A.1 Multiplying Binomials

MULTIPLYING BINOMIALS, THE FOIL METHOD

The product of two binomials initially results in four products, four terms. Sometimes, two of those terms are like terms and can be combined. One technique for multiplying binomials is called the **FOIL method**.

When multiplying two binomials, such as $(a + b)(c + d)$, there are four pairs of terms in the following positions:

Pair of terms	Position	Represented by	
• a and c	**First** terms in each binomial When we distribute, $a \cdot c$ is called the **F**irst product.	**F**	**F** $(\boldsymbol{a} + b)(\boldsymbol{c} + d)$
• a and d	the outermost, or **Outer**, terms in the entire product When we distribute, $a \cdot d$ is called the **O**uter product.	**O**	**O** $(\boldsymbol{a} + b)(c + \boldsymbol{d})$
• b and c	the innermost, or **Inner**, terms in the entire product When we distribute, $b \cdot c$ is called the **I**nner product.	**I**	**I** $(a + \boldsymbol{b})(\boldsymbol{c} + d)$
• b and d	the **Last** terms in each binomial When we distribute, $b \cdot d$ is called the **L**ast product.	**L**	**L** $(a + \boldsymbol{b})(c + \boldsymbol{d})$

The **FOIL** method doesn't tell us anything new; it simply gives us a way to speak about the four individual products that result. The results are the same as when we apply the distributive property.

In multiplying $\mathbf{(2x + 3)(4x - 5)}$, we get four initial products. Based on the **FOIL** method, those products are:

F	O	I	L
First Product	Outer Product	Inner Product	Last Product
$2x \cdot 4x = \mathbf{8x^2}$	$2x \cdot (-5) = \mathbf{-10x}$	$3 \cdot 4x = \mathbf{12x}$	$3 \cdot (-5) = \mathbf{-15}$

$$8x^2 \quad - \quad 10x \quad + \quad 12x \quad - \quad 15$$

$$= \quad \mathbf{8x^2 + 2x - 15}$$

If the **O**uter and **I**nner products are like terms, then they can be combined, making the **FOIL** method more like **F + (O + I) + L**. When the sum of **O** and **I** are done mentally, this becomes a one-step process.

Note: If the power of the first term is more than 1, then the power must also be considered when multiplying binomials. This is demonstrated in Example 1.

Example 1: Multiply.

a) $(v + 5)(3v + 2)$ b) $(2x^2 - 3)(6x^2 - 5)$ c) $(w^3 - 5)(2w^3 + 4)$

Procedure: Find the product using the **FOIL** method. These may be done in one step. Simplify by combining like terms.

Answer:

a) $(v + 5)(3v + 2)$	b) $(2x^2 - 3)(6x^2 - 5)$	c) $(w^3 - 5)(2w^3 + 4)$
F + (O + I) + L	**F + (O + I) + L**	**F + (O + I) + L**
$= 3v^2 + 17v + 10$	$= 12x^4 - 28x^2 + 15$	$= 2w^6 - 6w^3 - 20$

You Try It 1 Practice using the **FOIL** method along with the one-step technique to multiply. Use Example 1 as a guide.

a) $(3k - 5)(k - 4)$ b) $(2x^2 + 3)(4x^2 + 1)$ c) $(3c^3 + 7)(2c^3 - 1)$

Caution: The **FOIL** method does not replace anything you have already learned about multiplying polynomials. It is a technique that applies only to multiplying two binomials.

SQUARING A BINOMIAL

To square a binomial, such as $(2x - 5)^2$, we can, as an option, write it as the product of the same binomial:

$(2x - 5)^2 = (2x - 5)(2x - 5)$. Using the **FOIL** method, the product is

$$\begin{array}{lcccc} & \textbf{F} & \textbf{O} & \textbf{I} & \textbf{L} \\ = & 4x^2 & \underline{-\ 10x} & \underline{-\ 10x} & +\ 25 \\ = & 4x^2 & \mathbf{-\ 20x} & +\ 25 & \end{array}$$

Because the Outer and Inner products are identical, their sum is just twice the Outer product. In other words,

$(2x - 5)^2 = (2x - 5)(2x - 5)$. Using the **FOIL** method and doubling the Outer product, we get

$$\begin{array}{lccc} & \text{F} & 2 \cdot \text{O} & \text{L} \\ = & 4x^2 & -\ 2(10x) & +\ 25 \\ = & 4x^2 & -\ 20x & +\ 25 \end{array}$$

In general, the square of a binomial fits one of these patterns:

The Square of a Binomial

1. $(a + b)^2 = a^2 + 2ab + b^2$ **2.** $(a - b)^2 = a^2 - 2ab + b^2$

Example 2: Multiply these squared binomials.

a) $(x + 7)^2$ b) $(3x - 4)^2$ c) $(2x^2 + 5)^2$

Procedure: Multiply directly using **F + 2 · O + L**

Answer: a) $x^2 + 14x + 49$ b) $9x^2 - 24x + 16$ c) $4x^4 + 20x^2 + 25$

Caution: One common mistake in using this one-step technique is forgetting to double the **O**uter product, which is the middle term in the resulting trinomial.

