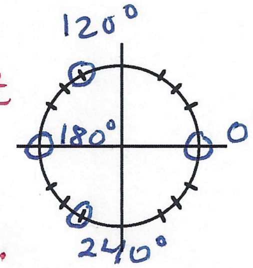
4. Solve the equation for $0° \le \theta < 360°$.

These equations have a restricted solving interval, and the solutions must be in degrees.



a) $\sin(2\theta) = -\sin\theta$ ← this equation has two different arguments, so we must use the double angle identity for sine to get all arguments to just θ.

$2\sin\theta\cos\theta = -\sin\theta$

At this second step, there might be a temptation to divide each side by $\sin\theta$, but doing so is incorrect; we might be dividing out potential solutions. Instead, add $\sin\theta$ to each side.

$2\sin\theta\cos\theta + \sin\theta = 0$

Now factor out $\sin\theta$ on the left side.

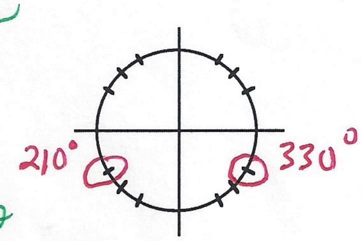
$\sin\theta(2\cos\theta + 1) = 0$

$\sin\theta = 0$ or $2\cos\theta + 1 = 0$

$\cos\theta = -\frac{1}{2}$

$$\boxed{\theta = 0°, 180°, 120°, 240°}$$

b) $2\cos^2\theta + 5\sin\theta + 1 = 0$ ← This equation has two different functions added together. We can use $\cos^2\theta = 1 - \sin^2\theta$ to write it in terms of $\sin\theta$ only.



$2(1 - \sin^2\theta) + 5\sin\theta + 1 = 0$

$2 - 2\sin^2\theta + 5\sin\theta + 1 = 0$

$-2\sin^2\theta + 5\sin\theta + 3 = 0$

$-1(-2\sin^2\theta + 5\sin\theta + 3) = 0 \cdot (-1)$

$2\sin^2\theta - 5\sin\theta - 3 = 0$ ← this is quadratic in form; replace $\sin\theta$ with a variable, w.

$2w^2 - 5w - 3 = 0$ Product $= -6$

$2w^2 - 6w + w - 3 = 0$ Sum $= -5$

$-6 \quad +1$

$2w(w-3) + 1(w-3) = 0$

$(2w+1)(w-3) = 0$

$(2\sin\theta + 1)(\sin\theta - 3) = 0$

$2\sin\theta + 1 = 0$ or $\sin\theta = 3$

$\sin\theta = -\frac{1}{2}$

This is impossible; $\sin\theta \le 1$

$$\boxed{\theta = 210°, 330°}$$

Sec. 6.3 Multiple Angles

5. Solve the equation for $0° \le \theta < 360°$.

i) adjust the solving interval to fit the argument
ii) let arg = argument and solve using the altered interval
iii) Adjust the solution set by solving for θ or x.

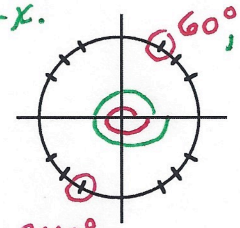
$\sqrt{3}\cot(2\theta) - 1 = 0$

(i) $0° \le \theta < 360°$

$\times 2 \quad \times 2 \quad \times 2$

$0° \le 2\theta < 720°$

$0° \le \text{arg} < 720°$



(ii) $\sqrt{3}\cot(\text{arg}) - 1 = 0$

$\cot(\text{arg}) = \frac{1}{\sqrt{3}}$

$\cot(\text{arg}) = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\cot(\text{arg}) = \frac{\sqrt{3}}{3}$

$\frac{2\theta}{2} = \text{arg} = \frac{60°}{2}, \frac{240°}{2}, \frac{420°}{2}, \frac{600°}{2}$

(iii) adjust the solution set by dividing each side by 2

$\boxed{\theta = 30°, 120°, 210°, 300°}$

6. Solve the equation for $0 \le x < 2\pi$.

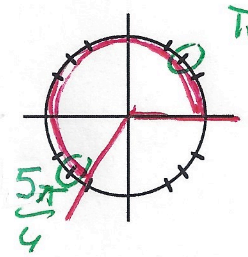
$\tan\left(\frac{2}{3}x\right) - 1 = 0$

(i) $0 \le x < 2\pi$

$\times \frac{2}{3} \quad \times \frac{2}{3} \quad \times \frac{2}{3}$

$0 \le \frac{2}{3}x < \frac{4\pi}{3}$

$0 \le \text{arg} < \frac{4\pi}{3}$



(ii) $\tan(\text{arg}) - 1 = 0$

$\tan(\text{arg}) = 1$

$\text{arg} = \frac{\pi}{4}, \frac{5\pi}{4}$

$\frac{2}{3}x = \frac{\pi}{4}, \frac{5\pi}{4}$

$\frac{3}{2} \cdot \frac{2}{3}x = \left(\frac{\pi}{4}, \frac{5\pi}{4}\right) \cdot \frac{3}{2}$

(iii) Adjust the solution set by multiplying each side by $\frac{3}{2}$.

$\boxed{x = \frac{3\pi}{8}, \frac{15\pi}{8}}$