A.3 Solving Quadratic Equations

INTRODUCTION

A **quadratic equation** is an equation in which the degree of the polynomial is 2.

> The *standard form* for a quadratic equation is
>
> $ax^2 + bx + c = 0$ or $0 = ax^2 + bx + c$

Standard form means that one side of this equation is a polynomial in descending order and the other side is 0. Some quadratic equations are not yet written in standard form but can be rewritten into standard form. Here are some examples of quadratic equations:

$-2y^2 + 6y = 0$ $\qquad$ $v^2 - 7v = -10$ $\qquad$ $(x + 3)(x - 2) = 14$

THE ZERO PRODUCT PRINCIPLE

The product of any two non-zero numbers will always be either positive or negative. The only way a product can be zero is if one of the factors is zero. This idea leads us to the Zero Product Principle.

> **The Zero Product Principle**
>
> If the product of two numbers is 0, then one of the numbers must be 0.
>
> **If $A \cdot B = 0$, then either $A = 0$ or $B = 0$.**

The Zero Product Principle can be applied to any two factors that multiply to get 0. For example, if $(x + 7)(x - 5) = 0$, then one of the factors must be 0:

either $x + 7 = 0$ or $x - 5 = 0$

In other words, either $\mathbf{x = -7}$ or $\mathbf{x = 5}$

To apply the Zero Product Principle, one side of the equation must be 0 and the other side must be in a factored form. For example, we cannot apply the Zero Product Principle on $x^2 + x - 6 = 0$ until the left side of the equation is factored:

$$x^2 + x - 6 = 0$$

$$(x + 3)(x - 2) = 0$$

Either $x + 3 = 0$ or $x - 2 = 0$

Either $x = -3$ or $x = 2$

It is common to combine the solutions into a single solution set: $\boxed{x = -3, 2}$

Example 1: Solve each equation.

a) $x^2 + 2x - 35 = 0$ b) $-2r^2 - 8r = 0$ c) $0 = 4y^2 - 13y + 3$

Procedure: Factor the polynomial and apply the Zero Product Principle. Verify the answers and write the solutions in a solution set.

Answer:

a)

$x^2 + 2x - 35 = 0$	Factor the left side. Use the Factor Game.
$(x + 7)(x - 5) = 0$	Set each factor equal to 0.
$x + 7 = 0$ or $x - 5 = 0$	Solve each linear equation.
$x = -7$ or $x = 5$	Write the solutions in a solution set.
$x = \mathbf{-7, 5}$	

b)

$-2r^2 - 8r = 0$	Factor the left side. Extract $-2r$.
$-2r(r + 4) = 0$	Set each factor equal to 0.
$-2r = 0$ or $r + 4 = 0$	Solve each linear equation.
$r = 0$ or $r = -4$	A solution may be 0.
$r = \mathbf{0, -4}$	

c)

$0 = 4y^2 - 13y + 3$	Factor the right side. Use the Factor Game.
$0 = (4y - 1)(y - 3)$	Set each factor equal to 0.
$4y - 1 = 0$ or $y - 3 = 0$	Solve each linear equation.
$y = \frac{1}{4}$ or $y = 3$	A solution may be an integer or a fraction.
$y = \mathbf{\frac{1}{4}, 3}$	

Notice that, in Example 1 c), 0 is on the left side. That is okay. The Zero Product Principle works the same way as long as 0 is on one side or the other.

You Try It 1 Solve each equation. Use Example 1 as a guide.

a) $m^2 + 3m - 28 = 0$

b) $-5x^2 + 20x = 0$

c) $4w^2 - 5w - 6 = 0$

d) $9k^2 - 16 = 0$

To solve a quadratic equation, the polynomial must first be set equal to 0.

Consider $x^2 + 3x + 2 = 12$. As written, the polynomial is not yet equal to 0, so we cannot yet factor. Instead we must add -12 to each side to create 0 on the right side. Then we may factor and solve:

$$x^2 + 3x + 2 = 12$$

First, add -12 to each side so that 0 is on the right side:

$$-12 = -12$$

Now factor the polynomial:

$$x^2 + 3x - 10 = 0$$

$$(x + 5)(x - 2) = 0$$

Write the two linear equations and solve:

$$x + 5 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = -5 \quad \text{or} \quad x = 2$$

$$x = \mathbf{-5, 2}$$

Example 2: Solve each equation.

a) $k^2 - 5k + 2 = -4$ b) $(y + 6)(2y - 3) = -28$ c) $4x - x^2 = 4$

Procedure: Get 0 (zero) on one side of the equation. Factor the polynomial and apply the Zero Product Principle. Verify the answers and write the solutions in a solution set.

