CHART OF UNIT CIRCLE SINE AND COSINE VALUES

Recall (Section 3.5) these charts (below) of sine and cosine values from around the unit circle. Notice how they rise and fall—increase and decrease—within the unit circle quadrants.

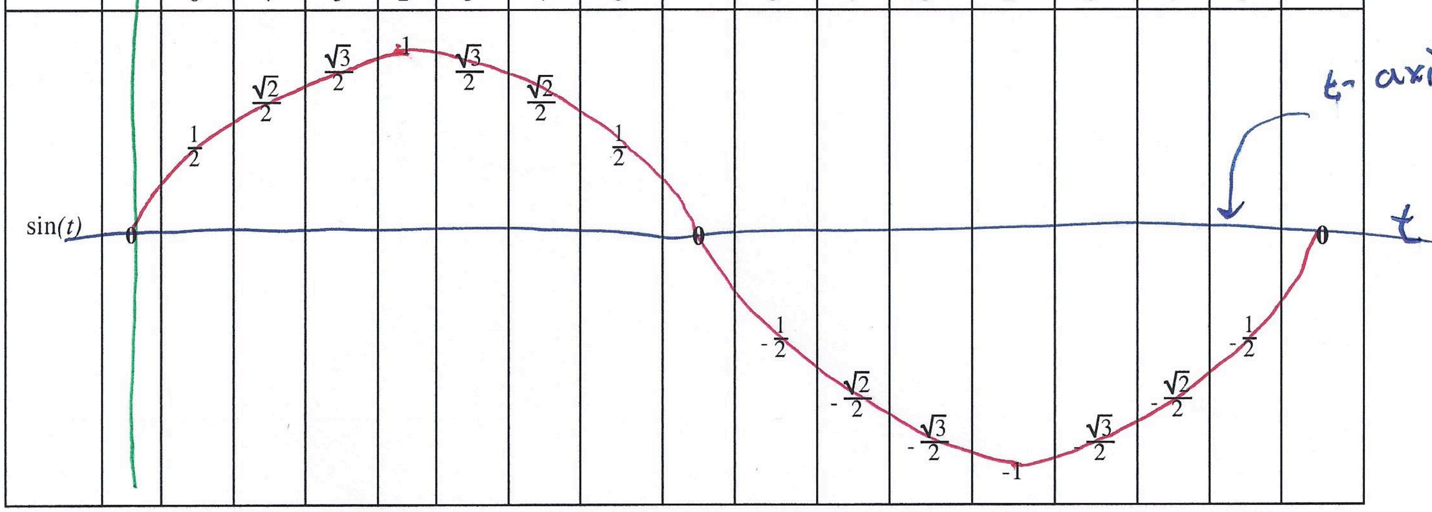
Note: At the top of the chart we see the unit circle quadrants—and their corresponding radian values—spread out in linearly, not around a circle. These are here only as a reference.

