A.5 Complex Fractions

A **complex fraction** is any fraction in which the numerator or the denominator, or both, contain one or more fractions. Examples of complex fractions include

$$\frac{5}{\frac{7}{8}} \qquad \frac{\frac{2}{3}}{6} \qquad \frac{1}{\frac{5}{2}} \qquad \frac{\frac{x}{2y}}{\frac{3}{y^2}} \qquad \frac{\frac{2}{3}+\frac{5}{6}}{\frac{5}{4}-\frac{1}{2}} \qquad \frac{\frac{1}{x}-\frac{3}{2x}}{1+\frac{5}{x^2}}$$

SIMPLIFYING COMPLEX FRACTIONS, METHOD 1

If a complex fraction contains no addition or subtraction, then it can be simplified by first writing the fraction as division.

For example, $\frac{\frac{2}{3}}{\frac{5}{4}}$ can be rewritten as $\frac{2}{3} \div \frac{5}{4}$. This can be simplified by inverting the second fraction and multiplying: $\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$, which cannot simplify any further.

Example 1: Simplify each complex fraction by first rewriting it using the division symbol.

a) $\frac{5}{\frac{7}{8}}$ b) $\frac{w}{\frac{x}{w}}$ c) $\frac{\frac{2}{3}}{6}$ d) $\frac{\frac{x}{2y}}{\frac{3}{y^2}}$

Procedure: First rewrite each using the division symbol and then invert and multiply.

Answer:

a) $\frac{5}{\frac{7}{8}} = 5 \div \frac{7}{8} = \frac{5}{1} \cdot \frac{8}{7} = \frac{40}{7}$

b) $\frac{w}{\frac{x}{w}} = w \div \frac{x}{w} = \frac{w}{1} \cdot \frac{w}{x} = \frac{w^2}{x}$

c) $\frac{\frac{2}{3}}{6} = \frac{2}{3} \div \frac{6}{1} = \frac{2}{3} \cdot \frac{1}{6} = \frac{2}{18} = \frac{1}{9}$

d) $\frac{\frac{x}{2y}}{\frac{3}{y^2}} = \frac{x}{2y} \div \frac{3}{y^2} = \frac{x}{2y} \cdot \frac{y^2}{3} = \frac{xy^2}{6y} = \frac{xy}{6}$

You Try It 1 Simplify each complex fraction by first rewriting it using the division symbol. Use Example 1 as a guide.

a) $\dfrac{\frac{2}{3}}{\frac{7}{5}}$ b) $\dfrac{10}{\frac{5}{4}}$ c) $\dfrac{\frac{x}{y}}{\frac{3x}{y^2}}$

SIMPLIFYING COMPLEX FRACTIONS, METHOD 2

A second method of simplifying complex fractions is to clear all denominators directly, while in its complex form. We do so by multiplying the whole fraction by a carefully chosen value of 1, such as $\frac{7}{7}$, $\frac{x}{x}$, or $\frac{3w^2}{3w^2}$.

For example, in the complex fraction $\dfrac{\frac{4}{5}}{\frac{3}{2}}$, we can clear the denominators, 2 and 5, by using the common multiplier of 10, multiplying the whole fraction by $\frac{10}{10}$. (10 is the least common denominator for $\frac{4}{5}$ and $\frac{3}{2}$). In this case, $\frac{10}{10}$ is better written as $\dfrac{\frac{10}{1}}{\frac{10}{1}}$ so that we can more easily multiply:

$$\frac{\frac{4}{5}}{\frac{3}{2}} \cdot \frac{\frac{10}{1}}{\frac{10}{1}} = \frac{\frac{4}{5} \cdot \frac{10}{1}}{\frac{3}{2} \cdot \frac{10}{1}} = \frac{\frac{40}{5}}{\frac{30}{2}} = \frac{8}{15}$$

Note: We use **LCD** to abbreviate *least common denominator*.

Let’s put Method 2 into practice.

Example 2: Simplify each complex fraction by using Method 2.

a) $\dfrac{5}{\frac{7}{8}}$ b) $\dfrac{\frac{7}{8}}{\frac{3}{4}}$ c) $\dfrac{\frac{3}{x}}{\frac{2}{x^2}}$ d) $\dfrac{\frac{x}{2y}}{\frac{3}{y^2}}$

Procedure: First recognize the least common denominator (LCD), then use it as the common multiplier. In part a), write the numerator as a fraction, $\frac{5}{1}$. Follow each step carefully.

