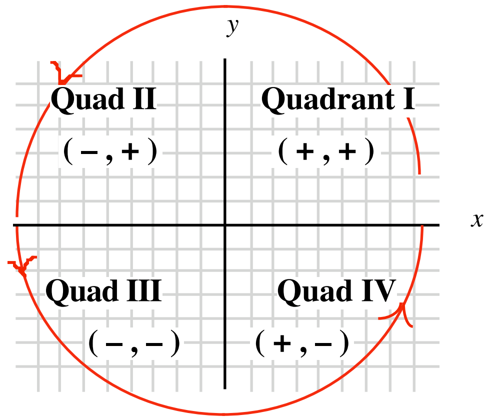
Section 2.4 Points in the *x*-*y*-plane

QUADRANTS

The x-axis and the y-axis divide the x-y-plane into four distinct regions called **quadrants**. We label the quadrants using Roman numerals I, II, III and IV.

Quadrant I:	upper right:	x^+ and y^+
Quadrant II:	upper left:	x^- and y^+
Quadrant III:	lower left:	x^- and y^-
Quadrant IV:	lower right:	x^+ and y^-

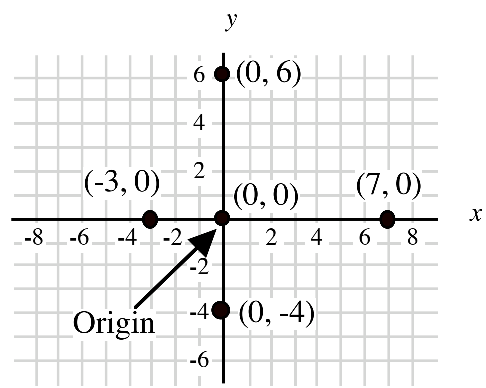


AXIAL POINTS

A point on an axis is called an **axial point**. One special feature of an axial point is <u>at least one of the coordinates must be 0</u>.

Axial points are not in any of the quadrants. For example, (7, 0) is not in Quadrant I, and it is not in Quadrant IV. It is *on the positive x-axis.*

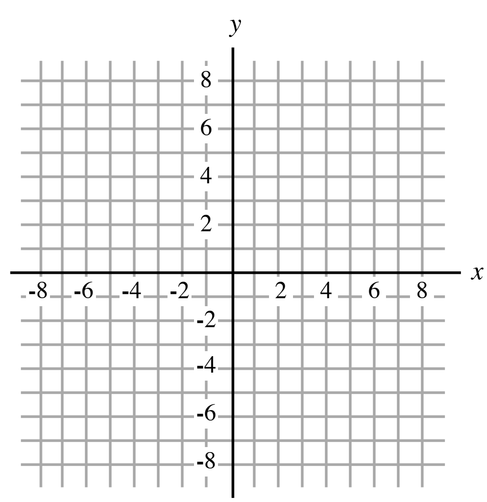
The origin (0, 0) is also an axial point.



GRAPH A LINE

In-Class Example 1: Use a table of values to graph the line given by $y = \frac{2}{3}x - 4$.

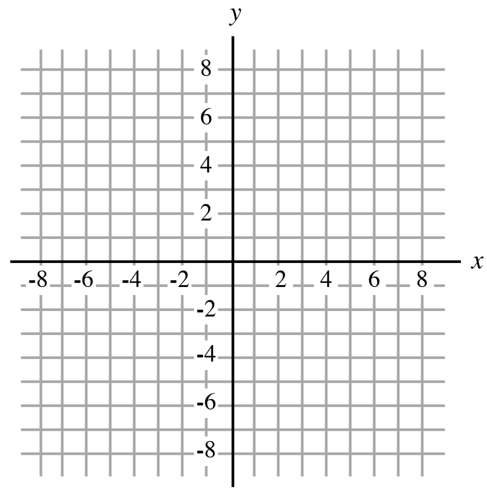
Choose an x-value	$y = \frac{2}{3}x - 4$	(x, y)



GRAPH A PARABOLA

In-Class Example 2: Use a table of values to graph the parabola given by $y = x^2 + 2x - 3$. Find the line of symmetry and the vertex of the parabola.

