**6.** Graph **one full period** of each function. Mark each important value along the $x$- and $y$-axes.

a) $f(x) = -1 + 2\cos\left(\frac{3}{4}x\right)$ $\quad g(x) = 2\cos\left(\frac{3}{4}x\right)$

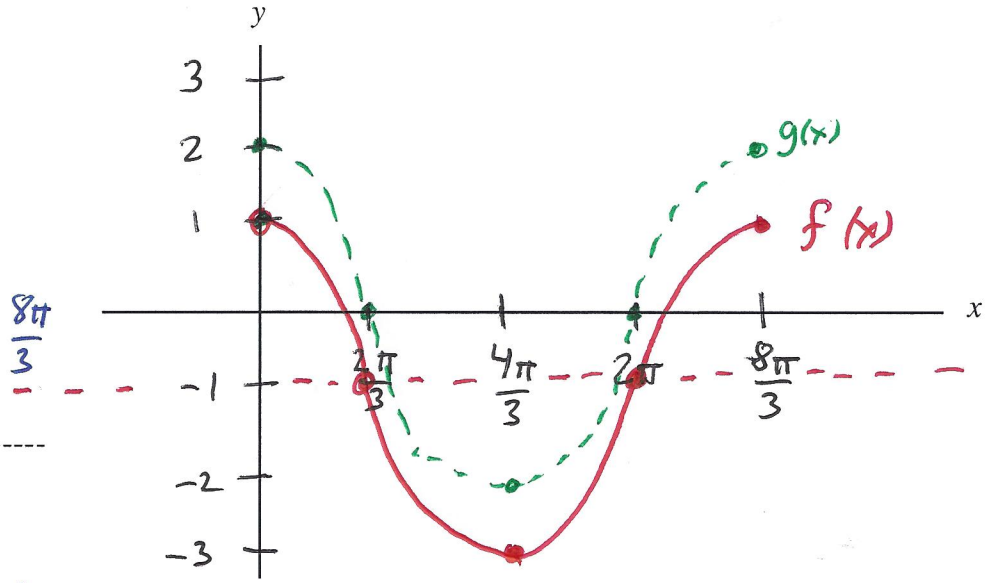
*i)* Amplitude = 2

*ii)* Reflected? No

*iii)* Frequency = $\frac{3}{4}$

*iv)* Period =

$\frac{2\pi}{\frac{3}{4}} = \frac{2\pi}{1} \cdot \frac{4}{3} = \frac{8\pi}{3}$

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*v)* Vertical shift:

down 1; $y = -1$

**b)** $f(x) = 3\cos\left(\frac{6}{5}x + \frac{\pi}{4}\right)$ $\quad g(x) = 3\cos\left(\frac{6}{5}x\right)$

*i)* First graph ~~$g(x) = \cos(3x)$~~ (oops) as a guide.

Ampl: = 3 $\quad$ Reflected? No

Freq = $\frac{6}{5}$ $\quad$ Period = $\frac{2\pi}{1} \cdot \frac{5}{6} = \frac{5\pi}{3}$

*ii)* Horizontal shift:

Argument = 0

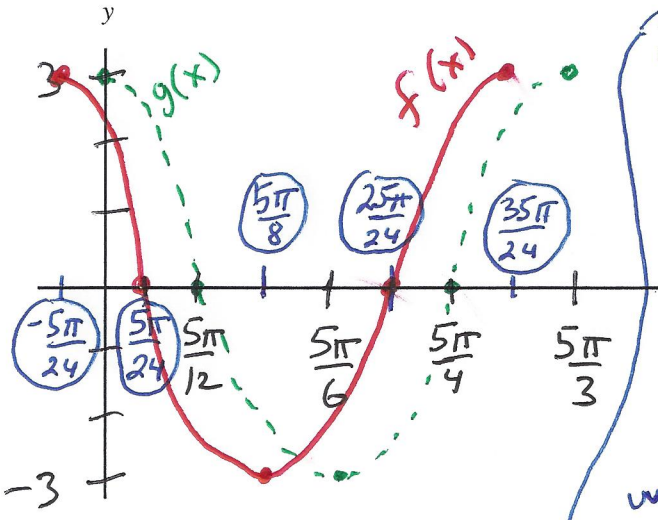
$\frac{6}{5}x + \frac{\pi}{4} = 0$

$\frac{5}{6} \cdot \frac{6}{5}x = -\frac{\pi}{4} \cdot \frac{5}{6}$

$x = \frac{-5\pi}{24}$

*iii)* Use this chart to identify the shifted $x$-values and draw the graph.