Another common mistake is to square the first and last terms but forget to find the **O**uter product at all. This incorrectly leads to the resulting product having no middle term.

You Try It 2 Multiply these perfect square binomials. Use Example 2 as a guide.

a) $(x + 8)^2$ b) $(4w - 9)^2$

c) $(5y^2 + 4)^2$ d) $(6x^3 - 2)^2$

CONJUGATES

Two binomials are **conjugates** if their *first terms are exactly the same* but their second terms are opposites, as in $(a + b)$ and $(a - b)$. Conjugates always come in pairs. For example, $(x + 3)$ is not a conjugate without $(x - 3)$.

When multiplying conjugates using the **FOIL** method, the **O**uter and **I**nner terms are always opposites and combine to 0, so the product of a pair of conjugates is always the difference of squares:

$(a + b)(a - b) = a^2 - b^2$.

For example,

$$\begin{aligned} &(x - 3)(x + 3) \\ &= x^2 + 3x - 3x - 9 \\ &= x^2 + 0x - 9 \\ &= x^2 - 9 \end{aligned}$$

$$\begin{aligned} &(2x + 5)(2x - 5) \\ &= 4x^2 - 10x + 10x - 25 \\ &= 4x^2 + 0x - 25 \\ &= 4x^2 - 25 \end{aligned}$$

Example 3: Identify the conjugate of the given binomial and then multiply the pair of conjugates.

a) $(2r - 3)$ b) $(6y + 7)$ b) $(3y^2 + 5)$

Procedure: The product of a pair of conjugates is always the difference of squares.

Answer:

a) Conjugate is $(2r + 3)$

$$\begin{aligned} &(2r - 3)(2r + 3) \\ &\quad a^2 - b^2 \\ &= (2r)^2 - (3)^2 \\ &= 4r^2 - 9 \end{aligned}$$

b) Conjugate is $(6y - 7)$

$$\begin{aligned} &(6y + 7)(6y - 7) \\ &\quad a^2 - b^2 \\ &= (6y)^2 - (7)^2 \\ &= 36y^2 - 49 \end{aligned}$$

b) Conjugate is $(3y^2 - 5)$

$$\begin{aligned} &(3y^2 - 5)(3y^2 + 5) \\ &\quad a^2 - b^2 \\ &= (3y^2)^2 - (5)^2 \\ &= 9y^4 - 25 \end{aligned}$$

You Try It 3 Identify the binomial conjugate of the given binomial and then multiply the pair of conjugates. Use Example 3 as a guide.

a) $(x - 8)$ b) $(4x + 9)$ c) $(10w^2 - 2)$

You Try It Answers

YTI 1: a) $3k^2 - 17k + 20$ b) $8x^4 + 14x^2 + 3$
c) $6c^6 + 11c^3 - 7$

YTI 2: a) $x^2 + 16x + 64$ b) $16w^2 - 72v + 81$
c) $25y^4 + 40y^2 + 16$ d) $36x^6 - 24x^3 + 4$

YTI 3: a) $(7x + 8)$; $49x^2 - 64$ b) $(4x^2 - 9)$; $16x^4 - 81$
c) $(10w^3 + 2)$; $100w^6 - 4$

A.1 Focus Exercises

Multiply and simplify. Write each answer in descending order.

1. $(2x - 3)(3x + 8)$

2. $(5y - 6)(3y - 4)$

3. $(3u + 4)(4u - 3)$

4. $(7w - 10)(5w + 3)$

5. $(2n^2 + 5)(6n^2 + 1)$

6. $(x^2 + 6)(3x^2 - 4)$

7. $(2r^3 - 3)(5r^3 - 1)$

8. $(3r^3 - 5)(6r^3 + 2)$

9. $(x + 7)^2$

10. $(w - 9)^2$

11. $(5y - 3)^2$

12. $(3m + 4)^2$

13. $(3c^2 - 5)^2$

14. $(4p^2 + 3)^2$

15. $(a^3 + 10)^2$

16. $(2x^3 - 5)^2$

Identify the binomial conjugate of the given binomial and then multiply the pair of conjugates.

17. $(x + 5)$

18. $(y - 6)$

19. $(r - 8)$

20. $(p + 10)$

21. $(2c + 9)$

22. $(4w - 7)$

23. $(x^2 - 8)$

24. $(6y^2 - 1)$