Answer:

a) LCD = 8: $\dfrac{\frac{5}{1}}{\frac{7}{8}} \cdot \dfrac{\frac{8}{1}}{\frac{8}{1}} = \dfrac{\frac{5}{1}\cdot\frac{8}{1}}{\frac{7}{8}\cdot\frac{8}{1}} = \dfrac{\frac{5}{1}\cdot\frac{8}{1}}{\frac{7}{1}\cdot\frac{1}{1}} = \dfrac{40}{7}$

b) LCD = 8: $\dfrac{\frac{7}{8}}{\frac{3}{4}} \cdot \dfrac{\frac{8}{1}}{\frac{8}{1}} = \dfrac{\frac{7}{8}\cdot\frac{8}{1}}{\frac{3}{4}\cdot\frac{8}{1}} = \dfrac{\frac{7}{1}\cdot\frac{1}{1}}{\frac{3}{1}\cdot\frac{2}{1}} = \dfrac{7}{6}$

c) LCD = x^2: $\dfrac{\frac{3}{x}}{\frac{2}{x^2}} \cdot \dfrac{\frac{x^2}{1}}{\frac{x^2}{1}} = \dfrac{\frac{3}{x}\cdot\frac{x^2}{1}}{\frac{2}{x^2}\cdot\frac{x^2}{1}} = \dfrac{\frac{3}{1}\cdot\frac{x}{1}}{\frac{2}{1}\cdot\frac{1}{1}} = \dfrac{3x}{2}$

d) LCD = $2y^2$: $\dfrac{\frac{x}{2y}}{\frac{3}{y^2}} \cdot \dfrac{\frac{2y^2}{1}}{\frac{2y^2}{1}} = \dfrac{\frac{x}{2y}\cdot\frac{2y^2}{1}}{\frac{3}{y^2}\cdot\frac{2y^2}{1}} = \dfrac{\frac{x}{1}\cdot\frac{y}{1}}{\frac{3}{1}\cdot\frac{2}{1}} = \dfrac{xy}{6}$

You Try It 2 Simplify each complex fraction by using Method 2. Use Example 2 as a guide.

a) $\dfrac{\frac{5}{8}}{\frac{3}{2}}$ b) $\dfrac{\frac{4}{9}}{\frac{8}{3}}$ c) $\dfrac{\frac{4y^2}{w^2}}{\frac{8y}{3w}}$

WHEN A COMPLEX FRACTION IS, WELL ... MORE COMPLEX

If a complex fraction contains addition or subtraction, we cannot use Method 1 without first creating a single fraction in both the numerator and denominator. As an alternative, we can use Method 2 and multiply the complex fraction by the LCD of *all* of the smaller fractions within.

For example, the complex fraction $\dfrac{\frac{2}{3} + \frac{5}{6}}{\frac{5}{4} - \frac{1}{2}}$ has four denominators within it: 3, 6, 4, and 2. The LCD is 12, so the common multiplier for the entire fraction is 12, and we use it to clear all of the denominators within the complex fraction.

Multiply the whole fraction by $\dfrac{12}{12}$, better written as $\dfrac{\frac{12}{1}}{\frac{12}{1}}$, as shown here:

$$= \frac{\frac{2}{3} + \frac{5}{6}}{\frac{5}{4} - \frac{1}{2}} \cdot \frac{\frac{12}{1}}{\frac{12}{1}}$$

Multiply $\frac{12}{1}$ to both the numerator and the denominator.
Distribute $\frac{12}{1}$ to each term in the numerator and to each term in the denominator.

$$= \frac{\frac{2}{3} \cdot \frac{12}{1} + \frac{5}{6} \cdot \frac{12}{1}}{\frac{5}{4} \cdot \frac{12}{1} - \frac{1}{2} \cdot \frac{12}{1}}$$

Simplify each product of fractions within by dividing out common factors.

$$= \frac{8 + 10}{15 - 6}$$

Again, simplify in the numerator and in the denominator.

$$= \frac{18}{9} = \boxed{2}$$

Simplify this fraction.