Answer:

a)

$k^2 - 5k + 2 = -4$	Add 4 to each side to create 0 on the right side.
$k^2 - 5k + 6 = 0$	Factor the left side. Use the Factor Game.
$(k - 2)(k - 3) = 0$	Set each factor equal to 0.
$k - 2 = 0$ or $k - 3 = 0$	Solve each linear equation.
$k = 2$ or $k = 3$	Combine the solutions.
$k = \mathbf{2, 3}$	

b)

$(y + 6)(2y - 3) = -28$	First, multiply out the left side.
$2y^2 + 9y - 18 = -28$	Add 28 to each side to create 0 on the right side.
$2y^2 + 9y + 10 = 0$	Factor the left side. Use the Factor Game.
$(2y + 5)(y + 2) = 0$	Set each factor equal to 0.
$2y + 5 = 0$ or $y + 2 = 0$	Solve each linear equation.
$y = -\frac{5}{2}$ or $y = -2$	Combine the solutions.
$y = \mathbf{-\frac{5}{2}, 2}$	

c)

$4x - x^2 = 4$	Because the x^2 term on the left is negative, add x^2 and $-4x$ to each side to create 0 on the left.
$0 = x^2 - 4x + 4$	Factor the left side. Use the Factor Game. This is a perfect square trinomial.
$0 = (x - 2)^2$	There is only one solution. Set the single factor equal to 0 and solve.
$x - 2 = 0$	
$x = \mathbf{2}$	

<u>**You Try It 2**</u> Solve each equation. Use Example 2 as a guide. *Check your answers to verify that they are solutions.*

a) $x^2 + 5x - 8 = 28$

b) $6w = 5w^2 - 8$

c) $y^2 + 2y = 8y - 9$

d) $(x - 6)^2 = x + 6$

e) $(3v - 1)(v - 5) = -15$

f) $(x - 1)(6x + 5) = x + 3$

THE SQUARE ROOT PROPERTY OF EQUATIONS

Consider the equation $x^2 = 25$. There are two ways that we can approach this. Either technique we choose, though, should result in the same solution set.

Technique 1: Solve by factoring

$x^2 = 25$ Add -25 to each side

$x^2 - 25 = 0$ Factor

$(x - 5)(x + 5) = 0$ Solve.

$\boxed{x = 5, -5}$ Two Solutions.

Technique 2: Solve by taking the square root

$x^2 = 25$

$\sqrt{x^2} = \sqrt{25}$

This appears to have only one solution, $x = 5$. However, we know ithere are two solutions, and we can represent them as $\boxed{x = \pm 5}$

This is an example of the **Sqaure Root Property**: If $x^2 = a$

then $\sqrt{x^2} = \sqrt{a}$

and $x = \pm\sqrt{a}$

Example 3: Solve $(y + 4)^2 = 36$ using the Square Root Property of Equations.

Procedure: Take the square root of each side. The following step must include the ± symbol in front of the evaluated square root.

Answer:

$(y + 4)^2 = 36$ Take the square root of each side.

$\sqrt{(y + 4)^2} = \sqrt{36}$ For the next step, place ± in front of the 6

$y + 4 = \pm 6$ Set $y + 4$ equal to 6 and then to -6.

$y + 4 = 6$ or $y + 4 = -6$ Solve each linear equation.

$y = 2$ or $y = -10$ Combine the solutions.

$y =$ **2, -10**

You Try It 3 Solve each equation using the Square Root Property of Equations.. Use Example 3 as a guide. *Check your answers to verify that they are solutions.*

a) $(x - 5)^2 = 16$

b) $(2y + 3)^2 = 49$

You Try It Answers

YTI 1:	a) $m = -7, 4$	b) $x = 0, 4$	c) $w = -\frac{3}{4}, 2$	d) $k = -\frac{4}{3}, \frac{4}{3}$
YTI 2:	a) $x = -9, 4$	b) $w = \frac{-4}{5}, 2$	c) $y = 3$	d) $x = 3, 10$
YTI 3:	a) $x = 1, 9$	b) $y = 2, -5$		

A.3 Focus Exercises

Solve each equation. Check your answers to verify that they are solutions.

1. $6x^2 - 54x = 0$

2. $3r^2 + 12r = 0$

3. $p^2 - p - 90 = 0$

4. $2n^2 - 13n + 15 = 0$

5. $3p^2 + p = 10$

6. $w^2 + 2w - 4 = 59$

7. $-3x^2 + 4x = -15$

8. $y^2 - y = 18 - 4y$

9. $v(8v + 2) = 15$

10. $(x - 3)(x + 7) = -16$

11. $x^2 = 81$

12. $w^2 = 6$

13. $(m - 9)^2 = 4$

14. $(4x + 1)^2 = 25$