\sin\left(-\frac{\pi}{2}\right)^{\frac{2}{2}} \sin\left(\frac{\pi}{2}\right)$ In general, any value that doesn't make size 0 is a countermompte
 - make sine 0 is a counterexample.
- 17. One option: $x = \frac{\pi}{4}$

$$\sin\left(2\cdot\frac{\pi}{4}\right)^{?} = \sin\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{\pi}{4}\right)$$

In general, any value that doesn't make sine or cosine 0 is a counterexample.

Section 5.3 Sum and Difference Formulas

- 1. One option: $A = 45^{\circ}$ and $B = 135^{\circ}$ $\cos(45^{\circ} + 135^{\circ}) \stackrel{?}{=} \cos(45^{\circ}) + \cos(135^{\circ})$
- 3. $\frac{\sqrt{6} + \sqrt{2}}{4}$ 5. $\frac{-\sqrt{6} \sqrt{2}}{4}$ 7. $2 \sqrt{3}$ 9. $\frac{\sqrt{3}}{2}$ 11. $\frac{\sqrt{2}}{2}$ 13. $-\frac{\sqrt{3}}{3}$ 15. $-\frac{\sqrt{2}}{2}$ 17. 1
- 19. Undefined

Answers for the proofs #21-28 are not shown, but hints are given.

- 21. Start with sin(-x) = sin(0 x)Then use the difference identity for sine to simplify the right side.
- **25.** On the left side, use the difference identity for sine; simplify.
- **23.** On the left side, carefully use the sum and difference identities for sine. Simplify.
- **27.** On the left side, first write the expression in terms of sine and cosine; then use the appropriate difference identity. Simplify.

15. One option: $x = \pi$

 $(1 + \cos \pi)^2 = 1 + \cos^2 \pi$

In general, any value that doesn't make cosine 0 is a counterexample.

Section 5.4 Double Angle Formulas

1. a)
$$\sin(2A) = \frac{-4\sqrt{5}}{9}$$

b) $\cos(2A) = \frac{1}{9}$
c) $\cos(2A) = \frac{1}{9}$
c) $\cos(2A) = \frac{4}{9}$
c) $\cos(2A) = \frac{4}{5}$
c) $\cos(2A) = \frac{4}{5}$
c) $\cos(2A) = \frac{5}{13}$
c) $\sin(2A) = \frac{-4}{3}$
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Note about #18.

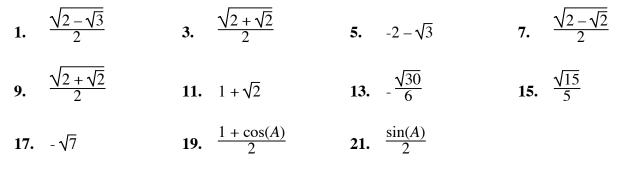
The author made an error on this exercise. As written, $2\sin(63.5^\circ)\cos(63.5^\circ)$ becomes $\sin(127^\circ)$, which is not one of our familiar angle measures. Instead, the author meant to write the exercise as $2\sin(67.5^\circ)$. Please correct this in your textbook.

19. $-\frac{1}{4}$ **21.** $\frac{1}{2}$ **23.** $1-\frac{\sqrt{2}}{2}$ **25.** $-\frac{\sqrt{3}}{2}$ -1

For #27-29, full answers are not shown. Instead, hints are given as to how to start the proof:

- 27. On the left side, multiply out the quantity and simplify.
- **29.** On the left side, use the double angle identity for sine and one of the double angle identities for cosine; simplify. Note: any one of the three cosine identities for cosine will work (eventually), but one is a better choice than the others.

Section 5.5 Half-Angle Formulas



#23-25, hints are given:

- **23.** Use the half angle identity on the left-hand side. Separate the fractions.
- **25.** Use the expressions found in #19 and #20.