Choose an x-value	$y = x^2 + 2x - 3$	(x, y)

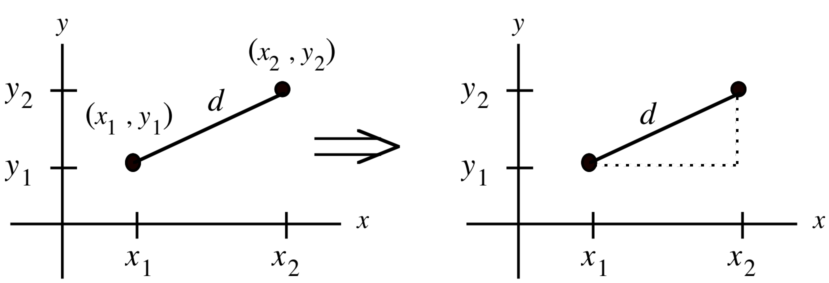


THE DISTANCE FORMULA

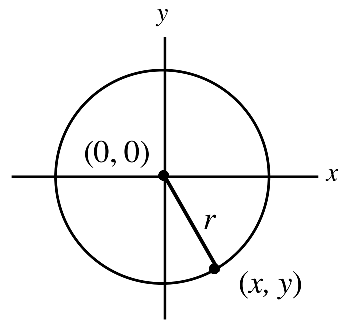
In the x-y-plane, the distance between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In-Class Example 3:
Use the Pythagorean Theorem to develop the distance formula using the diagrams below.



EQUATION OF A CIRCLE CENTERED AT THE ORIGIN



A circle with radius, r, centered at the origin, (0, 0), has equation

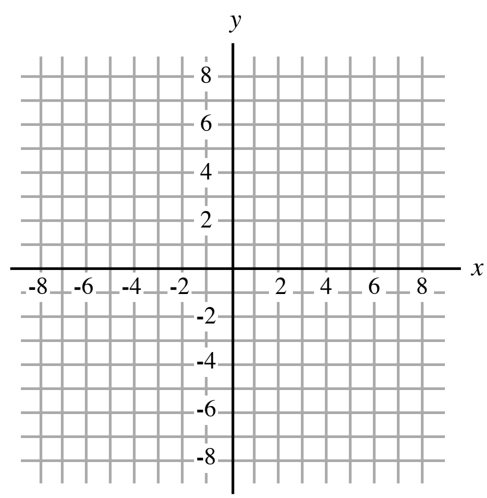
$$x^2 + y^2 = r^2.$$

This is found by choosing a general point anywhere on the circle and calling it (x, y). The distance between this point and the center must be the radius. We use the distance formula to find the equation of the circle:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Replace d with r; use the origin for (x_1, y_1). And (x, y) for (x_2, y_2).
$r = \sqrt{(x - 0)^2 + (y - 0)^2}$	Simplify inside each grouping.
$r = \sqrt{(x)^2 + (y)^2}$	Square each side.
$r^2 = \left(\sqrt{(x)^2 + (y)^2}\right)^2$	Simplify.
$r^2 = x^2 + y^2$	This is commonly written as $x^2 + y^2 = r^2$

GRAPH A CIRCLE

You Try It 1 Consider a circle centered at the origin that passes through (4, -3).



a) Find the radius of the circle *(Use the distance formula between the point (4, -3) and the origin.)*

b) Draw its graph. *(First plot axial points; next plot the given point; then plot points symmetric to the given point.)*

c) What is the equation of the circle?

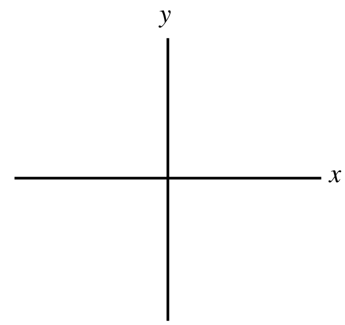
The Unit Circle

In mathematics, “unit” typically means “one” (1).