In-Class Example 1: In each chart, for sin(t) and for cos(t), lightly draw

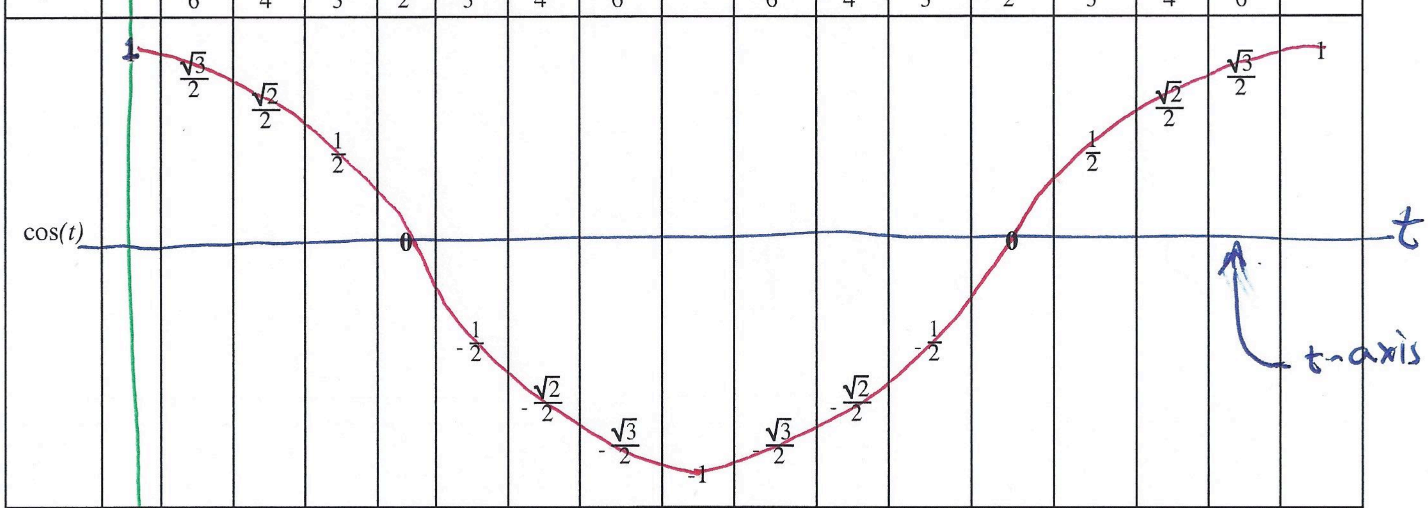
(a) a vertical axis through the value at 0^r and in green
(b) a horizontal axis through the 0's (zeros); in blue
(c) Lightly trace each function from 0^r to $2\pi^r$. in red

These quadrants represent the unit circle quadrants stretched along the t-axis.

Unit Circle →	0°	Quadrant I			90°	Quadrant II			180°	Quadrant III			270°	Quadrant IV			360°
t^r	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

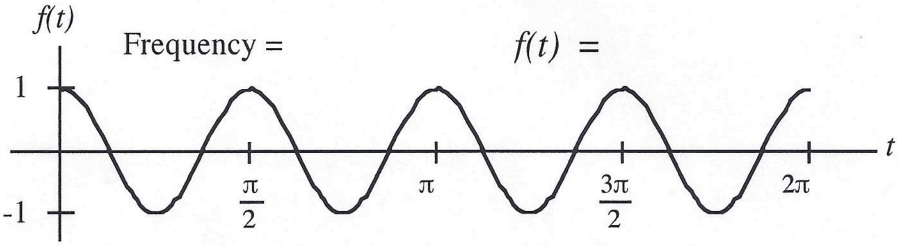


Unit Circle →	0°	Quadrant I			90°	Quadrant II			180°	Quadrant III			270°	Quadrant IV			360°
t^r	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

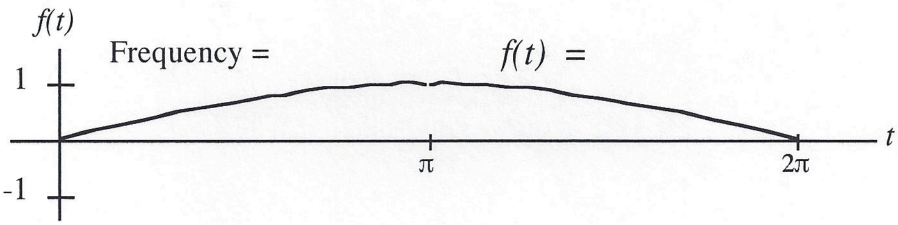


You Try It 2 Given the graph, identify the frequency of the wave and write the function. Also identify the period of the function.

a)



b)



