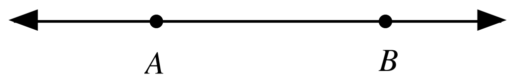
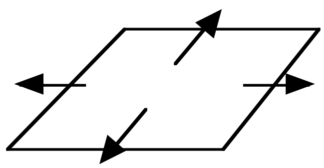
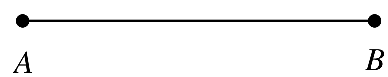
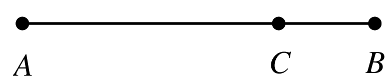
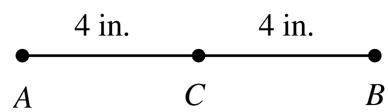
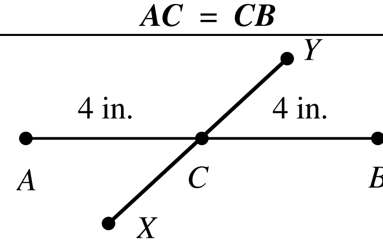
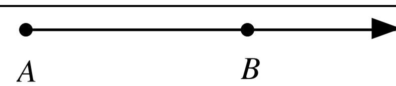
Section 1.1 Basic Definitions of Geometry

UNDEFINED TERMS

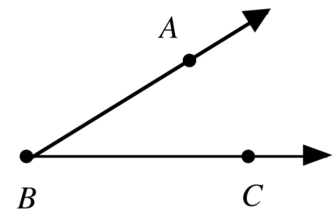
The following terms will remain basically undefined: ***point***, ***line***, ***between***, ***set***, ***plane***. We assume we all share an understanding of what these terms mean, but here are the basic ideas:

A ***point*** is represented by a dot to indicate its location, but it really has no size or dimension.	
A ***line*** means a straight line, not a curved line. A line that passes through points A and B is denoted $\overleftrightarrow{AB}$	
A **plane** is a flat surface, much like a floor or a wall; technically, a plane is infinite in all directions.	
A ***line segment*** has two endpoints and has finite length. We use the notation $\overline{AB}$ (or $\overline{BA}$) to represent the line segment with endpoints A and B.	
Because a line segment has a beginning and an end, it has a definite length. We represent the *length* of $\overline{AB}$ as just ***AB*** (with no bar over it).	 ***AB*** = 8
The point C is ***between*** A and B and if (i) C is on $\overline{AB}$ and (ii) C is not also one of the endpoints.	
The point C is the ***midpoint*** of $\overline{AB}$ if (i) C is between A and B and (ii) new line segments $\overline{AC}$ and $\overline{CB}$ have the same length.	 ***AC*** = ***CB***
A line or line segment that crosses $\overline{AB}$ at its midpoint, C, is called a ***segment bisector***. To *bisect* an object means to divide it into two equally measured parts.	
A ***ray***, or **half-line**, has a starting point and extends indefinitely from that point. A ray that starts at A and passes through B is denoted $\overrightarrow{AB}$	 Note: $\overrightarrow{BA}$ is *not* the same ray as $\overrightarrow{AB}$.

ANGLES

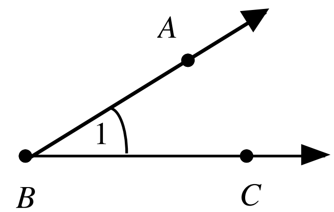
An ***angle***, ABC, which we may denote by $\angle ABC$, is the *union* of two line segments, $\overline{BA}$ and $\overline{BC}$, or two rays, emanating from the same point, B, called the ***vertex***.

When naming an angle, the vertex is always the middle letter.



Sometimes we use a number to represent an angle. For example, we can label this angle as $\angle ABC$ or as $\angle 1$;

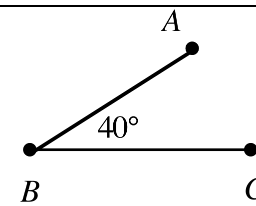
Note: The number used to label an angle, such as 1 or 2, typically has nothing to do with size or measure of the angle; the number is just a label.



Angles are measured in *degrees*. We use $m\angle ABC$ to represent the measure of $\angle ABC$.

