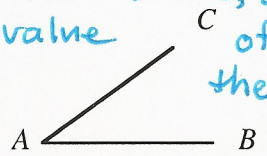
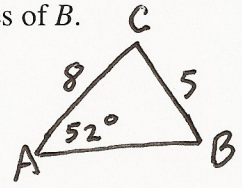
In the ambiguous case (#8), with $\triangle ABC$ as shown here, we always calculate $\angle B$ first. If we find an acute value of B, it doesn't necessarily mean that an obtuse value of B exists for that triangle. In other words, after finding the acute value of B, we must see if the obtuse value of B makes the $\triangle$ work.

8. Each refers to triangle *ABC*, which is not necessarily a right triangle. Each of these is of the Angle-Side-Side variety so the ambiguous case applies. From the given information, find all possible triangles, if any exist.



Use this incomplete triangle as the model for your work.

a) If $A = 52°$, $b = 8$ inches, and $a = 5$ inches, find all possible measures of *B*.



$\frac{\sin B}{8} = \frac{\sin 52°}{5}$

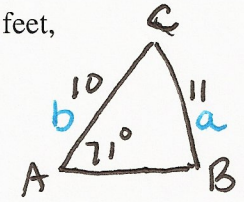
$\sin B = \frac{8 \cdot \sin 52°}{5}$

$\sin B = 1.2608$

Not a possible triangle

This is not possible because sin B can never be greater than 1.

b) If $A = 71°$, $b = 10$ feet, and $a = 11$ feet, find all possible measures of *B*.



$\frac{\sin B}{10} = \frac{\sin 71°}{11}$

$\sin B = \frac{10 \cdot \sin 71°}{11}$

$\sin B = .8596$

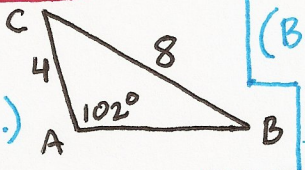
$B = 59.3°$

The obtuse value of B is 120.7°, but this also is not possible because the sum of $\angle A$ and $\angle B$ would be more than 180°.

Because $a > b$, $\angle A$ must be larger than $\angle B$. This means that $\angle B$ cannot be obtuse.

c) If $A = 102°$, $b = 4$ meters, and $a = 8$ meters, find all possible measures of *C*.

(There is no way $\angle B$ can have an obtuse value here.)



$\frac{\sin B}{4} = \frac{\sin 102°}{8}$

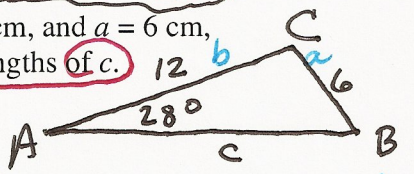
$\sin B = \frac{4 \cdot \sin 102°}{8}$

$\sin B = .4891$

$B = 29.3°$ Because $A = 102°$, this is the only possible value of B, so there is only one possible value of angle C: $C = 180° - (102° + 29.3°)$

$C = 48.7°$

d) If $A = 28°$, $b = 12$ cm, and $a = 6$ cm, find all possible lengths of *c*.



(Because $b > a$, $\angle B$ will have an obtuse value if an acute value exists.)

① $\frac{\sin B}{12} = \frac{\sin 28°}{6}$

$\sin B = \frac{12 \cdot \sin 28°}{6}$

$\sin B = .9389$

$\sim B = 69.9° \sim$

so $\sim C = 82.1° \sim$

② Use Law of Sines to find side c:

$\frac{c}{\sin 82.1°} = \frac{6}{\sin 28°}$

$c = \frac{6 \cdot \sin 82.1°}{\sin 28°} \Rightarrow$ $c = 12.7$ cm

Answer #1

③ The obtuse value of B is $180° - 69.9°$,

$\sim B = 110.1° \sim$

so $\sim C = 41.9° \sim$

④ 2nd value of c:

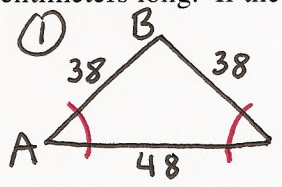
$\frac{c}{\sin 41.9°} = \frac{6}{\sin 28°}$

$c = \frac{6 \cdot \sin 41.9°}{\sin 28°}$

$c = 8.5$ cm

Answer #2

9. The two congruent sides of an isosceles triangle are each 38 centimeters long. If the base is 48 centimeters long, what is the measure of each base angle?



