
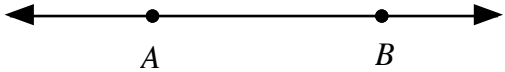
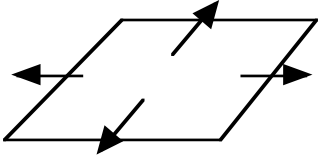
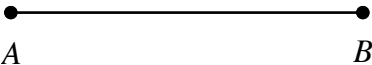
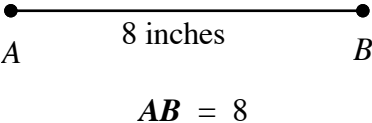
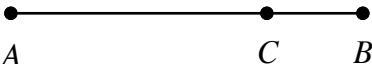
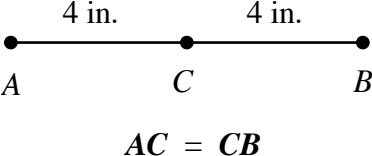
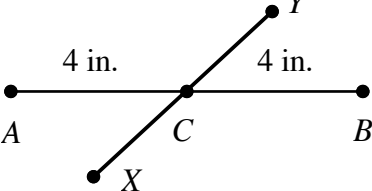
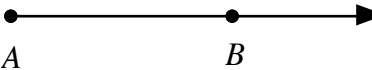


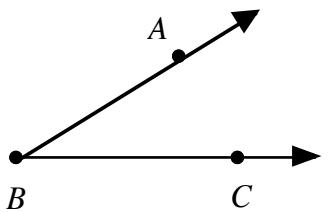
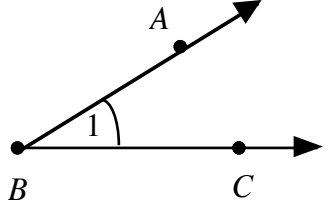
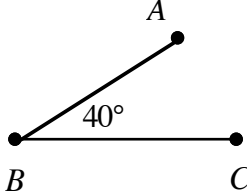
Section 1.1 Basic Definitions of Geometry

UNDEFINED TERMS

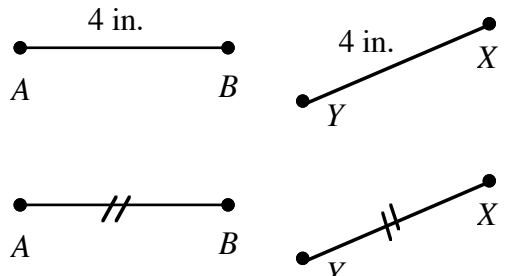
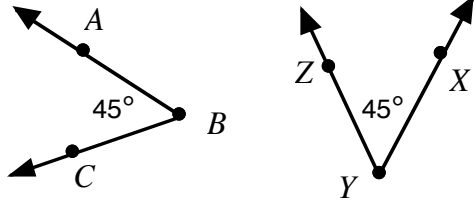
The following terms will remain basically undefined: *point*, *line*, *between*, *set*, *plane*. We assume we all share an understanding of what these terms mean, but here are the basic ideas:

<p>A point is represented by a dot to indicate its location, but it really has no size or dimension.</p>	
<p>A line means a straight line, not a curved line. A line that passes through points <i>A</i> and <i>B</i> is denoted \overleftrightarrow{AB}</p>	
<p>A plane is a flat surface, much like a floor or a wall; technically, a plane is infinite in all directions.</p>	
<p>A line segment has two endpoints and has finite length. We use the notation \overline{AB} (or \overline{BA}) to represent the line segment with endpoints <i>A</i> and <i>B</i>.</p>	
<p>Because a line segment has a beginning and an end, it has a definite length. We represent the <i>length</i> of \overline{AB} as just <i>AB</i> (with no bar over it).</p>	
<p>The point <i>C</i> is between <i>A</i> and <i>B</i> and if (i) <i>C</i> is on \overline{AB} and (ii) <i>C</i> is not also one of the endpoints.</p>	
<p>The point <i>C</i> is the midpoint of \overline{AB} if (i) <i>C</i> is between <i>A</i> and <i>B</i> and (ii) new line segments \overline{AC} and \overline{CB} have the same length.</p>	
<p>A line or line segment that crosses \overline{AB} at its midpoint, <i>C</i>, is called a segment bisector. To <i>bisect</i> an object means to divide it into two equally measured parts.</p>	
<p>A ray, or half-line, has a starting point and extends indefinitely from that point. A ray that starts at <i>A</i> and passes through <i>B</i> is denoted \overrightarrow{AB}</p>	 <p>Note: \overrightarrow{BA} is <i>not</i> the same ray as \overrightarrow{AB}.</p>

