Chapter 5, Trigonometric Identities

Focus Exercise Answers, Odds only

Section 5.1 Proving Trigonometric Identities

1. $\sin\theta$ **3.** $\cos\theta$ **5.** $\dfrac{\cos^2\theta}{\sin\theta}$

7. $\dfrac{\cos\theta}{\sin^2\theta}$ or $\csc\theta\cot\theta$ **9.** $\csc\theta$ **11.** $\dfrac{\sin\theta + 1}{\cos\theta}$ **13.** 0

15. LCD $= \cos^2\theta$; $\dfrac{\cos\theta + 1}{\cos^2\theta}$ **17.** LCD $= \sin\theta\cos\theta$; $\dfrac{\sin\theta}{\cos\theta}$

19. $\sin\theta - \cos^2\theta$ **21.** $\cos^2\theta + 3\cos\theta + 2$

For #23-29, full answers are not shown. Instead, strategies are given as to how to work the proof:

#23-27: On the left side, write the expression in terms of sine and cosine only; simplify using the trigonometric reciprocal and ratio identities.

#29: On the left side, write the expression in terms of sine and cosine, combine the fractions, and use a Pythagorean Identity to arrive at a conclusion.

Section 5.2 Verifying Trigonometric Identities

For #1-11, full answers are not shown. Instead, hints are given as to how to start the proof:

1. On the left side, rearrange the terms to see an important identity.

3. On the right side, convert to sine/cosine. Simplify.

5. **One option:** On the right side, use a Pythagorean identity. Factor and simplify.
Alternative: On the left side, convert to sine/cosine and simplify.

7. On the left side, get common denominators. Simplify.

9. On the left side, distribute and simplify; on scratch paper, simplify the right side to give guidance on how to complete the proof.

11. **One option:** On the left side, Convert to sine/cosine. Simplify the complex fraction.
Alternative: On the left side, split the fraction and simplify.

For #13-17, one option is shown along with the first step. For some, hints might be given as to alternative answers. You are encouraged to choose a different value than the one shown.

13. One option: $\theta = \frac{\pi}{2}$

$\sin\left(-\frac{\pi}{2}\right) \stackrel{?}{=} \sin\left(\frac{\pi}{2}\right)$

In general, any value that doesn't make sine 0 is a counterexample.

15. One option: $x = \pi$

$(1 + \cos\pi)^2 \stackrel{?}{=} 1 + \cos^2\pi$

In general, any value that doesn't make cosine 0 is a counterexample.

17. One option: $x = \frac{\pi}{4}$

$\sin\left(2\cdot\frac{\pi}{4}\right) \stackrel{?}{=} \sin\left(\frac{\pi}{4}\right) \cdot \cos\left(\frac{\pi}{4}\right)$

In general, any value that doesn't make sine or cosine 0 is a counterexample.

Section 5.3 Sum and Difference Formulas

1. One option: $A = 45°$ and $B = 135°$

$\cos(45° + 135°) \stackrel{?}{=} \cos(45°) + \cos(135°)$

3. $\frac{\sqrt{6}+\sqrt{2}}{4}$ **5.** $\frac{-\sqrt{6}-\sqrt{2}}{4}$ **7.** $2 - \sqrt{3}$ **9.** $\frac{\sqrt{3}}{2}$

11. $\frac{\sqrt{2}}{2}$ **13.** $-\frac{\sqrt{3}}{3}$ **15.** $-\frac{\sqrt{2}}{2}$ **17.** 1

19. Undefined

Answers for the proofs #21-28 are not shown, but hints are given.

21. Start with $\sin(-x) = \sin(0 - x)$ Then use the difference identity for sine to simplify the right side.

23. On the left side, carefully use the sum and difference identities for sine. Simplify.

25. On the left side, use the difference identity for sine; simplify.

27. On the left side, first write the expression in terms of sine and cosine; then use the appropriate difference identity. Simplify.

Section 5.4 Double Angle Formulas

1. a) $\sin(2A) = \frac{-4\sqrt{5}}{9}$ b) $\cos(2A) = \frac{1}{9}$

3. a) $\sin(2A) = \frac{3}{5}$ b) $\cos(2A) = \frac{4}{5}$

5. a) $\sin(2A) = \frac{-12}{13}$ b) $\cos(2A) = \frac{5}{13}$

7. $\tan(2A) = -4\sqrt{5}$ **9.** $\tan(2A) = \frac{-4}{3}$ **11.** $\frac{\sqrt{3}}{2}$ **13.** $\frac{\sqrt{2}}{2}$

15. $-\frac{1}{2}$ **17.** -1

Note about #18.
The author made an error on this exercise. As written, 2sin(63.5°) cos(63.5°) becomes sin(127°), which is not one of our familiar angle measures. Instead, the author meant to write the exercise as 2sin(67.5°). Please correct this in your textbook.

19. $-\frac{1}{4}$ **21.** $\frac{1}{2}$ **23.** $1 - \frac{\sqrt{2}}{2}$ **25.** $-\frac{\sqrt{3}}{2} - 1$

For #27-29, full answers are not shown. Instead, hints are given as to how to start the proof:

27. On the left side, multiply out the quantity and simplify.

29. On the left side, use the double angle identity for sine and one of the double angle identities for cosine; simplify. Note: any one of the three cosine identities for cosine will work (eventually), but one is a better choice than the others.

Section 5.5 Half-Angle Formulas

1. $\frac{\sqrt{2-\sqrt{3}}}{2}$ **3.** $\frac{\sqrt{2+\sqrt{2}}}{2}$ **5.** $-2-\sqrt{3}$ **7.** $\frac{\sqrt{2-\sqrt{2}}}{2}$

9. $\frac{\sqrt{2+\sqrt{2}}}{2}$ **11.** $1+\sqrt{2}$ **13.** $-\frac{\sqrt{30}}{6}$ **15.** $\frac{\sqrt{15}}{5}$

17. $-\sqrt{7}$ **19.** $\frac{1+\cos(A)}{2}$ **21.** $\frac{\sin(A)}{2}$

#23-25, hints are given:

23. Use the half angle identity on the left-hand side. Separate the fractions.
25. Use the expressions found in #19 and #20.