For #9 and #10, use the proportion $\frac{S}{2\pi r} = \frac{\theta}{360^\circ}$

9. The radius of a circle is 4 inches, and the measure of Arc_{AB} is $\frac{20\pi}{3}$. Determine the measure of the central angle that subtends Arc_{AB}.

r

S

find θ

$$\frac{\frac{20\pi}{3}}{2\cdot\pi\cdot 4} = \frac{\theta}{360^\circ}$$

$$* \quad \frac{\frac{20\pi}{3}}{\frac{8\pi}{1}} = \frac{\theta}{360^\circ}$$

$$* \quad \frac{5}{6} = \frac{\theta}{360^\circ}$$

multiply each side by 360°:

$$\frac{360^\circ}{1}\cdot\frac{5}{6} = \frac{\theta}{360^\circ}\cdot\frac{360^\circ}{1}$$

$$60^\circ\cdot 5 = \theta$$

$$\boxed{\theta = 300^\circ}$$

* Work off to the side:

$$\frac{\frac{20\pi}{3}}{\frac{8\pi}{1}}$$

$$= \frac{\cancel{20\pi}^{5}}{3}\cdot\frac{1}{\cancel{8\pi}_{2}}$$

$$= \frac{5}{6}$$

10. The radius of a circle is 6 inches, and the measure of a central angle, $m\angle AOB$ is 80°. Determine the length of the arc it subtends.

r

θ

Let's use x instead of S

$$\frac{x}{2\cdot\pi\cdot 6} = \frac{80^\circ}{360^\circ}$$ ← divide out 40°

$$\frac{x}{12\pi} = \frac{2}{9}$$ ← multiply each side by 12π

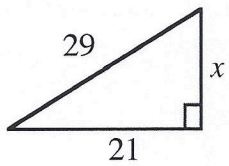
$$\frac{12\pi}{1}\cdot\frac{x}{12\pi} = \frac{2}{9}\cdot\frac{12\pi}{1}$$

$$x = \frac{24\pi}{9}$$

$$\boxed{x = \frac{8\pi}{3}}$$

Find the value of x in the given triangle and simplify completely.

11.



Use the Pythagorean theorem $a^2 + b^2 = c^2$

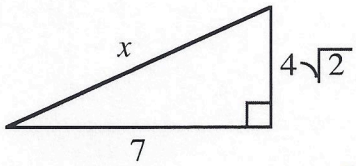
$$x^2 + 21^2 = 29^2$$

$$x^2 + 441 = 841$$

$$\sqrt{x^2} = \sqrt{400}$$

$$\boxed{x = 20}$$

12.



$$7^2 + (4\sqrt{2})^2 = x^2$$

$$49 + 16\cdot 2 = x^2$$

$$49 + 32 = x^2$$

$$81 = x^2$$

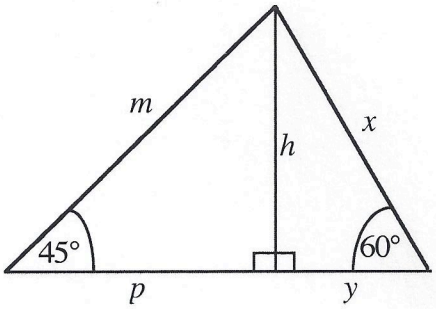
$$\boxed{x = 9}$$

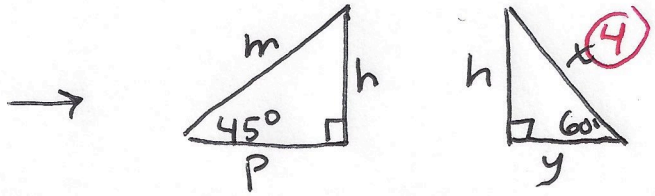
For each of these, I separate the triangles and re-orient them. This is recommended, but not required.

Find the values of the other side measures of these triangles. Simplify completely.

It is important, though, to label the new triangles correctly.

13. a) $x = 4$





I find the values in this order:

30-60-90: x, y, h

45-45-90: h, p, m

$x = 4$

$y = 2$

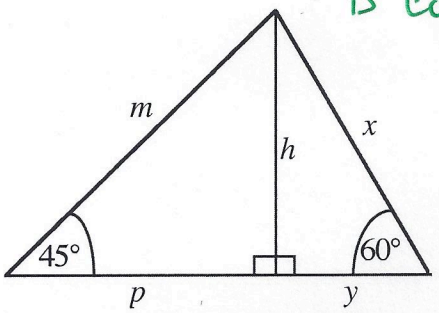
$h = 2\sqrt{3}$

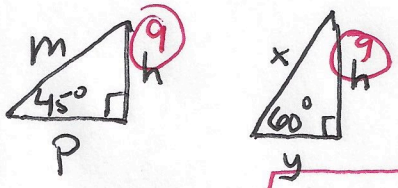
$p = 2\sqrt{3}$

$m = 2\sqrt{3}\cdot\sqrt{2} = 2\sqrt{6}$

In this example, I am orienting the 30-60-90 Δ a little differently, but the labeling is consistent.