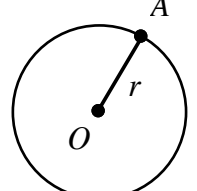
ANGLES

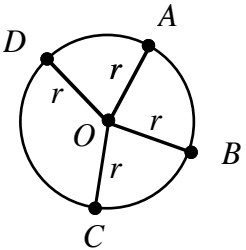
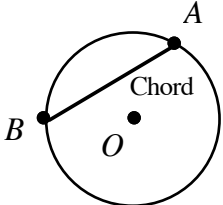
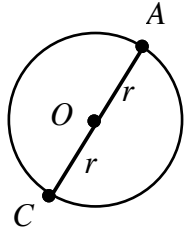
<p>An angle, ABC, which we may denote by $\angle ABC$, is the <i>union</i> of two line segments, \overline{BA} and \overline{BC}, or two rays, emanating from the same point, B, called the vertex.</p> <p>When naming an angle, the vertex is always the middle letter.</p>	
<p>Sometimes we use a number to represent an angle. For example, we can label this angle as $\angle ABC$ or as $\angle 1$;</p> <p>Note: The number used to label an angle, such as 1 or 2, typically has nothing to do with size or measure of the angle; the number is just a label.</p>	
<p>Angles are measured in <i>degrees</i>. We use $m\angle ABC$ to represent the measure of $\angle ABC$.</p> <p>In the diagram at right, $m\angle ABC = 40^\circ$.</p> <p>Note: The degree symbol must be written when the measure of an angle is used.</p>	

CONGRUENCE

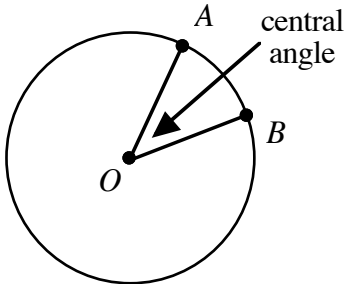
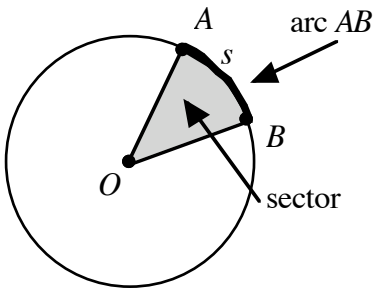
<p>Congruence means <i>same size</i>. The symbol for congruence is \cong. Two line segments are congruent if they have the same length $\overline{AB} \cong \overline{XY}$.</p> <p>(We can also write this as $AB = XY$.)</p> <p>Note: It is common to indicate that two line segments are congruent by placing the same number of <i>dashes</i> on them, as shown at right.</p>	
<p>Congruent angles have the same <i>angle measure</i>.</p> <p>$\angle ABC \cong \angle XYZ$</p> <p>Notice the subtle difference between congruence, \cong, and equal, $=$. We use \cong between two physical items and $=$ between two measures.</p>	

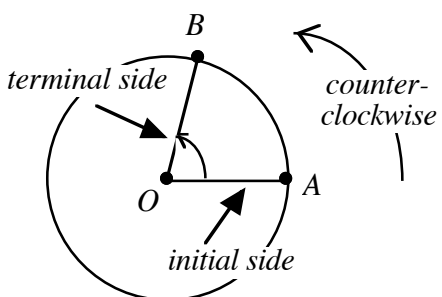
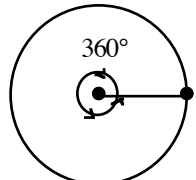
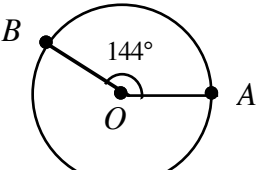
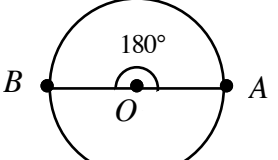
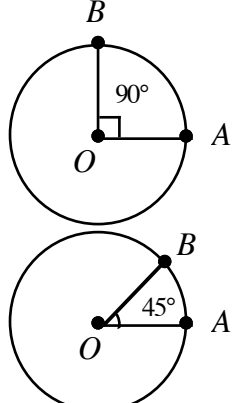
CIRCLES