In the diagram at right, $m\angle ABC = 40°$.

Note: The degree symbol <u>must</u> be written when the measure of an angle is used.

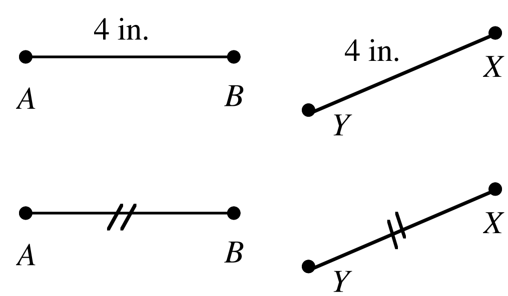


CONGRUENCE

Congruence means *same size*. The symbol for congruence is $\cong$. Two line segments are congruent if they have the same length $\overline{AB} \cong \overline{XY}$.

(We can also write this as $\boldsymbol{AB} = \boldsymbol{XY}$.)

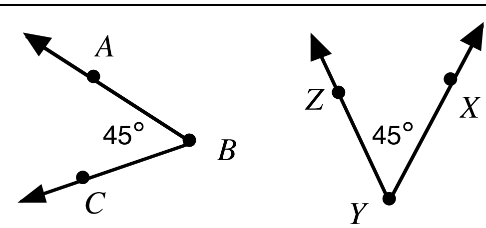
Note: It is common to indicate that two line segments are congruent by placing the same number of *dashes* on them, as shown at right.



Congruent angles have the same *angle measure*.

$\angle ABC \cong \angle XYZ$ and $m\angle ABC = m\angle XYZ$

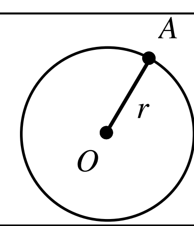
Notice the subtle difference between congruence, $\cong$, and equal, =.
We use $\cong$ between two physical items and = between two measures.

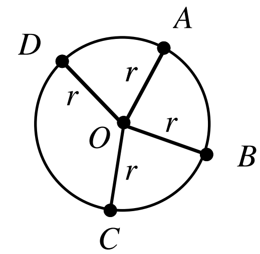
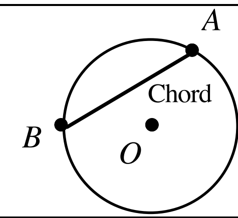
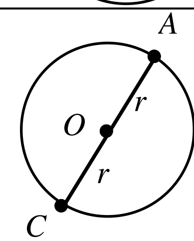


CIRCLES (*Circles are discussed in greater detail in Section 1.4.)*

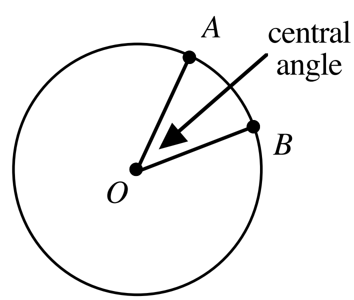
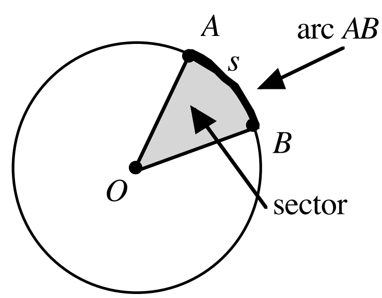
A ***circle*** is the set of all points in a plane that are equidistant (the same distance) from a fixed point, the ***center***, labeled O in the diagram at right.

This equal distance, represented by $\overline{OA}$, is called the ***radius***, and has length r.



There are many radii (pronounced *ray′–dee–eye*, the plural of radius) in the circle, and each one has the center as one of its endpoints. **Note:** "Radius" has two meanings: 1) A line segment from the center of a circle to a point on the circle, and 2) the length of a radius.	
A line segment in which both endpoints are on the circle is called a ***chord*** of the circle.	
A chord that passes through the center, O, is called a ***diameter*** of the circle. In this case, O is the midpoint of $\overline{AC}$. Each diameter has the length of two radii: $\boldsymbol{d = 2r}$.	