13. b) $h = 9$





I use this order

45-45-90: h, p, m

30-60-90: h, y, x

$h = 9$

$p = 9$

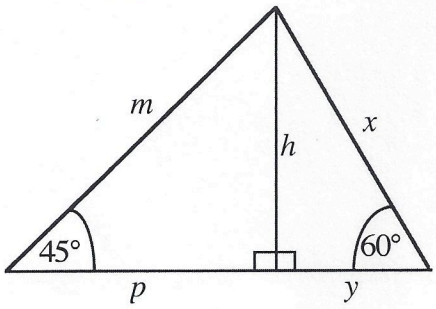
$m = 9\sqrt{2}$

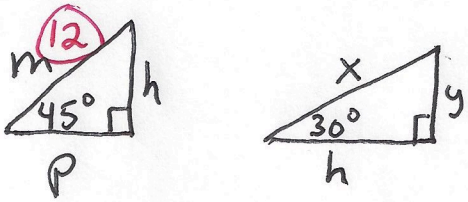
$y = \frac{9}{\sqrt{3}} = \frac{9\sqrt{3}}{3} = 3\sqrt{3}$

$x = 2\cdot 3\sqrt{3} = 6\sqrt{3}$

Again, I re-orient the 30-60-90 Δ, just because I can.

13. c) $m = 12$





I use this order:

45-45-90: m, p, h

30-60-90: h, y, x

$m = 12$

$p = \frac{12}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$

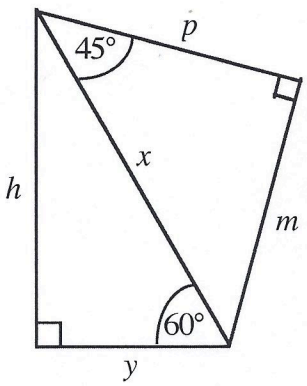
$h = 6\sqrt{2}$

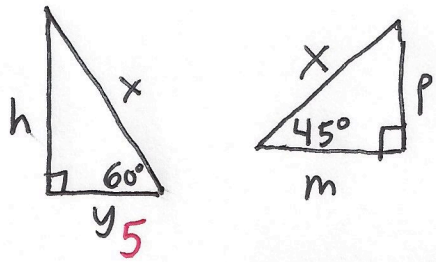
$y = \frac{6\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{6}}{3} = 2\sqrt{6}$

$x = 2\cdot 2\sqrt{6} = 4\sqrt{6}$

For each of these, I separate the triangles.
As before, this is recommended but not required.

14. a) $y = 5$





order:

30-60-90: y, h, x

45-45-90: x, p, m

$y = 5$

$h = 5\sqrt{3}$

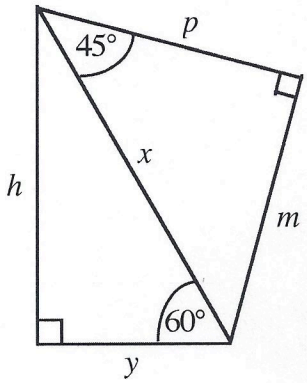
$x = 10$

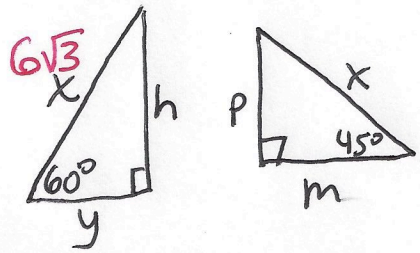
$p = \frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

$m = 5\sqrt{2}$

14. b) $x = 6\sqrt{3}$

the triangles are re-oriented and carefully labeled.





order:

30-60-90: x, y, h

45-45-90: x, p, m

$x = 6\sqrt{3}$

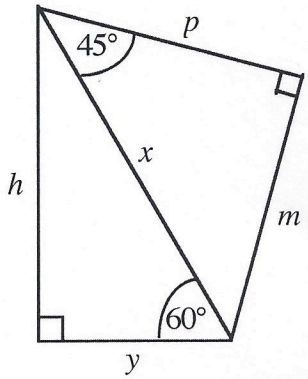
$y = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$

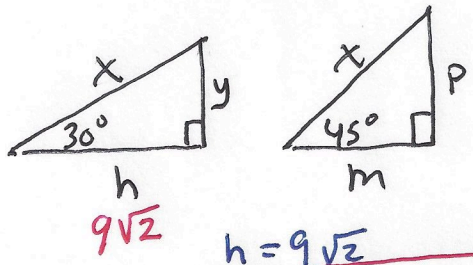
$h = 3\sqrt{3} \cdot \sqrt{3} = 3 \cdot 3 = 9$

$p = \frac{6\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{6}}{2} = 3\sqrt{6}$

$m = 3\sqrt{6}$

14. c) $h = 9\sqrt{2}$





order:

30-60-90: h, y, x

45-45-90: x, p, m

$h = 9\sqrt{2}$

$y = \frac{9\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{6}}{3} = 3\sqrt{6}$

$x = 2 \cdot 3\sqrt{6} = 6\sqrt{6}$

$p = \frac{6\sqrt{6}}{\sqrt{2}} = 6 \cdot \frac{\sqrt{6}}{2} = 6\sqrt{3}$

$m = 6\sqrt{3}$