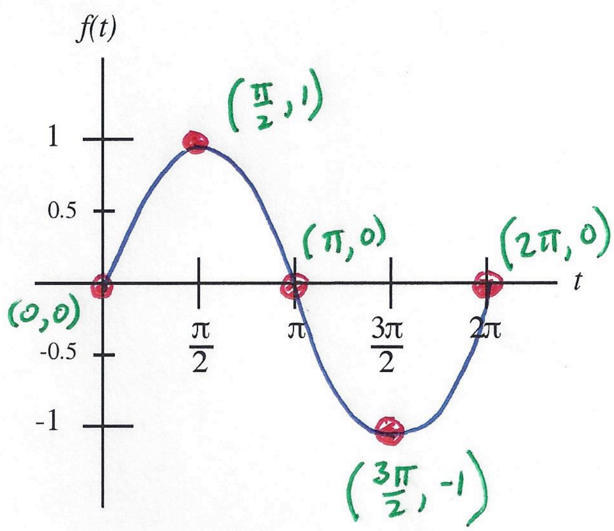
THE GRAPH OF THE PARENT SINE FUNCTION

Recall (Section 4.1) when graphing a parabola, we don't need to find every point. Once we identify the vertex and two or three points to the right of the vertex, we have enough information to plot symmetric partners and draw the graph.

This notion of finding only a few necessary points is extended to the graphs of sine and cosine. The most important points (the featured points) are the maxima, minima, and zeros. For the parent functions, these occur at the axial radian measures, as shown below.

In-Class Example 2: Graph the function $f(t) = \sin(t)$ for $0 \le t \le 2\pi$; use only axial values of t.

t	$f(t) = \sin(t)$	$(t, f(t))$	Feature
0	$f(0) = \sin(0) = 0$	$(0, 0)$	zero
$\frac{\pi}{2}$	$f\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$	$\left(\frac{\pi}{2}, 1\right)$	max
π	$f(\pi) = \sin(\pi) = 0$	$(\pi, 0)$	zero
$\frac{3\pi}{2}$	$f\left(\frac{3\pi}{2}\right) = \sin\left(\frac{3\pi}{2}\right) = -1$	$\left(\frac{3\pi}{2}, -1\right)$	min
2π	$f(2\pi) = \sin(2\pi) = 0$	$(2\pi, 0)$	zero



TRACING THE SINE WAVE

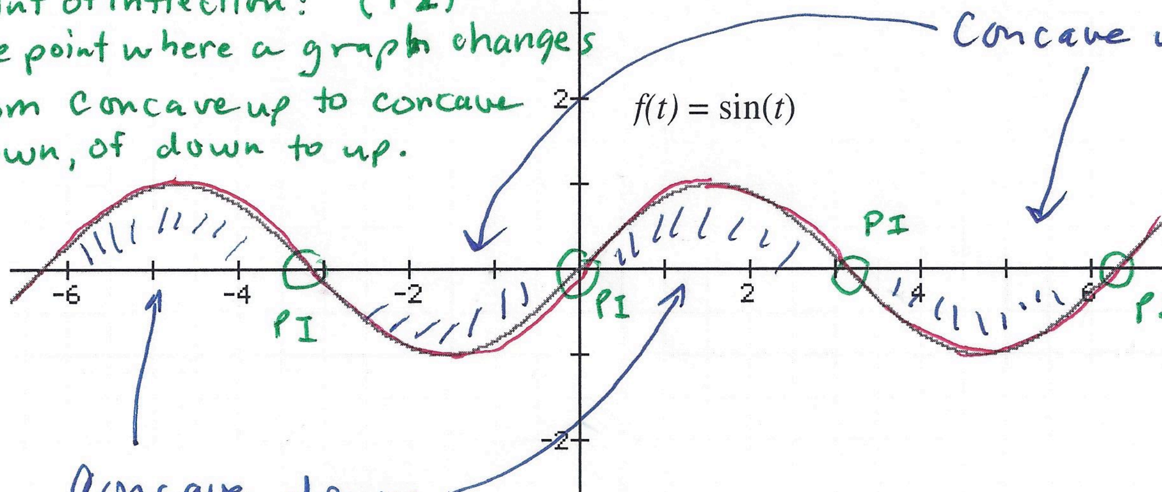
When drawing the sine wave, it is important to include the concavity and points of inflection.