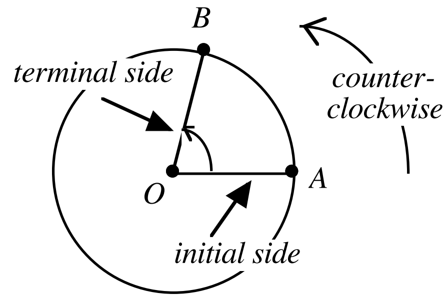
ANGLES IN CIRCLES (*Angles in circles are discussed in greater detail in Section 1.4.)*

An angle with its vertex at the center of a circle is called a ***central angle***. A central angle in a circle is formed using two distinct radii. In the diagram at right, the central angle is $\angle AOB$.	
A central angle creates a ***sector*** of the circle, which is a portion of the region inside the circle. This angle also creates an ***arc*** on the circle. We say that the central angle ***subtends*** the arc it creates; arc AB is sometimes labeled by $\overset{\frown}{AB}$. The measure of $\overset{\frown}{AB}$, labeled by $m\overset{\frown}{AB}$, and is sometimes called $\boldsymbol{s}$: $m\overset{\frown}{AB} = s$.	

It is common to represent a central angle starting with the "right-extended" horizontal radius, called the ***initial side***.

The angle then sweeps counter-clockwise to create the central angle. The radius at which it stops is called the ***terminal side***.

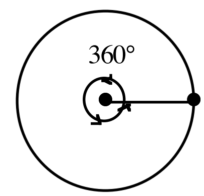
Note: This is discussed in greater detail in Section 1.4.



A full circle has been (arbitrarily) assigned an angular measure value of 360 ***degrees*** (or 360°).

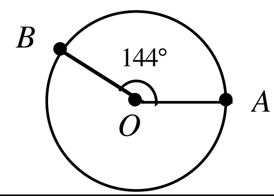
The reason 360 is a good choice is that it is evenly divisible by so many whole numbers: 2, 3, 4, 5, 6, 8, 9, 10, 12 and many more.

In this way we can easily identify the angle measures of these portions of a circle: $\frac{1}{2}$ of a circle, $\frac{1}{3}$ of a circle, $\frac{3}{4}$ of a circle, $\frac{2}{5}$ of a circle, and so on.

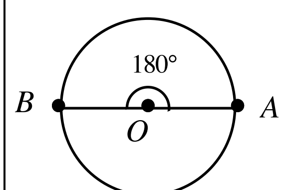


For example, $\frac{2}{5}$ of a circle measures 144° because $\frac{2}{5} \cdot 360^\circ = \frac{720^\circ}{5} = 144^\circ$.

$$m\angle AOB = 144^\circ$$



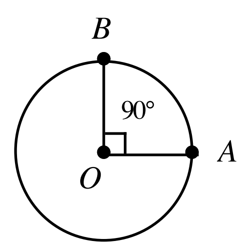
A central angle that subtends one-half of a circle has a measure of 180° (one-half of 360°).

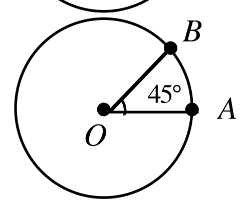


A central angle that subtends one-fourth of a circle has a measure of 90° (one-fourth of 360°).

A 90° angle is also called a ***right angle***.

One-half of a right angle is a 45° angle.

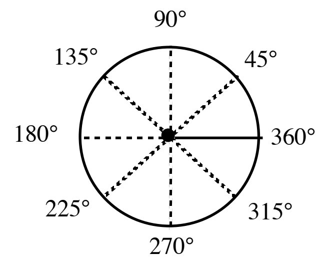




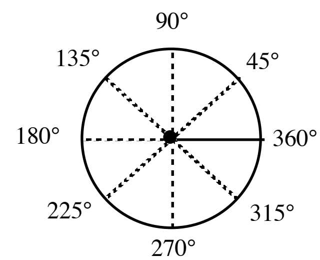
We can label the circle every 45° and use these values to help us estimate the placement of central angles throughout the circle.