<p>A circle is the set of all points in a plane that are equidistant (the same distance) from a fixed point, the center, labeled O in the diagram at right.</p> <p>This equal distance, represented by \overline{OA}, is called the radius, and has length r.</p>	
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<p>There are many radii (pronounced <i>ray'-dee-eye</i>, the plural of radius) in the circle, and each one has the center as one of its endpoints.</p> <p>Note: “Radius” has two meanings:</p> <ol style="list-style-type: none"> 1) A line segment from the center of a circle to a point on the circle, and 2) the length of a radius. 	
<p>A line segment in which both endpoints are on the circle is called a chord of the circle.</p>	
<p>A chord that passes through the center, O, is called a diameter of the circle. In this case, O is the midpoint of \overline{AC}. Each diameter has the length of two radii:</p> $d = 2r.$	

ANGLES IN CIRCLES

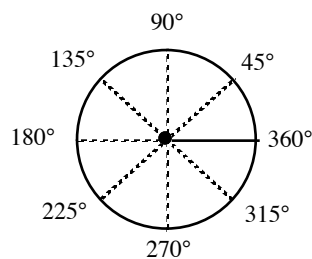
<p>An angle with its vertex at the center of a circle is called a central angle. A central angle in a circle is formed using two distinct radii;</p> <p>In the diagram at right, the central angle is $\angle AOB$.</p>	
<p>A central angle creates a sector of the circle, which is a portion of the region inside the circle.</p> <p>This angle also creates an arc on the circle. We say that the central angle subtends the arc it creates; arc AB is sometimes labeled by \widehat{AB}.</p> <p>The measure of \widehat{AB}, labeled by $m\widehat{AB}$, and is sometimes called s: $m\widehat{AB} = s$.</p>	

<p>It is common to represent a central angle starting with the “right-extended” horizontal radius, called the <i>initial side</i>.</p> <p>The angle then sweeps counter-clockwise to create the central angle. The radius at which it stops is called the <i>terminal side</i>.</p> <p>Note: This is discussed in greater detail in Section 1.4.</p>	
<p>A full circle has been (arbitrarily) assigned an angular measure value of 360 <i>degrees</i> (or 360°).</p> <p>The reason 360 is a good choice is that it is evenly divisible by so many whole numbers: 2, 3, 4, 5, 6, 8, 9, 10, 12 and many more.</p> <p>In this way we can easily identify the angle measures of these portions of a circle: $\frac{1}{2}$ of a circle, $\frac{1}{3}$ of a circle, $\frac{3}{4}$ of a circle, $\frac{2}{5}$ of a circle, and so on.</p>	
<p>For example, $\frac{2}{5}$ of a circle measures 144° because $\frac{2}{5} \cdot 360^\circ = \frac{720^\circ}{5} = 144^\circ$.</p> <p style="text-align: center;">$m\angle AOB = 144^\circ$</p>	
<p>A central angle that subtends one-half of a circle has a measure of 180° (one-half of 360°).</p>	
<p>A central angle that subtends one-fourth of a circle has a measure of 90° (one-fourth of 360°).</p> <p style="text-align: center;">A 90° angle is also called a <i>right angle</i>.</p> <p>One-half of a right angle is a 45° angle.</p>	

We can label the circle every 45° and use these values to help us estimate the placement of central angles throughout the circle.

For example, a central angle of 144° is between 135° and 180° , and it is closer to 135° .

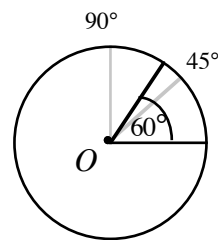
Take a moment to draw a 144° angle in the circle at right.



Example 1: Find the measure of the central angle that subtends *one-sixth* of a circle. Draw this central angle in a circle.

Answer: $\frac{1}{6}$ of a circle means $\frac{1}{6} \cdot 360^\circ = \frac{360^\circ}{6} = 60^\circ$, or

$$\frac{1}{6} \cdot 360^\circ = \frac{1}{6} \cdot \frac{360^\circ}{1} = \frac{1}{1} \cdot \frac{60^\circ}{1} = 60^\circ$$



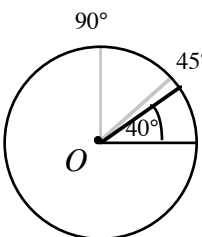
To draw a 60° central angle in a circle, use both the 45° and 90° angle as guides. The diagram shows an estimate of where a 60° central angle fits.