A **unit circle** is centered at the origin and has a radius of one, $r = 1$ (unit).

You Try It 2 What is the equation of the unit circle?

Use this *x*-*y*-plane to draw a unit circle.



VERIFYING POINTS ON THE UNIT CIRCLE

We can verify that a given point is on the unit circle by placing the *x*- and *y*-values into the unit circle equation, $x^2 + y^2 = 1$.

For example, verify that $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is on the unit circle:

$$x^2 + y^2 = 1$$

$$\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$$

$$\frac{1}{4} + \frac{3}{4} = 1$$

$$\frac{4}{4} = 1$$ True. So, $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is on the unit circle.

You Try It 3 Verify that each point is on the unit circle.

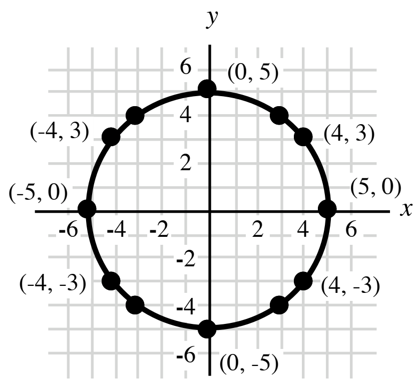
a) $\left(\frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\right)$

b) $\left(\frac{3\sqrt{5}}{7}, \frac{2}{7}\right)$

You Try It Answers

YTI 1 a) radius, $r = 5$

b)



c) Equation: $x^2 + y^2 = 25$

YTI 2 Unit Circle Equation: $x^2 + y^2 = 1$

YTI 3 a) $\left(\frac{1}{\sqrt{5}}\right)^2 + \left(\frac{-2}{\sqrt{5}}\right)^2$

$= \frac{1}{5} + \frac{4}{5} = \frac{5}{5} = 1$

b) $\left(\frac{3\sqrt{5}}{7}\right)^2 + \left(\frac{2}{7}\right)^2$

$= \frac{9 \cdot 5}{49} + \frac{4}{49}$

$= \frac{45}{49} + \frac{4}{49} = \frac{49}{49} = 1$

Section 2.4 Focus Exercises

Identify which quadrant(s) fits the given description.

1. The y-coordinate is negative.

2. The x-coordinate is positive.

3. The ratio $\frac{y}{x}$ is negative.

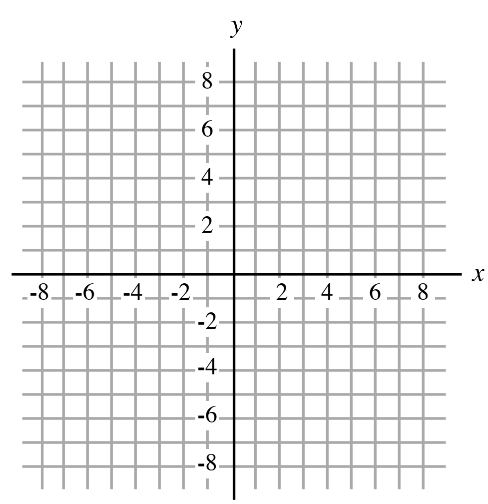
4. The ratio $\frac{y}{x}$ is positive.

5. The x-coordinate is negative.

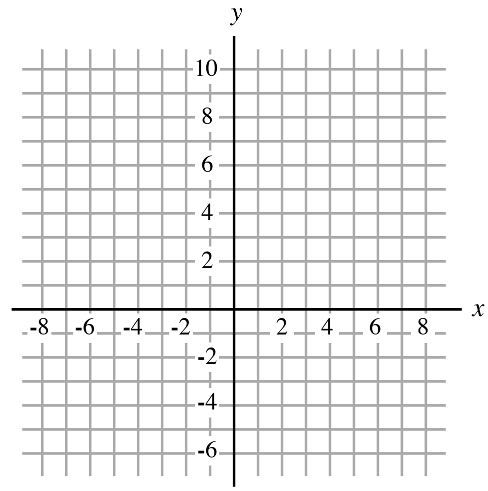
6. The y-coordinate is positive.