| Feature | Original $x$ | Shifted $x$ |
|---|---|---|
| max | $0 - \frac{5\pi}{24}$ | $\frac{-5\pi}{24}$ |
| zero | $\frac{10\pi}{24} - \frac{5\pi}{24}$ | $\frac{5\pi}{24}$ * |
| min | $\frac{20\pi}{24} - \frac{5\pi}{24}$ | $\frac{15\pi}{24} = \frac{5\pi}{8}$ |
| zero | $\frac{30\pi}{24} - \frac{5\pi}{24}$ | $\frac{25\pi}{24}$ |
| max | $\frac{40\pi}{24} - \frac{5\pi}{24}$ | $\frac{35\pi}{24}$ |




To use the chart at left, we need to subtract $\frac{5\pi}{24}$ from each featured $x$-value. Let's write the original $x$-values with a common denominator of 24.

* Note: $\frac{5\pi}{24}$ is half-way between the $y$-axis and $\frac{5\pi}{12}$.

7. Graph **two full periods** of each function. Mark each important value along the $x$- and $y$-axes.

a) $(x) = -2\cos\left(\frac{4}{5}x\right)$

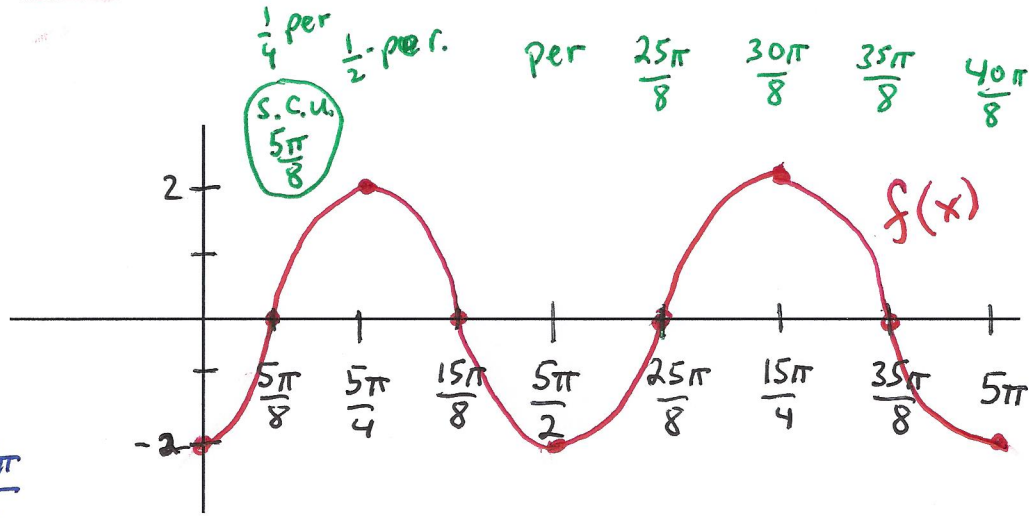
a) Ampl: 2

b) Refl: yes

c) Freq: $\frac{4}{5}$

d) Per:

$\frac{2\pi}{\frac{4}{5}} = \frac{2\pi}{1} \cdot \frac{5}{4} = \frac{5\pi}{2}$




b) $(x) = -2\sin\left(\frac{2}{3}x\right)$

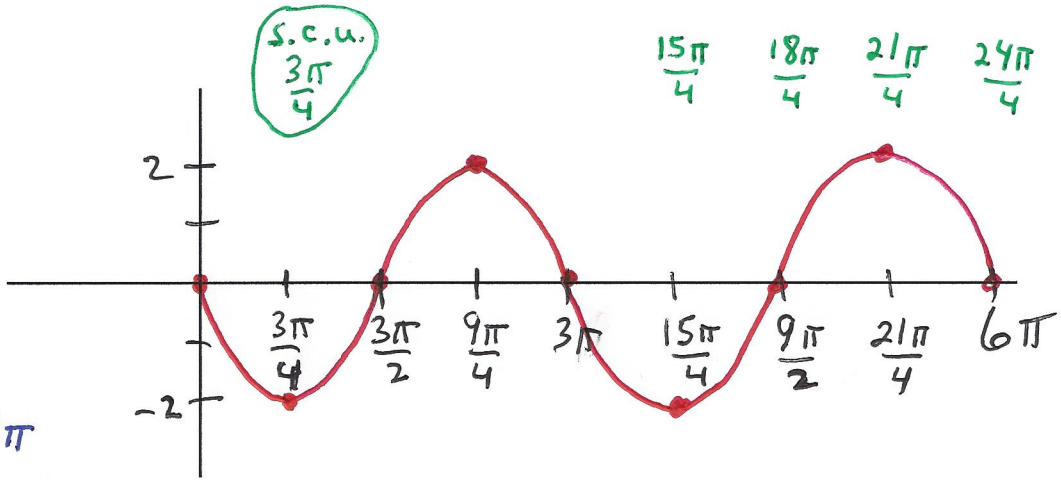
a) Ampl: 2

b) Refl: yes

c) Freq: $\frac{2}{3}$

d) Per:

$\frac{2\pi}{\frac{2}{3}} = \frac{2\pi}{1} \cdot \frac{3}{2} = 3\pi$




c) $f(x) = 3\csc\left(\frac{6}{5}x\right)$

For the guideline graph:

$g(x) = 3\sin\left(\frac{6}{5}x\right)$

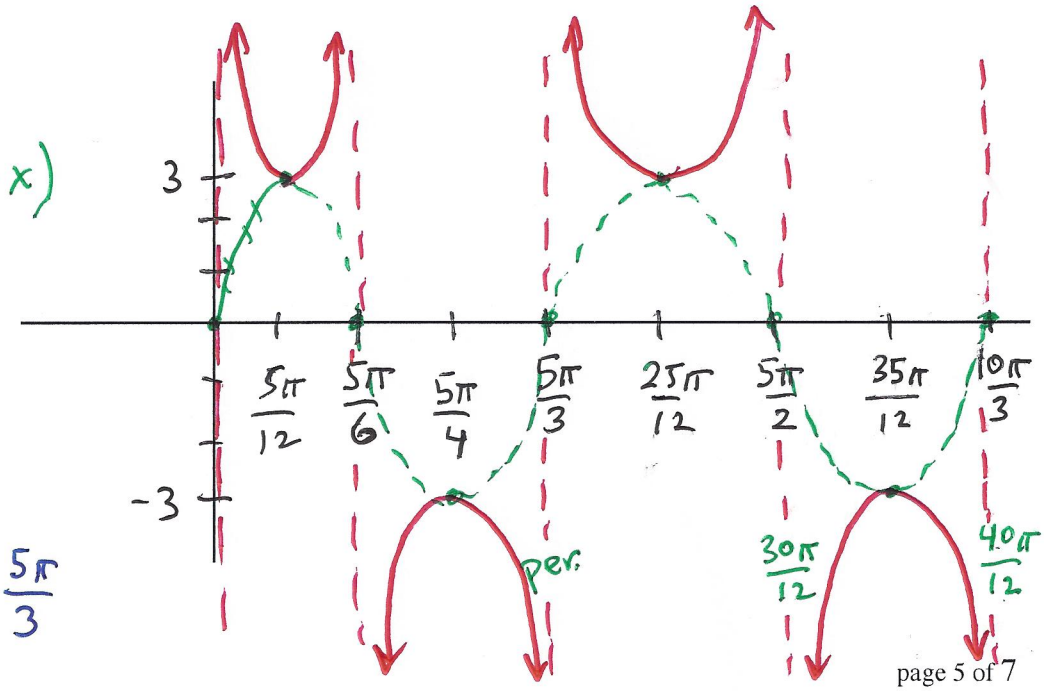
a) Ampl: 3

b) Refl: No

c) Freq: $\frac{6}{5}$

d) Per:

$= \frac{2\pi}{\frac{6}{5}} = \frac{2\pi}{1} \cdot \frac{5}{6} = \frac{5\pi}{3}$



**8.** Graph **two full branches** of each function.

Because $\tan\theta = \frac{\sin\theta}{\cos\theta}$, we use $g(x) = \cos$ as a guide line graph to help us find the asymptotes of tangent: whenever cosine $= 0$, tangent is undefined.