Your work might look a little more like this:

$$\frac{\frac{2}{3} + \frac{5}{6}}{\frac{5}{4} - \frac{1}{2}} \cdot \frac{\frac{12}{1}}{\frac{12}{1}} = \frac{\frac{2}{{}_1\cancel{3}} \cdot \frac{\cancel{12}^{4}}{1} + \frac{5}{{}_1\cancel{6}} \cdot \frac{\cancel{12}^{2}}{1}}{\frac{5}{{}_1\cancel{4}} \cdot \frac{\cancel{12}^{3}}{1} - \frac{1}{{}_1\cancel{2}} \cdot \frac{\cancel{12}^{6}}{1}} = \frac{8 + 10}{15 - 6} = \frac{18}{9} = 2$$

... or you might distribute and cancel mentally as you go, such as ...

$$\frac{\frac{2}{3} + \frac{5}{6}}{\frac{5}{4} - \frac{1}{2}} \cdot \frac{\frac{12}{1}}{\frac{12}{1}} = \frac{8 + 10}{15 - 6} = \frac{18}{9} = 2$$

Example 3: Simplify the complex fraction by using Method 2, described above.

$$\frac{\frac{4}{x} - \frac{3}{2x}}{1 + \frac{5}{x^2}}$$

Procedure: Make every term a fraction, if it isn't already. That will help in the multiplication process. Next, find the least common denominator and use it to create a form of 1.

Answer: First, the common denominator is $2x^2$. Let's go right to the distribution step where each small fraction is multiplied by the $\frac{2x^2}{1}$.

$$\frac{\frac{4}{x}\cdot\frac{2x^2}{1} - \frac{3}{2x}\cdot\frac{2x^2}{1}}{\frac{1}{1}\cdot\frac{2x^2}{1} + \frac{5}{x^2}\cdot\frac{2x^2}{1}} = \frac{\frac{4}{1}\cdot\frac{2x}{1} - \frac{3}{1}\cdot\frac{x}{1}}{\frac{1}{1}\cdot\frac{2x^2}{1} + \frac{5}{1}\cdot\frac{2}{1}} = \frac{8x - 3x}{2x^2 + 10} = \frac{5x}{2x^2 + 10}$$

You Try It 3 Simplify each complex fraction using Method 2. Use Example 3 as a guide.

a) $\dfrac{\frac{3}{8} + \frac{1}{6}}{2 - \frac{1}{4}}$

b) $\dfrac{\frac{v}{4} - \frac{1}{v}}{\frac{1}{2} - \frac{1}{v}}$

You Try It Answers

You Try It 1: a) $\frac{10}{21}$ b) 8 c) $\frac{y}{3}$

You Try It 2: a) $\frac{5}{12}$ b) $\frac{1}{6}$ c) $\frac{3y}{2w}$

You Try It 3: a) $\frac{13}{42}$ b) $\frac{v + 2}{2}$ or $\frac{v}{2} + 1$

A.5 Focus Exercises

Simplify each complex fraction using any method.

1. $\dfrac{\frac{10}{9}}{\frac{25}{12}}$

2. $\dfrac{12}{\frac{6}{5}}$

3. $\dfrac{\frac{3m}{p}}{\frac{5m}{2p}}$

4. $\dfrac{\frac{u}{3v}}{6u}$

Simplify each complex fraction using Method 2.

5. $\dfrac{1 - \frac{5}{2}}{1 + \frac{2}{3}}$

6. $\dfrac{\frac{5}{12} + \frac{1}{3}}{\frac{2}{3} - \frac{1}{4}}$

7. $\dfrac{\frac{2}{3} - \frac{1}{2}}{1 - \frac{1}{6}}$

8. $\dfrac{1 + \frac{2}{y}}{\frac{y}{3} + \frac{2}{3}}$

9. $\dfrac{\frac{1}{6} - \frac{1}{2w}}{\frac{1}{3w} - \frac{1}{w^2}}$

10. $\dfrac{1 - \frac{1}{y^2}}{\frac{1}{y} + \frac{1}{y^2}}$

11. $\dfrac{\frac{1}{6} + \frac{4}{3x} + \frac{2}{x^2}}{1 + \frac{2}{x}}$

12. $\dfrac{\frac{1}{2} - \frac{3}{x} + \frac{4}{x^2}}{1 - \frac{2}{x}}$