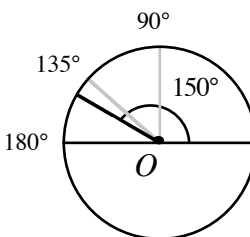
Example 2: Draw a central angle that has the given number of degrees. Also, on each circle, state what portion of a full circle the number of degrees is.

a) 40°

b) 150°

Answer: We can set up a ratio (fraction) comparing the small central angle to the entire circle:

a)  $\frac{40}{360} = \frac{4}{36} = \frac{1}{9}$
 So 40° is *one-ninth* of a circle.

b)  $\frac{150}{360} = \frac{15}{36} = \frac{5}{12}$
 So 150° is *five-twelfths* of a circle.

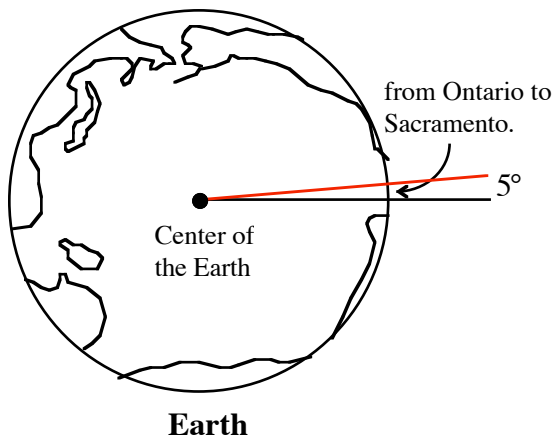
DEGREES, MINUTES AND SECONDS

There are 360° in a full circle, so 1° is a small angle. However, “small” is a relative term, and there are times when even a single degree can be a comparatively large angle. For this reason, 1° is broken into 60 smaller units, called **minutes**, and each minute is broken into 60 smaller units, called **seconds**.

When writing the measure of an angle in degrees, minutes and seconds, we use a single “tick” to represent *minutes*, and a double tick to represent *seconds*. For example, we can represent

$$51 \text{ degrees, } 22 \text{ minutes and } 48 \text{ seconds as } 51^\circ 22' 48''.$$

The circumference of the earth is about 24,900 miles (and the radius is about 3,960 miles).



The arc subtended by $5^\circ \approx 345$ miles, the approximate earth distance between Ontario, CA and Sacramento, CA.

$$5^\circ \approx 345 \text{ miles means } 1^\circ \approx 69 \text{ miles}$$

1 minute (one-sixtieth of a degree) is about 1.15 miles:

$$1' \approx 1.15 \text{ miles}$$

1 second (one-sixtieth of a minute) is about 101 feet:

$$1'' \approx 101 \text{ feet}$$

Even with the advent of calculators, the fractional portions of a degree—minutes and seconds—are still used in some global positions systems (GPS). For example, the distance between 43° and 44° Longitude (or latitude) is about 69 miles, the earth distance between San Bernardino and Santa Monica.

Here is a look at the relationships between these fractional portions of a degree:

Relationship	Abbreviation	Also ...
1 degree = 60 minutes	$1^\circ = 60'$	$1' = \frac{1}{60}^\circ$
1 minute = 60 seconds	$1' = 60''$	$1'' = \frac{1}{60}'$ $1'' = \frac{1}{3,600}^\circ$

Example 3: a) $17' = \frac{17}{60}^\circ$

b) $38'' = \frac{38}{3,600}^\circ$

Angles may be measured in degrees, minutes, and seconds (**DMS**), or they may be measured in degrees and decimal fractions of degrees. Converting from DMS to decimals (and vice-versa) often requires the use of a calculator, and that discussion is presented in Chapter 7 of our textbook.

ADDING AND SUBTRACTING IN DMS

Sometimes we must add or subtract angle measures that are written in DMS. Just as in adding and subtracting like terms in algebra, we add or subtract degrees with degrees, minutes with minutes, and seconds with seconds.

Example 4: Add or subtract as indicated.

a) $58^\circ 14' 33'' + 72^\circ 26' 15''$ b) $103^\circ 47' 52'' - 81^\circ 35' 44''$

Answer: It is often easier to align each angle measure vertically before adding or subtracting.

a)
$$\begin{array}{r} 58^\circ 14' 33'' \\ + 72^\circ 26' 15'' \\ \hline 130^\circ 40' 48'' \end{array}$$
 b)
$$\begin{array}{r} 103^\circ 47' 52'' \\ - 81^\circ 35' 44'' \\ \hline 22^\circ 12' 08'' \end{array}$$

Notice, in Example 4a), we are able to add the minutes and seconds without concern for either of them being 60 or more.

