(a) $\sin(15°)$ can be thought of as either the difference of angles $(45° - 30°)$ or as half of $30°$ (half angle formula)

12. Using the sum, difference, double angle, or half angle formulas, evaluate the following.

The sine of a difference formula is easier to calculate:

a) $\sin(15°)$

$= \sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 45° \cdot \sin 30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \rightarrow \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

b) $2 \sin\left(\frac{7\pi}{12}\right) \cos\left(\frac{7\pi}{12}\right)$ This is the double angle for sine.

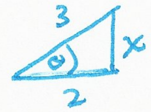
$= \sin\left(2 \cdot \frac{7\pi}{12}\right)$

$= \sin\left(\frac{7\pi}{6}\right) = \boxed{-\frac{1}{2}}$

13. Given that $\sec\theta = \frac{3}{2}$ and $0° < \theta < 90°$, evaluate the following.

Quad I

$\cos\theta = \frac{2}{3}$. We can use a triangle to find $\sin\theta$:



$x^2 + 4 = 9$

$x^2 = 5$

$x = \sqrt{5}$

So, $\sin\theta = \frac{\sqrt{5}}{3}$

a) $\cos(2\theta)$

$= 2\cos^2\theta - 1$

$= 2\left(\frac{2}{3}\right)^2 - 1$

$= 2 \cdot \frac{4}{9} - 1$

$= \frac{8}{9} - 1 = \boxed{-\frac{1}{9}}$

b) $\sin(2\theta)$

$= 2\sin\theta \cdot \cos\theta$

$= 2 \cdot \frac{\sqrt{5}}{3} \cdot \frac{2}{3}$

$= \boxed{\frac{4\sqrt{5}}{9}}$

14. Evaluate each. Denominators do not need to be rationalized.

Parts (a) and (b) are very simple.

a) $\tan\left(\tan^{-1}\frac{2}{3}\right) = \boxed{\frac{2}{3}}$

b) $\csc\left(\csc^{-1} 3\right) = \boxed{3}$

c) $\sin\left(\cos^{-1}\frac{1}{4}\right)$

Let $\theta = \cos^{-1}\left(\frac{1}{4}\right)$

$\cos\theta = \frac{1}{4}$

$x^2 + 1 = 16$

$x^2 = 15$

$x = \sqrt{15}$

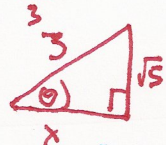
$= \sin\theta$

$= \boxed{\frac{\sqrt{15}}{4}}$

d) $\tan\left(\sin^{-1}\frac{\sqrt{5}}{3}\right)$

Let $\theta = \sin^{-1}\frac{\sqrt{5}}{3}$

$\sin\theta = \frac{\sqrt{5}}{3}$



$x^2 + 5 = 9$

$x^2 = 4$

$x = 2$

$= \tan(\theta)$

$= \boxed{\frac{\sqrt{5}}{2}}$

15. Prove each identity.

a) $\csc x - \cot x \cos x = \sin x$

$\frac{1}{\sin x} - \frac{\cos x}{\sin x} \cdot \cos x =$

$\frac{1}{\sin x} - \frac{\cos^2 x}{\sin x} =$

$\frac{1 - \cos^2 x}{\sin x} =$

$\frac{\sin^2 x}{\sin x} =$

$\sin x =$ QED

b) $\frac{1 - \tan^2 x}{1 + \tan^2 x} = \cos(2x)$

$\frac{\cos^2 x}{\cos^2 x} \cdot \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}} =$ multiply by $\frac{\cos^2 x}{\cos^2 x}$ to clear fractions.

$\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} =$

$\frac{\cos(2x)}{1} =$

$\cos(2x) =$ QED

16. Prove the identity $\dfrac{\csc x}{\cot x} - \dfrac{\cot x}{\csc x} = \sin x \tan x$

$$\frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}} - \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}} =$$

multiply each complex fraction by $\frac{\sin x}{\sin x}$ to clear the denominators.

$$\frac{\sin x}{\sin x} \cdot \frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}} - \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}} \cdot \frac{\sin x}{\sin x} =$$

$$\frac{1}{\cos x} - \frac{\cos x}{1} =$$ now get a common denominator of $\cos x$

$$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} =$$

$$\frac{1-\cos^2 x}{\cos x} =$$

$$\frac{\sin^2 x}{\cos x} =$$

$$\sin x \cdot \frac{\sin x}{\cos x} =$$

$$\sin x \cdot \tan x = \text{QED}$$

17. Solve each equation for $0° \le \theta < 360°$.

ⓐ Factor out $\tan\theta$.

a) $\tan^2\theta - \tan\theta = 0$

$\tan\theta(\tan\theta - 1) = 0$

$\tan\theta = 0$ or $\tan\theta - 1 = 0$

$0°, 180°$ $\quad$ $\tan\theta = 1$

$45°, 225°$

$$\boxed{\theta = 0°, 180°, 45°, 225°}$$

ⓑ write the identity for $\sin(2\theta)$:

b) $\sin 2\theta - \sin\theta = 0$

$2\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(2\cos\theta - 1) = 0$

$\sin\theta = 0$ or $2\cos\theta - 1 = 0$

$\cos\theta = \frac{1}{2}$

$$\boxed{\theta = 0°, 180°, 60°, 330°}$$

In #18, the argument includes a coefficient. This means we must adjust the restriction on x, temporarily, to find all possible values.

18. Solve each equation for $0 \le x < 2\pi$.

a) $1 + \sin(3x) = 0$

Argument is $3x$: $0 \le 3x < 6\pi$

$\sin(3x) = -1$

Argument = three values

$\frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}$

$3x = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}$

multiply each by $\frac{1}{3}$:

$$\boxed{x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

b) $\sec^2\left(\frac{1}{2}x\right) = 2$

Argument is $\frac{1}{2}x$: $0 \le \frac{1}{2}x < \pi$

$\sec\left(\frac{1}{2}x\right) = \pm\sqrt{2}$

this is the same as $\cos\left(\frac{1}{2}x\right) = \pm\frac{1}{\sqrt{2}} = \frac{\pm\sqrt{2}}{2}$

These occur at $\frac{\pi}{4}$ and $\frac{3\pi}{4}$

$\frac{1}{2}x = \frac{\pi}{4}, \frac{3\pi}{4}$

multiply each by 2:

$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$$