**a)** $(x) = -\tan(3x)$

For the guideline graph:

$g(x) = -\cos(3x)$ only one cycle.

a) Reflected? yes

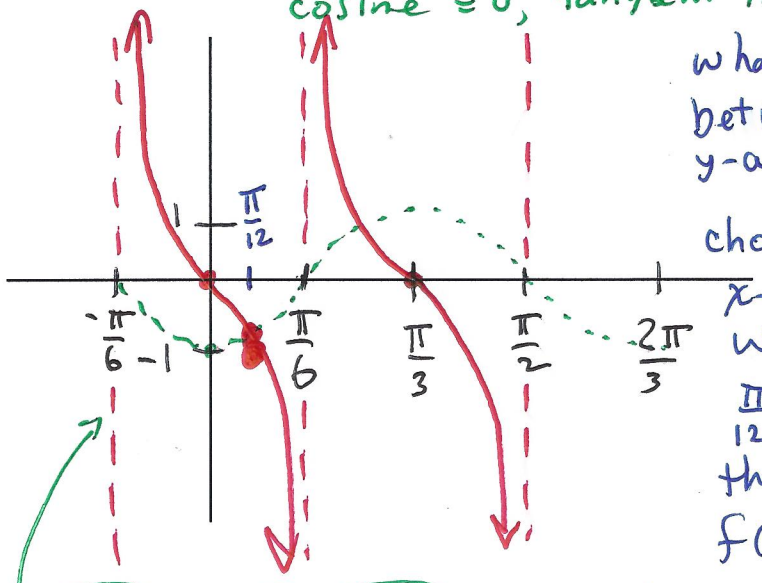
b) Freq: 3

c) Per: $\frac{2\pi}{3}$

d) What is the period of $f(x)$?

$Per_{tan} = \frac{1}{2} \cdot \frac{2\pi}{3}$

$= \frac{\pi}{3}$




what is happening between the y-axis and $\frac{\pi}{6}$?

choose the x-value half-way between, $\frac{\pi}{12}$, and put this into the $f(x)$ function.

$f\left(\frac{\pi}{12}\right) = -\tan\left(3 \cdot \frac{\pi}{12}\right)$

$= -\tan\left(\frac{\pi}{4}\right)$

$= -1$

we prefer to start at an asymptote, so we go one s.c.u. to the left of the y-axis.

**b)** $f(x) = \cot\left(\frac{1}{2}x\right)$

the denominator is sine, so the is sine.

For the guideline graph:

$g(x) = \sin\left(\frac{1}{2}x\right)$

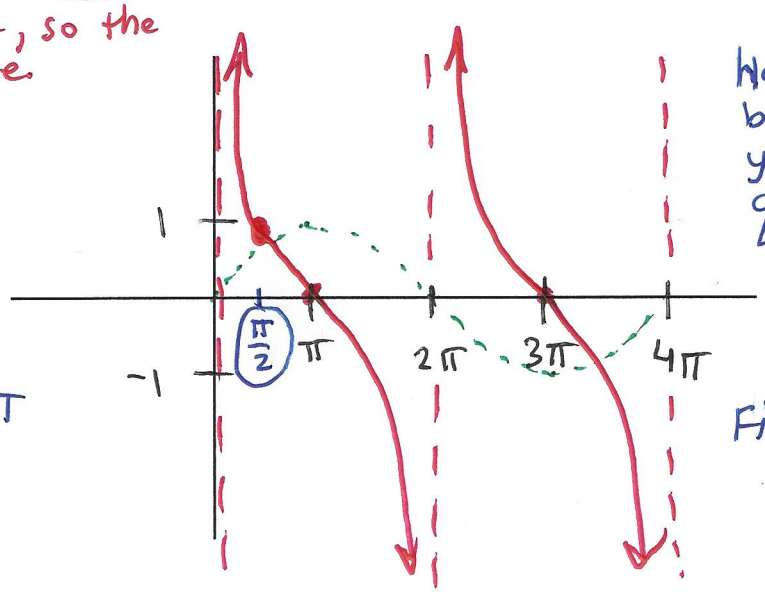
a) Reflected? No

b) Freq: $\frac{1}{2}$

c) Per: $\frac{2\pi}{\frac{1}{2}} = \frac{2\pi}{1} \cdot \frac{2}{1} = 4\pi$

d) What is the period of $f(x)$?

$Per_{cot} = \frac{1}{2} \cdot 4\pi = 2\pi$




Half-way between the y-axis and quarter per (s.c.u.) is $\frac{\pi}{2}$.

Find $f\left(\frac{\pi}{2}\right)$

$f\left(\frac{\pi}{2}\right) = \cot\left(\frac{1}{2} \cdot \frac{\pi}{2}\right)$

$= \cot\left(\frac{\pi}{4}\right)$

$= 1$