However, if we are to add, for example, 50 minutes to 17 minutes, we get 67 minutes. There is nothing wrong with 67 minutes, especially in measuring time. However, just as 67 minutes on a clock can be rewritten as 1 hour and 7 minutes, so, too, can 67 minutes of an angle be rewritten:

$67 \text{ minutes} = 1 \text{ degree and } 07 \text{ minutes: } 67' = 1^\circ 07'$

Also, a) $85 \text{ minutes} = 1 \text{ degree and } 25 \text{ minutes: } 85' = 1^\circ 25'$

b) $104 \text{ seconds} = 1 \text{ minute and } 44 \text{ seconds: } 104'' = 1' 44''$

Example 5: Rewrite each angle measure so that there are no more than 59 minutes and 59 seconds.

a) $30^\circ 82' 95''$ b) $75^\circ 59' 108''$ c) $103^\circ 48' 62''$

Procedure: Start by adjusting the number of seconds, if necessary. If the number of seconds is 60 or more, then (i) subtract 60 from the number of seconds, (ii) add 1 (one) to the number of minutes, and (iii) write the remainder as the new number of seconds. Next, adjust the number of minutes, if necessary.

$$\begin{array}{r} 14^\circ 82' 95'' \\ + 1' - 60'' \\ \hline 14^\circ 83' 35'' \\ + 1^\circ - 60' \\ \hline 15^\circ 23' 35'' \end{array}$$

Answer:

	<u>Given angle measure</u>	<u>with adjusted seconds</u>	<u>with adjusted minutes</u>
a)	$30^\circ 82' 95''$	$= 30^\circ 83' 35''$	$= 31^\circ 23' 35''$
b)	$75^\circ 59' 108''$	$= 75^\circ 60' 48''$	$= 76^\circ 00' 48''$
c)	$103^\circ 48' 62''$	$= 103^\circ 49' 02''$	(no change)

Referring back to Example 4b), we are able to subtract minutes and seconds without concern for either of them being negative because the top number in each case is larger than the bottom number.

To prevent us from getting a negative value for either the minutes or seconds, it may be necessary to adjust the top number by a technique some call “borrowing” from one measure to another. (It might be more appropriately called “donating” because the first number never expects to be “repaid.”)

Consider, for example, $13^\circ 10' - 8^\circ 25'$. Setting this up as a vertical subtraction, we see that we cannot subtract the minutes directly:

$$\begin{array}{r} 13^\circ 10' \\ - 8^\circ 25' \\ \hline \end{array}$$

Instead, we must *borrow* 1° from 13° (or have 13° *donate* 1°) and add it—in the form of $60'$ —to the $10'$ already there. This means that $13^\circ 10' = (13 - 1)^\circ + (60 + 10)' = 12^\circ 70'$, and now we can subtract directly:

$$\begin{array}{r} 12^\circ 70' \\ - 8^\circ 25' \\ \hline 4^\circ 45' \end{array}$$

Example 6: Add or subtract as indicated.

a) $58^\circ 44' 33'' + 25^\circ 51' 35''$ b) $91^\circ 20' 38'' - 65^\circ 35' 44''$

Answer: Align each angle measure vertically. Adjust the answer as necessary.

a)
$$\begin{array}{r} 58^\circ 44' 33'' \\ + 25^\circ 51' 35'' \\ \hline 83^\circ 95' 68'' \end{array} \quad \begin{array}{l} \text{Adjust the seconds:} \\ \Rightarrow 83^\circ 96' 08'' \end{array} \quad \begin{array}{l} \text{Adjust the minutes:} \\ \Rightarrow 84^\circ 36' 08'' \end{array}$$

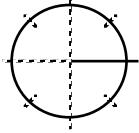
b) We must adjust the top number so that both the minutes and seconds are greater than the minutes and seconds in the bottom number.

$$\begin{array}{r} \text{Make } 1' \text{ into } 60'' \\ 91^\circ 20' 38'' \\ - 65^\circ 35' 44'' \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} \text{Make } 1^\circ \text{ into } 60' \\ 91^\circ 19' 98'' \\ - 65^\circ 35' 44'' \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} \text{Now subtract:} \\ 90^\circ 79' 98'' \\ - 65^\circ 35' 44'' \\ \hline 25^\circ 44' 54'' \end{array}$$

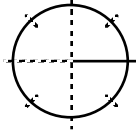
Section 1.1 Focus Exercises

What degree measure represents the given portion of a circle? Draw a central angle that has that same number of degrees.

