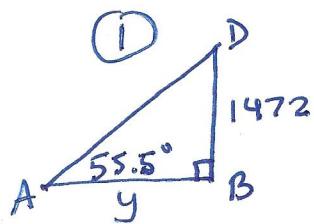


6. The height of the Empire State Building in New York City, from the ground to the top of its antenna, is 1,472 feet. From a point several blocks away, the angle of elevation to the top of the antenna is 55.5° . From the same point, the angle of elevation to the roof of the building is 51.0° . How high is the roof top of the Empire State Building? (Round to the nearest foot.)

Strategy: First find y as a part of the larger right triangle, ABD . Second, use the value of y within the smaller right triangle, ABC , to find x .



$$\tan 55.5^\circ = \frac{1472}{y}$$

$$y \cdot \tan 55.5^\circ = 1472$$

$$y = \frac{1472}{\tan 55.5^\circ}$$

$$y = 1,011.67 \quad \text{don't round yet}$$



$$\tan 51.0^\circ = \frac{x}{1,011.67}$$

$$1,011.67 \cdot \tan 51.0^\circ = x$$

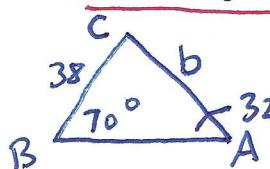
$$x = 1,249.3$$

$$x \approx 1,249 \text{ ft}$$

The roof of the Empire State Building is 1,249 ft. high.

7. Each refers to triangle ABC , which is not necessarily a right triangle. For each, determine whether to use the Law of Sines or the Law of Cosines to answer the question. The ambiguous case is not included in this set of exercises.

- a) If $A = 32^\circ$, $B = 70^\circ$, and $a = 38$ centimeters, find the length of b .



This is A-A-S.
use the Law of Sines to find b.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{b}{\sin 70^\circ} = \frac{38}{\sin 32^\circ}$$

$$b = \frac{38 \cdot \sin 70^\circ}{\sin 32^\circ}$$

$$b = 67.4 \text{ cm}$$

- b) If $A = 15^\circ$, $b = 8$ inches, and $c = 12$ inches, find the length of a .

$c = 12$ is the longest side, so $\angle C$ is the largest angle. $\angle C$ might not be obtuse, but it likely is. S-A-S use Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$a^2 = 8^2 + 12^2 - 2(8)(12) \cdot \cos(15^\circ)$$

$$a^2 = 64 + 144 - 192 \cdot (-0.9659)$$

$$a^2 = 64 + 144 - 185.46$$

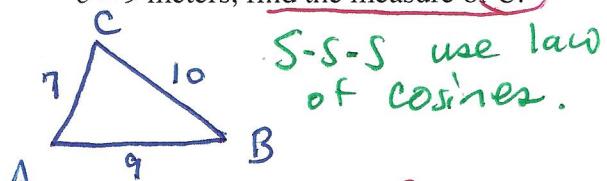
$$a^2 = 208 - 185.46$$

$$a^2 = 22.54$$

$$a = \sqrt{22.54}$$

$$a = 4.7 \text{ in.}$$

- c) If $a = 10$ meters, $b = 7$ meters, and $c = 9$ meters, find the measure of C .



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

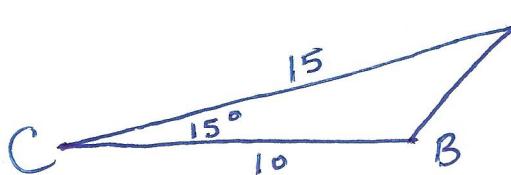
$$\cos C = \frac{10^2 + 7^2 - 9^2}{2(10)(7)}$$

$$\cos C = \frac{68}{140} = .4857$$

$$C = \cos^{-1}(0.4857)$$

$$\boxed{C = 60.9^\circ}$$

- e) If $C = 15^\circ$, $b = 15$ feet, and $a = 10$ feet, find the measure of B .



S-A-S use Law of cosines to find side c . we can then use either Law of sines or Law of cosines to find $\angle B$. Because B is the largest angle, possibly obtuse, it is best to use Law of cosines (again) to find B .

① Law of cosines to find side c :

$$c^2 = a^2 + b^2 - 2bc \cdot \cos C$$

$$c^2 = 10^2 + 15^2 - 2(10)(15) \cdot \cos(15^\circ)$$

$$c^2 = 100 + 225 - 300 \cdot (0.9659)$$

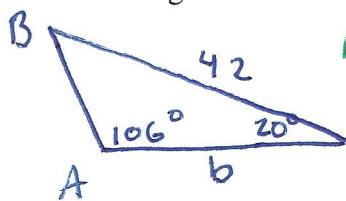
$$c^2 = 100 + 225 - 289.78$$

$$c^2 = 35.22 \leftarrow \text{we can use this value in step ②}$$

$$c = \sqrt{35.22}$$

$c = 5.9346$ {Because c is a rounded value, it is appropriate to use four decimal places for step ②}

- d) If $A = 106^\circ$, $C = 20^\circ$, and $a = 42$ inches, find the length of b .



A-A-S Law of Sines.

C First find $m\angle B$ and use it to find side b .

$$B = 180^\circ - (106^\circ + 20^\circ) = 54^\circ$$

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{b}{\sin 54^\circ} = \frac{42}{\sin 106^\circ}$$

$$b = \frac{42 \cdot \sin 54^\circ}{\sin 106^\circ}$$

$$\boxed{b = 35.3 \text{ in.}}$$

② Law of cosines to find $\angle B$:

② Law of cosines to find $\angle B$:

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{10^2 + 35.22^2 - 15^2}{2(10)(5.9346)}$$

$$\cos B = \frac{-89.78}{118.692}$$

$$\cos B = -0.7564$$

$$B = \cos^{-1}(-0.7564)$$

$$\boxed{B = 139.1^\circ}$$