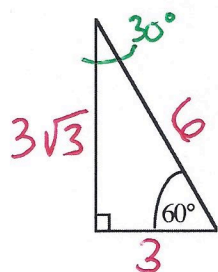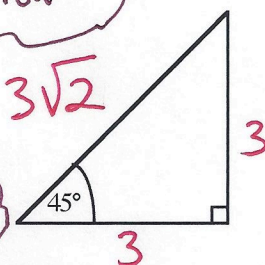First, the triangles shown in #22 are here as a guide. Use them only if they are helpful. To "mark them however you wish," does not mean to put any three numbers around the sides. These are the special right triangles, and you must use the 30-60-90 and 45-45-90 relationships. So, "...however you wish" means...

22. Use the given triangles and mark them however you wish to assist you in finding the following trig values. (Simplify and rationalize the denominator, if necessary.)

... to start with any side measure. I'll choose the shortest side to be 3, but you might choose it to be 1 (one).

Note: use "Soh-Cah-Toa" for the first three functions and reciprocals for the next three functions.

$\sin 30° = \dfrac{3}{6} = \boxed{\dfrac{1}{2}}$

$\sin 60° = \boxed{\dfrac{\sqrt{3}}{2}}$ ✱

$\sin 45° = \dfrac{3}{3\sqrt{2}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ ‡

$\cos 30° = \dfrac{3\sqrt{3}}{6} = \boxed{\dfrac{\sqrt{3}}{2}}$

$\cos 60° = \boxed{\dfrac{1}{2}}$

$\cos 45° = \dfrac{\sqrt{2}}{2}$

$\tan 30° = \dfrac{3}{3\sqrt{3}} = \dfrac{1}{\sqrt{3}} = \boxed{\dfrac{\sqrt{3}}{3}}$

$\tan 60° = \boxed{\sqrt{3}}$

$\tan 45° = \dfrac{3}{3} = 1$

(i) For the next three functions, use the reciprocals of the first three functions.

✱(ii) the trig functions of 60° have the same values as 30°; just use the co-functions.

$\cot 30° = \dfrac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$

$\cot 60° = \boxed{\dfrac{\sqrt{3}}{3}}$

$\cot 45° = 1$

$\sec 30° = \dfrac{2}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \boxed{\dfrac{2\sqrt{3}}{3}}$

$\sec 60° = \boxed{2}$

$\sec 45° = \dfrac{\sqrt{2}}{1} = \sqrt{2}$

$\csc 30° = \dfrac{2}{1} = \boxed{2}$

$\csc 60° = \boxed{\dfrac{2\sqrt{3}}{3}}$

$\csc 45° = \sqrt{2}$

‡ For 45°, notice that cofunctions are equivalent:

$\sin 45° = \cos 45°; \quad \tan 45° = \cot 45°; \quad \sec 45° = \csc 45°$

For #23 – 28, use the values that you got in #22.

Evaluate each. Simplify; rationalize the denominator, if necessary.

23. $2\sqrt{3}\ \sin 60°$

$$= 2\sqrt{3} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{2 \cdot \sqrt{9}}{2} = \sqrt{9} = \boxed{3}$$

24. $\sqrt{\cot 45°} = \sqrt{1} = \boxed{1}$

25. $\sqrt{\sec 60°}$

$$= \boxed{\sqrt{2}}$$

26. $(\tan 30°)^2$

$$= \left(\frac{\sqrt{3}}{3}\right)^2 = \frac{\sqrt{9}}{9} = \frac{3}{9} = \boxed{\frac{1}{3}}$$

27. $(\csc 45°)^2$

$$= \left(\sqrt{2}\right)^2 = \boxed{2}$$

28. $\tan 60° \cdot \cos 30°$

$$= \sqrt{3} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{9}}{2} = \boxed{\frac{3}{2}}$$

Write each in terms of sine and cosine only, and simplify completely. *(the answer doesn't need to be in $\sin\theta$ or $\cos\theta$.)*

29. $\sin\theta \cdot \cot\theta + \sec\theta$

$$= \frac{\sin\theta}{1} \cdot \frac{\cos\theta}{\sin\theta} + \frac{1}{\cos\theta}$$

$$= \frac{\cos\theta}{1} + \frac{1}{\cos\theta} \quad \text{get common denominators.}$$

$$= \frac{\cos\theta}{1} \cdot \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}$$

$$= \boxed{\frac{\cos^2\theta + 1}{\cos\theta}}$$

30. $\sec\theta - \tan\theta \cdot \sin\theta$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \cdot \sin\theta$$

$$= \frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{1 - \sin^2\theta}{\cos\theta} \leftarrow \text{Pythagorean identity}$$

$$= \frac{\cos^2\theta}{\cos\theta} \leftarrow \text{simplify.}$$

$$= \boxed{\cos\theta}$$