For example, a central angle of 144° is between 135° and 180°, and it is closer to 135°.

Take a moment to draw a 144° angle in the circle at right.



Example 1: Draw a central angle in a circle with the given number of degrees.

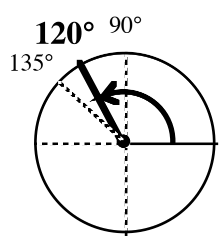


a) 120° b) 255° c) 330°

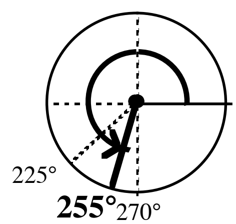
Procedure: Locate the given angle between two of the angle measures shown on the circle at right. Estimate the location, draw a radius and an arc (inside the circle) from initial side.

Answer:

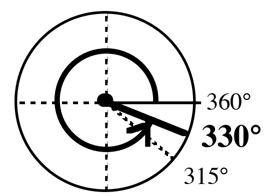
a) 120° is between 90° and 135°, closer to 135°.



b) 255° is between 225° and 270°, closer to 270°.

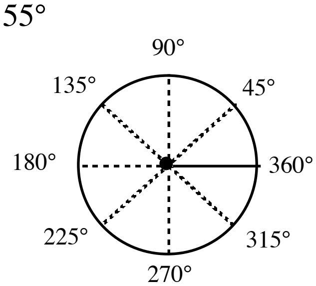


c) 330° is between 315° and 360°, closer to 315°.

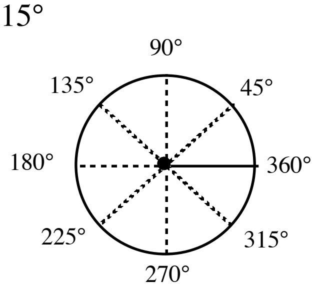


You Try It 1 For each, draw a central angle in a circle with the given number of degrees.

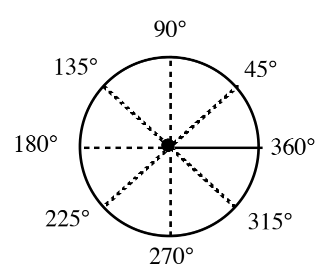
a) 155°



b) 215°



c) 290°

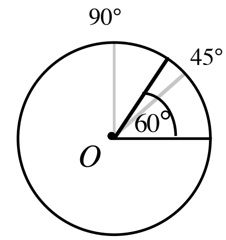


Note: Throughout the textbook, answers to You Try It exercises are found at the end of the section.

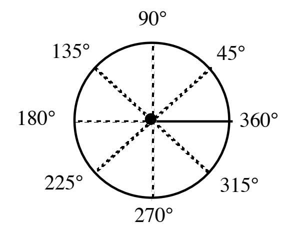
Example 2: Find the measure of the central angle that subtends ***one-sixth*** of a circle. Draw this central angle in a circle.

Procedure: $\frac{1}{6}$ of a circle means we multiply 360° by $\frac{1}{6}$. Then, draw the answer angle in standard form in a circle.

Answer: $\frac{1}{6} \cdot 360° = \frac{1}{6} \cdot \frac{360°}{1} = \frac{1}{1} \cdot \frac{60°}{1} = 60°$ →



You Try It 2 Find the measure of the central angle that subtends ***five-ninths*** of a circle. Draw this central angle in a circle.



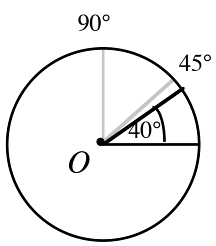
Example 3: Draw a central angle that has the given number of degrees. Also, on each circle, state what portion of a full circle the number of degrees is.

a) 40° b) 150°

Procedure: We can set up a ratio (fraction) comparing the central angle to the entire circle:

Answer:

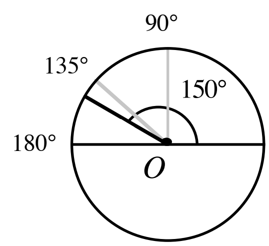
a)



$\frac{40}{360} = \frac{4}{36} = \frac{1}{9}$

So 40° is $\frac{1}{9}$ of a circle.

