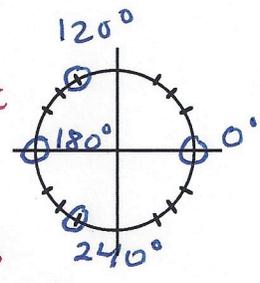


4. Solve the equation for $0^\circ \leq \theta < 360^\circ$.

These equations have a restricted solving interval, and the solutions must be in degrees.

a) $\sin(2\theta) = -\sin\theta$ ← This equation has two different arguments, so we must use the double angle identity for sine to get all arguments to just θ .



$$2\sin\theta\cos\theta = -\sin\theta$$

$$2\sin\theta\cos\theta + \sin\theta = 0$$

$$\sin\theta(2\cos\theta + 1) = 0$$

$$\sin\theta = 0 \text{ or } 2\cos\theta + 1 = 0$$

$$\cos\theta = -\frac{1}{2}$$

At this second step, there might be a temptation to divide each side by $\sin\theta$, but doing so is incorrect; we might be dividing out potential solutions. Instead, add $\sin\theta$ to each side.

Now factor out $\sin\theta$ on the left side.

$\theta = 0^\circ, 180^\circ, 120^\circ, 240^\circ$

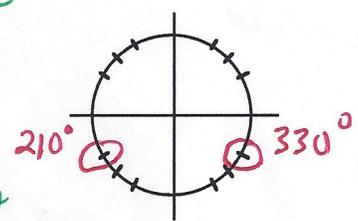
b) $2\cos^2\theta + 5\sin\theta + 1 = 0$

$$2(1 - \sin^2\theta) + 5\sin\theta + 1 = 0$$

$$2 - 2\sin^2\theta + 5\sin\theta + 1 = 0$$

$$-2\sin^2\theta + 5\sin\theta + 3 = 0$$

This equation has two different functions added together. We can use $\cos^2\theta = 1 - \sin^2\theta$ to write it in terms of $\sin\theta$ only.



$$-1(-2\sin^2\theta + 5\sin\theta + 3) = 0 \cdot (-1)$$

$$2\sin^2\theta - 5\sin\theta - 3 = 0$$

$$(2\sin\theta + 1)(\sin\theta - 3) = 0$$

$$2\sin\theta + 1 = 0 \text{ or } \sin\theta = 3$$

$$\sin\theta = -\frac{1}{2}$$

$\theta = 210^\circ, 330^\circ$

This is impossible; $\sin\theta \leq 1$

← this is quadratic in form; replace $\sin\theta$ with a variable, w .

$$2w^2 - 5w - 3 = 0 \quad \text{Product} = -6$$

$$2w^2 - 6w + w - 3 = 0 \quad \text{Sum} = -5 \quad \wedge$$

$$2w(w-3) + 1(w-3) = 0$$

$$(2w+1)(w-3) = 0$$

-6 + 1