

Chapter 6, Trigonometric Equations

Focus Exercise Answers

Sec. 6.1 Solving Trigonometric Equations

1. $\theta = 30^\circ, 330^\circ$ 2. $\theta = 60^\circ, 120^\circ$ 3. $x = \frac{2\pi}{3}, \frac{5\pi}{3}$ 4. $x = \frac{3\pi}{4}, \frac{5\pi}{4}$

5. $\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$ 6. $\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

7. $\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$ 8. $\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$

9. $x = 0, \pi, \frac{\pi}{4}, \frac{5\pi}{4}$ 10. $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

11. $x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$ 12. $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{5\pi}{4}$

13. $\left. \begin{array}{l} x = \frac{2\pi}{3} + 2\pi k \\ x = \frac{4\pi}{3} + 2\pi k \end{array} \right\} k \in \mathbf{Z}$ 14. $x = \pi + 2\pi k, k \in \mathbf{Z}$

15. $\left. \begin{array}{l} \theta = 135^\circ + 360^\circ k \\ \theta = 315^\circ + 360^\circ k \end{array} \right\} k \in \mathbf{Z}$ 16. $\left. \begin{array}{l} \theta = 210^\circ + 360^\circ k \\ \theta = 330^\circ + 360^\circ k \\ \theta = 270^\circ + 360^\circ k \end{array} \right\} k \in \mathbf{Z}$

or ... $\theta = 135^\circ + 180^\circ k$

Sec. 6.2 More Techniques for Solving Trigonometric Equations

1. $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ 2. $x = \frac{\pi}{2}, \frac{\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$

3. $\theta = 0^\circ, 180^\circ, 270^\circ$ 4. $\theta = 120^\circ, 240^\circ$

5. $\theta = 90^\circ + 180^\circ k, k \in \mathbf{Z}$ 6. $\theta = 45^\circ + 90^\circ k, k \in \mathbf{Z}$

7. $\left. \begin{array}{l} x = \frac{\pi}{2} + \pi k \\ x = \frac{7\pi}{6} + 2\pi k \\ x = \frac{11\pi}{6} + 2\pi k \end{array} \right\} k \in \mathbf{Z}$ 8. $x = \pi k, k \in \mathbf{Z}$

9. $x = 0, \pi, \frac{2\pi}{3}, \frac{5\pi}{3}$ 10. $x = \frac{\pi}{4}, \frac{5\pi}{4}$

11. *No Solution as written.**

12. $x = 0, \pi$ *as written.**

***Note:** The author made a couple of errors in writing #11 and #12. He used a plus sign when he meant to use a minus sign. Here are #11 and 12 as they were intended to be. The answers are at the end of this answer sheet, after Sec. 6.4 answers:

Solve for x , $0 \leq x < 2\pi$

11. $3\tan x - \cot x = 0$

12. $\sin(2x) - \tan x = 0.$

Another Note: For #13-15, the author did not write a solving interval; however, because the equations use the variable x , the solving interval is, by default, in terms of radians: $0 \leq x < 2\pi$.

13. $x = 0, \frac{2\pi}{3}$ (Note: $\frac{4\pi}{3}$ is extraneous)

14. $x = \frac{\pi}{6}, \frac{5\pi}{6}$ (Note: $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$ are extraneous)

The extraneous solutions come into play if you square each sides of the equation before solving.

15. $x = \frac{\pi}{4}$ (Note: Depending on how you solved the equatin, either $\frac{3\pi}{4}$ or $\frac{5\pi}{4}$ is extraneous)

Sec. 6.3 Altered Arguments

1. $\theta = 20^\circ, 200^\circ$

2. $\theta = 30^\circ, 120^\circ$

3. $\theta = 170^\circ, 350^\circ$

4. $\theta = 20^\circ, 200^\circ$

5. $x = \frac{\pi}{4}, \frac{7\pi}{4}$

6. No solution (Secant can't be $\frac{\sqrt{3}}{2}$.)

7. $x = \frac{7\pi}{8}, \frac{15\pi}{8}$

8. $x = \frac{\pi}{3}, \frac{4\pi}{3}$

9. $\theta = 60^\circ, 180^\circ, 300^\circ$

10. $\theta = 22.5^\circ, 112.5^\circ, 202.5^\circ, 292.5^\circ,$

11. $\theta = 15^\circ, 105^\circ, 195^\circ, 285^\circ$

12. $\theta = 30^\circ, 90^\circ, 150^\circ, 210^\circ, 270^\circ, 330^\circ$

Note: For #13-16, because the equations use the variable x , the solving interval should be in terms of radians: $0 \leq x < 2\pi$. The author accidentally wrote the solving interval in terms of degrees.

13. $x = \frac{\pi}{4}, \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}, \frac{21\pi}{12}$

14. $x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

15. $x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

16. $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

Sec. 6.4 Trigonometric Equations of Multiple Angle Measures

1. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

2. $x = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}$

3. $x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

4. $x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{6}, \frac{11\pi}{6}$

5. $\theta = 0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ$

6. $\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ, 90^\circ, 270^\circ$

7. $\theta = 90^\circ, 270^\circ$

8. $\theta = 45^\circ, 315^\circ$

9. $\left. \begin{array}{l} \theta = 10^\circ + 60^\circ k \\ \theta = 50^\circ + 60^\circ k \end{array} \right\} k \in \mathbf{Z}$

10. $\left. \begin{array}{l} \theta = 9^\circ + 72^\circ k \\ \theta = 27^\circ + 72^\circ k \end{array} \right\} k \in \mathbf{Z}$

11. $x = \frac{\pi}{4} + \frac{\pi}{2}k, k \in \mathbf{Z}$

12. $\left. \begin{array}{l} x = \frac{5\pi}{8} + \pi k \\ x = \frac{7\pi}{8} + \pi k \end{array} \right\} k \in \mathbf{Z}$

Answers to Sec. 6.2, corrected #11 and 12 (see above on page 2 of this answer sheet):

11. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

12. $x = 0, \pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$