b)

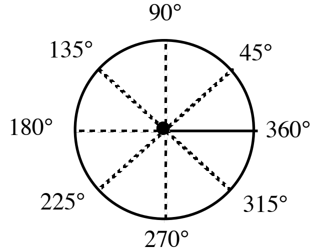


$\frac{150}{360} = \frac{15}{36} = \frac{5}{12}$

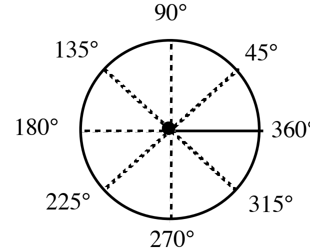
So 150° is $\frac{5}{12}$ of a circle.

You Try It 3 Draw a central angle that has the given number of degrees. Also, on each circle, state what portion of a full circle the number of degrees is.

a) 240°



b) 330°



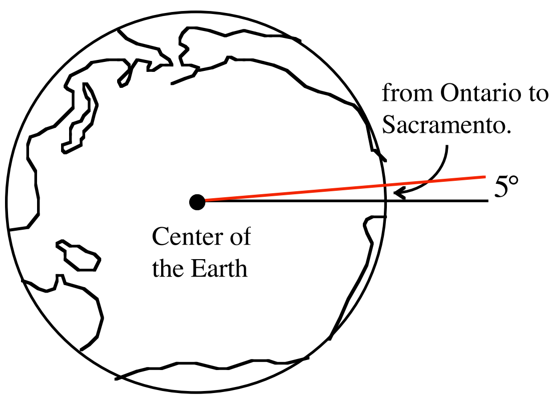
Degrees, Minutes and Seconds

There are 360° in a full circle, so 1° is a small angle. However, "small" is a relative term, and there are times when even a single degree can be a comparatively large angle. For this reason, 1° is broken into 60 smaller units, called **minutes**, and each minute is broken into 60 smaller units, called **seconds**.

When writing the measure of an angle in degrees, minutes and seconds, we use a single "tick" to represent *minutes*, and a double tick to represent *seconds*. For example, we can represent

51 *degrees*, 22 *minutes* and 48 *seconds* as $51^\circ\ 22'\ 48''$.

The circumference of the earth is about 24,900 miles (and the radius is about 3,960 miles).



Earth

The arc subtended by $5^\circ \approx$ 345 miles, the approximate earth distance between Ontario, CA and Sacramento, CA.

$5^\circ \approx 345$ miles means $1^\circ \approx 69$ miles

1 minute (one-sixtieth of a degree) is about 1.15 miles:

$1' \approx 1.15$ miles

1 second (one-sixtieth of a minute) is about 101 feet:

$1'' \approx 101$ feet

Even with the advent of calculators, the fractional portions of a degree—minutes and seconds—are still used in some global positions systems (GPS). For example, the distance between 43° and 44° Longitude (or latitude) is about 69 miles, the earth distance between San Bernardino and Santa Monica.

Here is a look at the relationships between these fractional portions of a degree:

Relationship	Abbreviation	Also ...	
1 degree = 60 minutes	$1^\circ = 60'$	$1' = \frac{1}{60}^\circ$	1 min $= \frac{1}{60}$ of a deg
1 minute = 60 seconds	$1' = 60''$	$1'' = \frac{1}{60}'$ $1'' = \frac{1}{3,600}^\circ$	1 sec $= \frac{1}{60}$ of a min 1 sec $= \frac{1}{3,600}$ of a deg

For example, $17' = \frac{17}{60}^\circ$ and $38'' = \frac{38}{3,600}^\circ$

You Try It 4 Write each number as a fraction of a degree. *Do not simplify the fraction.*

a) $9'$ b) $43'$ c) $27''$ d) $4''$

Angles may be measured in degrees, minutes, and seconds (**DMS**), or they may be measured in degrees and decimal fractions of degrees. Converting from DMS to decimals (and vice-versa) often requires the use of a calculator, and that discussion is presented in Chapter 7 of our textbook.