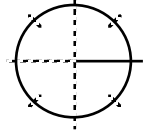
1. one-fourth of a circle



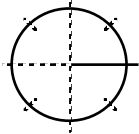
2. one-twelfth of a circle



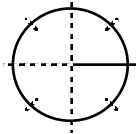
3. one-sixth of a circle



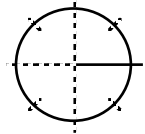
4. one-third of a circle



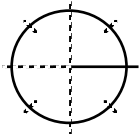
5. three-tenths of a circle



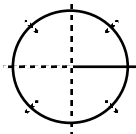
6. two-fifths of a circle



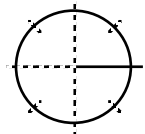
7. four-ninths of a circle



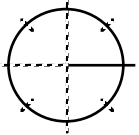
8. three-eighths of a circle



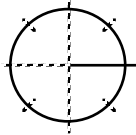
9. four-fifteenths of a circle



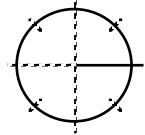
10. three-twentieths of a circle



11. eleven-thirtieths of a circle

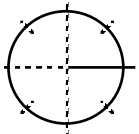


12. eleven-sixtieths of a circle

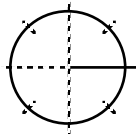


Draw a central angle that has the given number of degrees. Also, on each circle, state what portion of a full circle the number of degrees is.

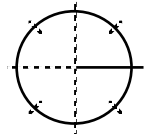
13. 30°



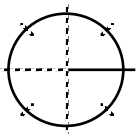
14. 72°



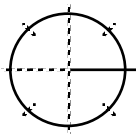
15. 45°



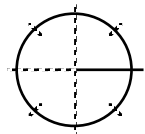
16. 18°



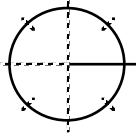
17. 120°



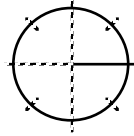
18. 135°



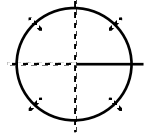
19. 180°



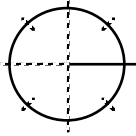
20. 80°



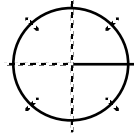
21. 75°



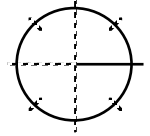
22. 160°



23. 225°



24. 100°



Add or subtract as indicated. Adjust the answer as necessary.

$$\begin{array}{r} 25. \quad 19^\circ 37' 21'' \\ + 38^\circ 16' 34'' \\ \hline \end{array}$$

$$\begin{array}{r} 26. \quad 91^\circ 09' 26'' \\ + 62^\circ 42' 44'' \\ \hline \end{array}$$

$$\begin{array}{r} 27. \quad 110^\circ 44' 35'' \\ + 42^\circ 23' 18'' \\ \hline \end{array}$$

$$\begin{array}{r} 28. \quad 60^\circ 24' 25'' \\ + 82^\circ 49' 56'' \\ \hline \end{array}$$

$$\begin{array}{r} 29. \quad 58^\circ 27' 19'' \\ + 31^\circ 32' 41'' \\ \hline \end{array}$$

$$\begin{array}{r} 30. \quad 124^\circ 42' 51'' \\ + 55^\circ 17' 09'' \\ \hline \end{array}$$

$$\begin{array}{r} 31. \quad 78^\circ 52' 19'' \\ - 45^\circ 37' 06'' \\ \hline \end{array}$$

$$\begin{array}{r} 32. \quad 105^\circ 17' 43'' \\ - 48^\circ 43' 15'' \\ \hline \end{array}$$

$$\begin{array}{r} 33. \quad 60^\circ 33' 06'' \\ - 48^\circ 50' 12'' \\ \hline \end{array}$$

$$\begin{array}{r} 34. \quad 79^\circ 08' 16'' \\ - 59^\circ 24' 35'' \\ \hline \end{array}$$

$$\begin{array}{r} 35. \quad 90^\circ 00' 00'' \\ - 29^\circ 14' 32'' \\ \hline \end{array}$$

$$\begin{array}{r} 36. \quad 180^\circ 00' 00'' \\ - 127^\circ 26' 47'' \\ \hline \end{array}$$