(1) $\angle A \cong \angle C$
Find $\angle A$ using Law of Cosines

(2) From the diagram:

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos A = \frac{48^2+38^2-38^2}{2(48)(38)}$$

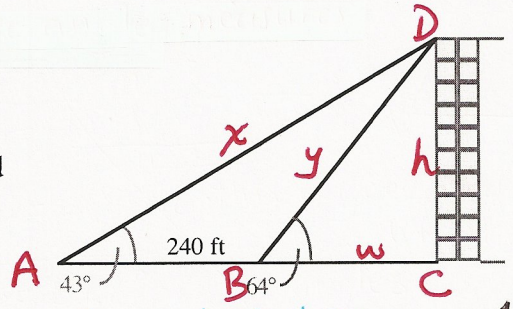
$$\cos A = \frac{2,304}{3,648}$$

$A = \cos^{-1}(2,304 \div 3648)$

$A = 50.8°$ ✓

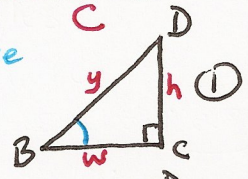
Each base angle measures 50.8°.

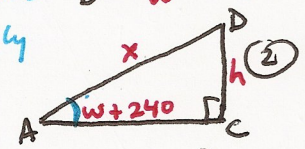
10. A woman standing near a building measures the angle of elevation to the top of the building to be 64°. She then walks 240 feet farther away from the building and measures the angle of elevation to the top of the building to be 43°. How tall is the building? (Round to the nearest foot.)

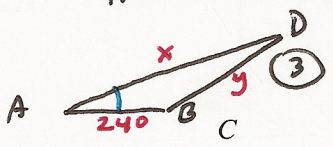


(4) The building is 411 feet tall.

I have labeled each unknown side value. In each of these three Δ's we can find all three angle measures. However, only ΔABD (3) has a known side value. So, we must use ΔABD to find either x or y (your choice) and then apply that value to either Δ(1) or Δ(2) to find h:







(1) Find the other two angle measures in Δ(3): $m\angle B = 116°$ and $m\angle D = 21°$.

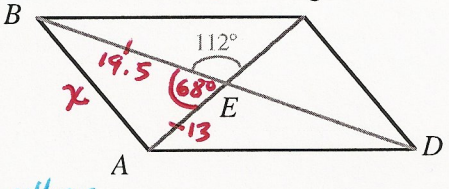
(2) $\frac{x}{\sin 116°} = \frac{240}{\sin 21°} \Rightarrow x = 602$

(3) In Δ(2) $\sin 43° = \frac{h}{602}$

$h = 411$ ft.

See answer above (4)

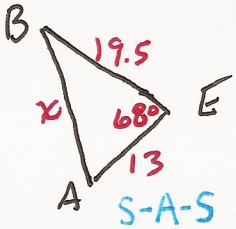
11. The diagonals of a parallelogram measure 26 inches (AC) and 39 inches (BD). If they meet at an angle of 112°, what is the length of the shorter side of the parallelogram?



(1) theorem from geometry: the diagonals of a parallelogram bisect each other.

(2) A shorter side of the parallelogram is $\overline{AB}$ and is opposite the acute $\angle AEB$, so find this measure first: $m\angle AEB = 68°$.

(3) $AC = 26$ in., so $AE = 13$ in.; $BD = 39$ in., so $BE = 19.5$ in.



S-A-S

(4) use the Law of Cosines to find AB, labeled x:

$$x^2 = 13^2 + 19.5^2 - 2(13)(19.5)\cos 68°$$

$$x^2 = 359.32$$

$x = 18.96 \approx 19.0$

The shorter side of the parallelogram is 19.0 inches long.