ADDING AND SUBTRACTING IN DMS

Sometimes we must add or subtract angle measures that are written in DMS. Just as in adding and subtracting like terms in algebra, we add or subtract degrees with degrees, minutes with minutes, and seconds with seconds.

Example 4: Add or subtract as indicated.

a) $58°\ 14'\ 33'' + 72°\ 26'\ 15''$ b) $103°\ 47'\ 52'' - 81°\ 35'\ 44''$

Procedure: It is often easier to align each angle measure vertically before adding or subtracting.

Answer: a)
$$\begin{array}{r} 58°\ 14'\ 33'' \\ +\ 72°\ 26'\ 15'' \\ \hline 130°\ 40'\ 48'' \end{array}$$
b)
$$\begin{array}{r} 103°\ 47'\ 52'' \\ -\ 81°\ 35'\ 44'' \\ \hline 22°\ 12'\ 08'' \end{array}$$

Notice, in Example 4a), we are able to add the minutes and seconds without concern for either of them being 60 or more.

However, if we are to add, for example, 50 minutes to 17 minutes, we get 67 minutes. There is nothing wrong with 67 minutes, especially in measuring time. However, just as 67 minutes on a clock can be rewritten as 1 hour and 7 minutes, so, too, can 67 minutes of an angle be rewritten:

67 minutes = 1 degree and 07 minutes: $67' = 1°\ 07'$

Also, a) 85 minutes = 1 degree and 25 minutes: $85' = 1°\ 25'$

b) 104 seconds = 1 minute and 44 seconds: $104'' = 1'\ 44''$

Example 5: Rewrite each angle measure so that there are no more than 59 minutes and 59 seconds.

a) $14° \ 82' \ 95''$ b) $75° \ 59' \ 108''$ c) $103° \ 48' \ 62''$

Procedure: Start by adjusting the number of seconds, if necessary. If the number of seconds is 60 or more, then (i) subtract 60 from the number of seconds, (ii) add 1 (one) to the number of minutes, and (iii) write the remainder as the new number of seconds. Next, adjust the number of minutes, if necessary.

$$\begin{array}{rrr} 14° & 82' & 95'' \\ & +1' & -60'' \\ \hline 14° & 83' & 35'' \\ +1° & -60' & \\ \hline 15° & 23' & 35'' \end{array}$$

Answer:

	Given angle measure	**with adjusted seconds**	**with adjusted minutes**
a)	$14° \ 82' \ \mathbf{95''}$	$= 14° \ \mathbf{83'} \ 35''$	$= 15° \ 23' \ 35''$
b)	$75° \ 59' \ \mathbf{108''}$	$= 75° \ \mathbf{60'} \ 48''$	$= 76° \ 00' \ 48''$
c)	$103° \ 48' \ \mathbf{62''}$	$= 103° \ 49' \ 02''$	(no change)

Referring back to Example 4b), $103° \ 47' \ 52'' - 81° \ 35' \ 44''$, we are able to subtract minutes and seconds directly (without concern for either of them being negative) because in each case the top number is larger than the bottom number.

To prevent us from getting a negative value for either the minutes or seconds, it may be necessary to adjust the top number by a technique some call "borrowing" from one measure to another. (It might be more appropriately called "donating" because the first number never expects to be "repaid.")

Consider, for example, $13° \ 10' - 8° \ 25'$. Setting this up as a vertical subtraction, we see that we cannot subtract the minutes directly:

$$\begin{array}{rr} 13° & \mathbf{10'} \\ -\ 8° & \mathbf{25'} \\ \hline \end{array}$$

Instead, we must *borrow* 1° from 13° (or have 13° *donate* 1°) and add it—in the form of $60'$—to the $10'$ already there. This means that $13° \ 10' = (13 - 1)° + (60 + 10)' = 12° \ 70'$, and now we can subtract directly:

$$\begin{array}{rr} 13° & \mathbf{10'} \\ -\ 8° & \mathbf{25'} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{rr} 12° & \mathbf{70'} \\ -\ 8° & \mathbf{25'} \\ \hline 4° & 45' \end{array}$$

Example 6: Add or subtract as indicated.