Curvature; these are not straight lines or line segments

In-Class Example 3: Trace the wave. Notice concavity and points of inflection. Also, mark radian values (in π) along each x-axis wherever the curve reaches a maximum, a minimum, or a zero.

Point of inflection: (PI) The point where a graph changes from concave up to concave down, or down to up.

Concave up

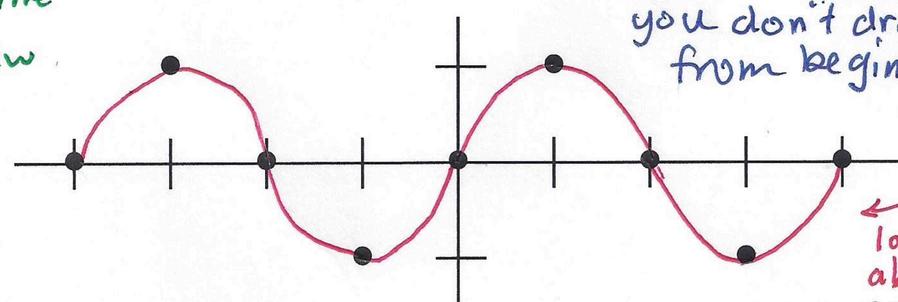


Concave down

In-Class Example 4: Draw the sine wave using the given max, min, and zero points. Keep in mind concavity and points of inflection.

It is recommended that you don't draw continuously from beginning to end...

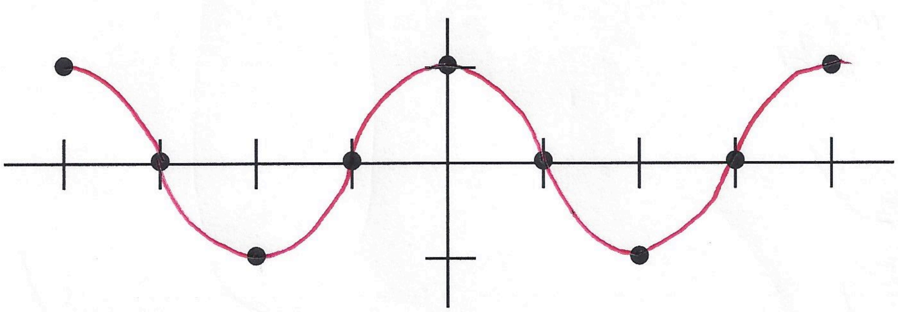
Do not draw line segments; draw smooth curves between each pair of points.



← this graph should look like the one above. If it doesn't, erase and start over.

In-Class Example 5: Draw the cosine wave using the given max, min, and zero points. Keep in mind concavity and points of inflection.

...instead, draw each piece of the curve from point to point.



THE GRAPH OF THE COSINE FUNCTION

In-Class Example 6: Graph the function $f(t) = \cos(t)$ for $0 \le t \le 2\pi$ Choose only axial values of t.

(a) use these ordered pairs to plot the points

(b) Carefully draw the cosine graph with good curvature.

t	$f(t) = \cos(t)$	$(t, f(t))$	Feature
0	$f(0) = \cos(0) = 1$	$(0, 1)$	max
$\frac{\pi}{2}$	$f\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$	$\left(\frac{\pi}{2}, 0\right)$	zero
π	$f(\pi) = \cos(\pi) = -1$	$(\pi, -1)$	min
$\frac{3\pi}{2}$	$f\left(\frac{3\pi}{2}\right) = \cos\left(\frac{3\pi}{2}\right) = 0$	$\left(\frac{3\pi}{2}, 0\right)$	zero
2π	$f(2\pi) = \cos(2\pi) = 1$	$(2\pi, 1)$	max