Given two vectors, find the angle, α, between them. Round α to the nearest whole number. Use the outline provided.

To be most accurate — by reducing round-off errors — we will keep the magnitudes in square root form and multiply them using the calculator.

13. $\mathbf{V} = \langle -4, 15 \rangle$ and $\mathbf{W} = \langle 9, -5 \rangle$

i) $|\mathbf{V}| = \sqrt{(-4)^2 + 15^2} = \sqrt{241}$

$|\mathbf{W}| = \sqrt{81 + 25} = \sqrt{106}$

ii) $\mathbf{V} \bullet \mathbf{W} = (-4)(9) + (15)(-5)$
$= -36 + (-75) = -111$

iii) $\cos\alpha = \dfrac{V \bullet W}{|V| \cdot |W|}$

$\cos\alpha = \dfrac{-111}{\sqrt{241} \cdot \sqrt{106}} = \dfrac{-111}{159.83}$

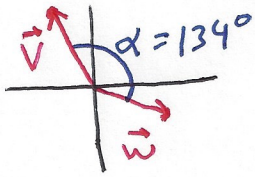
$\cos\alpha = -0.6945$

$\alpha = \cos^{-1}(-0.6945)$

$\alpha = 133.99°$

$\boxed{\alpha \approx 134°}$

This means the angle between vectors V and W is 134°, as shown here:



14. $\mathbf{S} = \langle 2, 9 \rangle$ and $\mathbf{T} = \langle 3, 1 \rangle$

i) $|\mathbf{S}| = \sqrt{4 + 81} = \sqrt{85}$

$|\mathbf{T}| = \sqrt{9 + 1} = \sqrt{10}$

ii) $\mathbf{S} \bullet \mathbf{T} = (2)(3) + (9)(1)$
$= 6 + 9 = 15$

iii) $\cos\alpha = \dfrac{S \bullet T}{|S| \cdot |T|}$

$\cos\alpha = \dfrac{15}{\sqrt{85} \cdot \sqrt{10}} = \dfrac{15}{29.1548}$

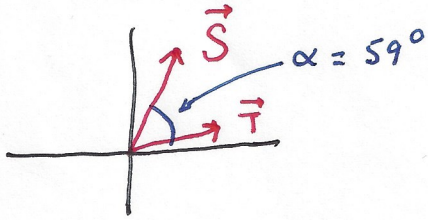
$\cos\alpha = 0.5145$

$\alpha = \cos^{-1}(0.5145)$

$\alpha = 59.036°$

$\boxed{\alpha \approx 59°}$

This graph is not required, but it is helpful to see:



15. $\mathbf{X} = \langle -8, 6 \rangle$ and $\mathbf{Y} = \langle -9, -12 \rangle$

i) $|\mathbf{X}| = \sqrt{64+36} = \sqrt{100} = 10$

$|\mathbf{Y}| = \sqrt{81+144} = \sqrt{225}$

ii) $\mathbf{X} \bullet \mathbf{Y} = (-8)(-9) + (6)(-12)$
$= 72 - 72 = 0$

iii) $\cos\alpha = \dfrac{X \bullet Y}{|X| \cdot |Y|}$

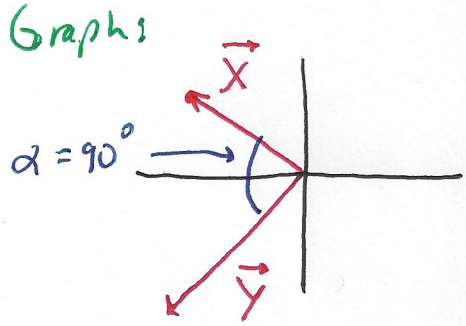
$\cos\alpha = \dfrac{0}{10 \cdot \sqrt{225}} = 0$

$\cos\alpha = 0$

$\alpha = \cos^{-1}(0)$

$\boxed{\alpha = 90^\circ}$

Graphs



16. $\mathbf{P} = \langle 6, -10 \rangle$ and $\mathbf{Q} = \langle -9, 15 \rangle$

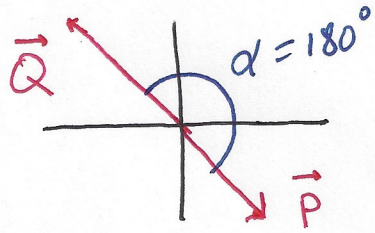
i) $|\mathbf{P}| = \sqrt{36+100} = \sqrt{136}$

$|\mathbf{Q}| = \sqrt{81+225} = \sqrt{306}$

ii) $\mathbf{P} \bullet \mathbf{Q} = (6)(-9) + (-10)(15)$
$= -54 - 150 = -204$

iii) $\cos\alpha = \dfrac{P \bullet Q}{|P| \cdot |Q|}$

$\cos\alpha = \dfrac{-204}{\sqrt{136} \cdot \sqrt{306}} = \dfrac{-204}{204}$

$\cos\alpha = -1$

$\boxed{\alpha = 180^\circ}$