a) $58° \, 44' \, 33'' + 25° \, 51' \, 35''$ b) $91° \, 20' \, 38'' - 65° \, 35' \, 44''$

Answer: Align each angle measure vertically. Adjust the answer as necessary.

a)
$$\begin{array}{r} 58° \; 44' \; 33'' \\ + \; 25° \; 51' \; 35'' \\ \hline 83° \; 95' \; 68'' \end{array}$$

Adjust the seconds: $\Rightarrow 83° \; 96' \; 08''$ *Adjust the minutes:* $\Rightarrow 84° \; 36' \; 08''$

b) We must adjust the top number so that both the minutes and seconds are greater than the minutes and seconds in the bottom number.

Make 1′ into 60″
$$\begin{array}{r} 91° \; 20' \; 38'' \\ - \; 65° \; 35' \; 44'' \\ \hline \end{array}$$

$\Rightarrow$ *Make 1° into 60′*
$$\begin{array}{r} 91° \; 19' \; 98'' \\ - \; 65° \; 35' \; 44'' \\ \hline \end{array}$$

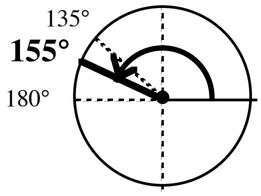
$\Rightarrow$ *Now subtract:*
$$\begin{array}{r} 90° \; 79' \; 98'' \\ - \; 65° \; 35' \; 44'' \\ \hline 25° \; 44' \; 54'' \end{array}$$

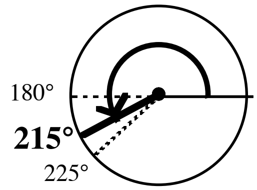
You Try It 5 Add or subtract as indicated.

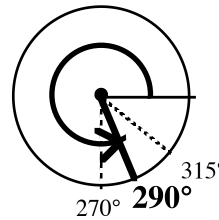
a) $31° \, 29' \, 42'' + 55° \, 48' \, 23''$ b) $67° \, 01' \, 19'' - 21° \, 28' \, 53''$

Section 1.1 You Try It Answers

YTI 1:

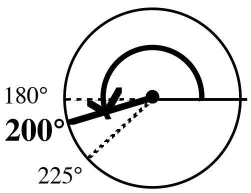
a) 

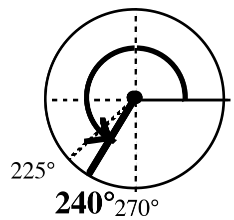
b) 

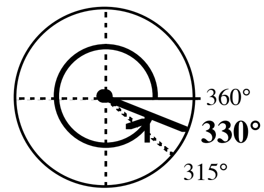
c) 

YTI 2: 200°.

YTI 3: a) 240° is $\frac{2}{3}$ of a circle. b) 330° is $\frac{11}{12}$ of a circle.







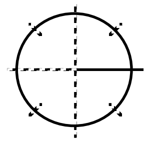
YTI 4: a) $\frac{9}{60}{}^{\circ}$ b) $\frac{43}{60}{}^{\circ}$ c) $\frac{27}{3{,}600}{}^{\circ}$ d) $\frac{4}{3{,}600}{}^{\circ}$

YTI 5: a) 87° 18′ 05″ b) 45° 32′ 26″

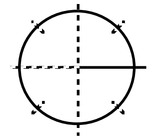
Section 1.1 Focus Exercises

What degree measure represents the given portion of a circle? Draw a central angle that has that same number of degrees. (Refer to Examples 1 and 2.)

