

# Trigonometric Identities & Formulas

## Ratio Identities:

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cot A = \frac{\cos A}{\sin A}$$

## Reciprocal Identities:

$$\csc A = \frac{1}{\sin A} \quad \sin A = \frac{1}{\csc A} \quad \tan A = \frac{1}{\cot A}$$

$$\sec A = \frac{1}{\cos A} \quad \cos A = \frac{1}{\sec A} \quad \cot A = \frac{1}{\tan A}$$

## Co-function Identities:

$$\sin(90^\circ - A) = \cos A$$

$$\cos(90^\circ - A) = \sin A$$

$$\tan(90^\circ - A) = \cot A$$

## Pythagorean Identities:

$$\sin^2 A + \cos^2 A = 1$$

$$\sin^2 A = 1 - \cos^2 A$$

$$\sin A = \pm \sqrt{1 - \cos^2 A}$$

$$\cos^2 A = 1 - \sin^2 A$$

$$\cos A = \pm \sqrt{1 - \sin^2 A}$$

$$\tan^2 A + 1 = \sec^2 A$$

$$\tan^2 A = \sec^2 A - 1$$

$$1 = \sec^2 A - \tan^2 A$$

$$1 + \cot^2 A = \csc^2 A$$

$$\cot^2 A = \csc^2 A - 1$$

$$1 = \csc^2 A - \cot^2 A$$

## Alternate Forms of the Pythagorean Identities:

## Odd/Even Functions (Negative Angle Identities):

$$\sin(-A) = -\sin A$$

$$\cos(-A) = \cos A$$

$$\tan(-A) = -\tan A$$

$$\csc(-A) = -\csc A$$

$$\sec(-A) = \sec A$$

$$\cot(-A) = -\cot A$$

## Sum and Difference Formulas:

$$\cos(A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$\cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$

$$\sin(A + B) = \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$\sin(A - B) = \sin A \cdot \cos B - \cos A \cdot \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$$

## Double Angle Formulas:

$$\sin(2A) = 2 \sin A \cdot \cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\cos(2A) = 2\cos^2 A - 1$$

$$\cos(2A) = 1 - 2\sin^2 A$$

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

**Half Angle Formulas:**

$$\sin\left(\frac{1}{2}A\right) = \pm\sqrt{\frac{1 - \cos A}{2}}$$

$$\tan\left(\frac{1}{2}A\right) = \frac{1 - \cos A}{\sin A}$$

$$\cos\left(\frac{1}{2}A\right) = \pm\sqrt{\frac{1 + \cos A}{2}}$$

$$\tan\left(\frac{1}{2}A\right) = \frac{\sin A}{1 + \cos A}$$

**Sum to Product Formulas:**

$$\sin A + \sin B = 2\sin\frac{A+B}{2} \cdot \cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\sin\frac{A-B}{2} \cdot \cos\frac{A+B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2} \cdot \cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2} \cdot \sin\frac{A-B}{2}$$

**Product to Sum Formulas:**

$$\sin A \cdot \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$\cos A \cdot \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$$

$$\cos A \cdot \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$\sin A \cdot \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$$