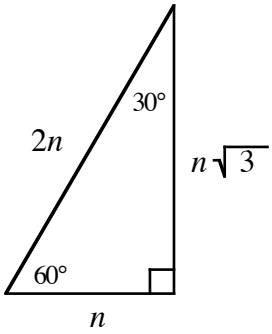
Section 2.3 Trigonometry of Special Right Triangles

THE 30°-60°-90° TRIANGLE

The most important thing to remember about the 30°-60°-90° triangle is that the shortest side is half of the hypotenuse. Also, the shortest side is always opposite the 30° angle. Because every 30-60-90 triangle is similar* to all other 30-60-90 triangles, it doesn't matter how big or how small the triangle is, the ratios—and the trig functions—are always the same.

It is common to represent 30-60-90 triangles with just the angles measures and not capitol letters at each vertex. At right is one way a 30-60-90 triangle can be labeled. Notice the shortest side is labeled n and the other two sides measures show their relationship to n.



Based on this diagram, we can find the sine, cosine and tangent values of both the 30° and 60° angles. And, once we have those values, we can also find their reciprocal values.

When working with trig functions, it is most common to rationalize the denominator. With that in mind, here are all six trig functions for these very important angles.

The main functions at 30°

$$\sin(30°) = \frac{\text{opp}_{30}}{\text{hyp}} = \frac{n}{2n} = \frac{1}{2}$$

$$\cos(30°) = \frac{\text{adj}_{30}}{\text{hyp}} = \frac{n\sqrt{3}}{2n} = \frac{\sqrt{3}}{2}$$

$$\tan(30°) = \frac{\text{opp}_{30}}{\text{adj}_{30}} = \frac{n}{n\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

The reciprocal functions at 30°

$$\csc(30°) = \frac{2}{1} = 2$$

$$\sec(30°) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cot(30°) = \frac{\sqrt{3}}{1} = \sqrt{3}$$

The main functions at 60°

$$\sin(60°) = \frac{\text{opp}_{60}}{\text{hyp}} = \frac{n\sqrt{3}}{2n} = \frac{\sqrt{3}}{2}$$

$$\cos(60°) = \frac{\text{adj}_{60}}{\text{hyp}} = \frac{n}{2n} = \frac{1}{2}$$

$$\tan(60°) = \frac{\text{opp}_{60}}{\text{adj}_{60}} = \frac{n\sqrt{3}}{n} = \sqrt{3}$$

The reciprocal functions at 60°

$$\csc(60°) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

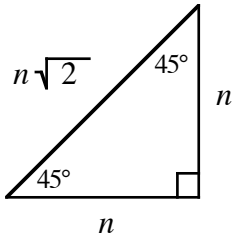
$$\sec(60°) = \frac{2}{1} = 2$$

$$\cot(60°) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

*Similar triangles have the same shape (same angle measures) but not necessarily the same size. However, the ratio of consecutive sides within a triangle will not change, regardless of the size.

THE 45°-45°-90° TRIANGLE

The most important thing to remember about the 45-45-90 triangle is that the two legs are congruent.



$$\sin(45°) = \frac{\text{opp}_{45}}{\text{hyp}} = \frac{n}{n\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \csc(45°) = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\cos(45°) = \frac{\text{adj}_{45}}{\text{hyp}} = \frac{n}{n\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \sec(45°) = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\tan(45°) = \frac{\text{opp}_{45}}{\text{adj}_{45}} = \frac{n}{n} = 1 \qquad \cot(45°) = \frac{1}{1} = 1$$

Whether you memorize these values or not, you should know how to generate them—by drawing special right triangles and finding the side relationships. In other words, learning how to *develop* these trigonometric ratios will serve you much better that just memorizing them.

Example 1: Evaluate each. Simplify; rationalize the denominator, if necessary.

a) $8\cos 30°$ b) $\frac{2}{3}\cos 30°$

c) $\sqrt{3}\cos 30°$ d) $(\cos 30°)^2$

Procedure: We know that $\cos 30° = \frac{\sqrt{3}}{2}$. Use the order of operations, one step at a time, to evaluate.

Answer: a) $8\cos 30° = \frac{8}{1} \cdot \frac{\sqrt{3}}{2} = 4\sqrt{3}$

b) $\frac{2}{3}\cos 30° = \frac{2}{3} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{3}$

c) $\sqrt{3}\cos 30° = \frac{\sqrt{3}}{1} \cdot \frac{\sqrt{3}}{2} = \frac{3}{2}$

d) $(\cos 30°)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

Section 2.3 Focus Exercises

1. Build a 30-60-90 triangle with 3 as the measure of the shortest side, and use that value to find the lengths of the two other sides. Then use that triangle to aid you in find the six trig functions of 30° and of 60°. For each, rationalize the denominator, if necessary.

2. Build a 45-45-90 triangle with 3 as the measure of one of the legs, and use that value to find the lengths of the two other sides. Then use that triangle to aid you in find the six trig functions of 45°. For each, rationalize the denominator, if necessary.

3. Does it make sense that sin (45°) = cos (45°)? Explain your answer.

4. Does it make sense that tan (45°) = 1? Explain your answer.

Evaluate each. Simplify; rationalize the denominator, if necessary.

5. $2\sin 30°$

6. $4\cos 30°$

7. $-5\tan 45°$

8. $-3\sec 45°$

9. $\sqrt{3}\ \cot 60°$

10. $-\sqrt{2}\ \csc 30°$

11. $\sqrt{\cos 60°}$

12. $\sqrt{\csc 30°}$

13. $(\cos 45°)^2$

14. $(\sin 60°)^2$

15. $(\cot 60°)^2$

16. $(\sec 30°)^2$