7. The x- and y-coordinates have the same sign.

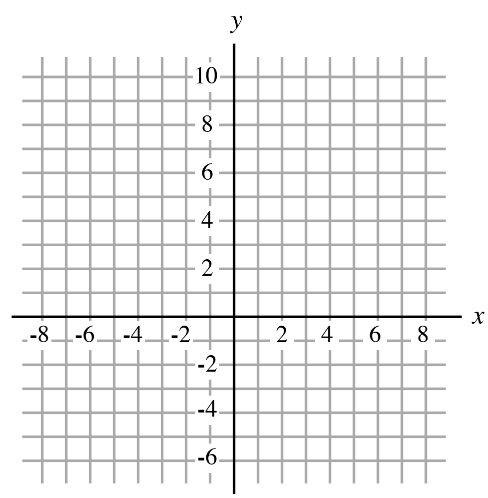
8. Given the equation $y = -\frac{3}{4}x + 5$, create a table of three x-y-values and graph the line.



9. Given the equation $y = x^2 - 6x + 5$, create a table of *five* x-y-values and graph the parabola.



10. Given the equation $y = -2x^2 + 8x$, create a table of *five* x-y-values and graph the parabola.



Find the distance between each pair of given points. Simplify the answer.

11. (1, 4) and (-3, 7)

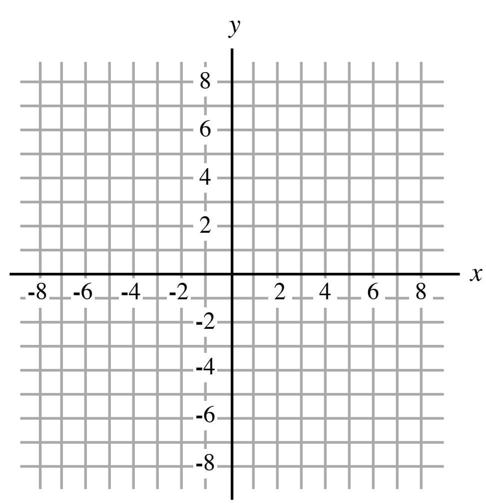
12. (2, -5) and (-2, -11)

13. The origin and (-5, 10)

14. The origin and (0, -7)

15. Consider a circle centered at the origin that passes through (6, -8).

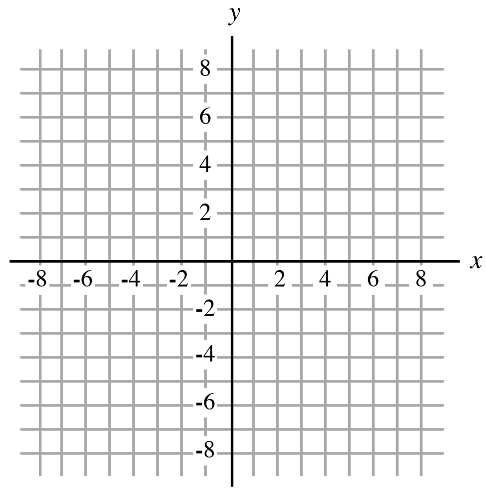
a) Find the radius of the circle



b) Draw its graph.

c) What is the equation of the circle?

16. Consider a circle centered at the origin that passes through (-4, 2).



a) Find the radius of the circle

b) Draw its graph.

c) What is the equation of the circle?

Verify that each given point is on the unit circle.

17. $\left(\frac{3}{5}, -\frac{4}{5}\right)$

18. $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

19. $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

20. $\left(-\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$