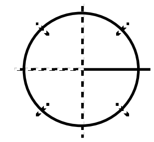
1. $\frac{1}{4}$ of a circle



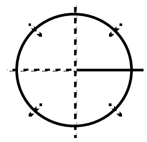
2. $\frac{1}{12}$ of a circle



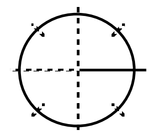
3. $\frac{1}{6}$ of a circle



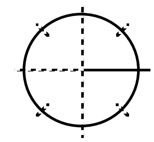
4. $\frac{1}{3}$ of a circle



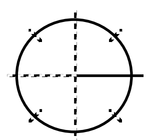
5. $\frac{3}{10}$ of a circle



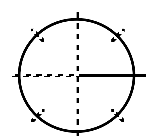
6. $\frac{2}{5}$ of a circle



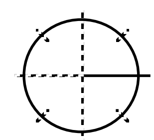
7. $\frac{4}{9}$ of a circle



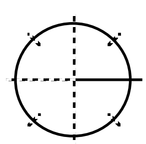
8. $\frac{3}{8}$ of a circle



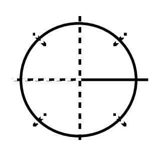
9. $\frac{4}{15}$ of a circle



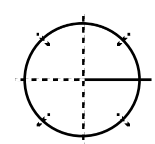
10. $\frac{3}{20}$ of a circle



11. $\frac{11}{30}$ of a circle

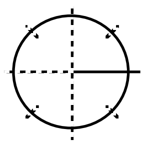


12. $\frac{11}{60}$ of a circle

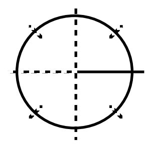


Draw a central angle that has the given number of degrees. Also, on each circle, state what portion of a full circle the number of degrees is. (Refer to Examples 1 and 3.)

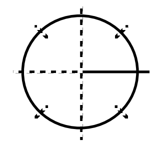
13. 30°



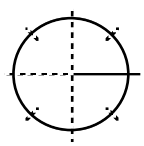
14. 72°



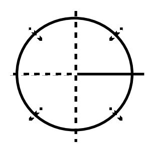
15. 45°



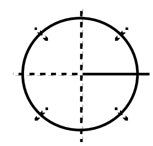
16. 18°



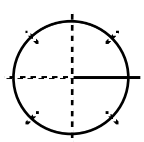
17. 120°



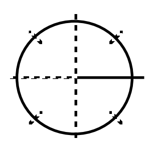
18. 135°



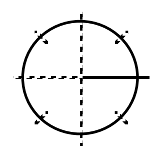
19. 180°



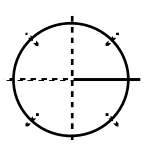
20. 80°



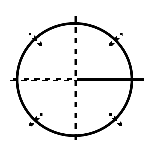
21. 75°



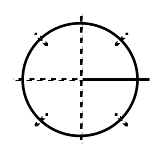
22. 160°



23. 225°



24. 100°



Add or subtract as indicated. Adjust the answer as necessary. (Refer to Examples 4, 5, and 6.)

25. $\begin{array}{r} 19° \; 37' \; 21'' \\ +\; 38° \; 16' \; 34'' \\ \hline \end{array}$

26. $\begin{array}{r} 91° \; 09' \; 26'' \\ +\; 62° \; 42' \; 44'' \\ \hline \end{array}$

27. $\begin{array}{r} 110° \; 44' \; 35'' \\ +\; 42° \; 23' \; 18'' \\ \hline \end{array}$

28. $\begin{array}{r} 60° \; 24' \; 25'' \\ +\; 82° \; 49' \; 56'' \\ \hline \end{array}$

29. $\begin{array}{r} 58° \; 27' \; 19'' \\ +\; 31° \; 32' \; 41'' \\ \hline \end{array}$

30. $\begin{array}{r} 124° \; 42' \; 51'' \\ +\; 55° \; 17' \; 09'' \\ \hline \end{array}$

31. $\begin{array}{r} 78° \; 52' \; 19'' \\ -\; 45° \; 37' \; 06'' \\ \hline \end{array}$

32. $\begin{array}{r} 105° \; 17' \; 43'' \\ -\; 48° \; 43' \; 15'' \\ \hline \end{array}$

33. $\begin{array}{r} 60° \; 33' \; 06'' \\ -\; 48° \; 50' \; 12'' \\ \hline \end{array}$

34. $\begin{array}{r} 79° \; 08' \; 16'' \\ -\; 59° \; 24' \; 35'' \\ \hline \end{array}$

35. $\begin{array}{r} 90° \; 00' \; 00'' \\ -\; 29° \; 14' \; 32'' \\ \hline \end{array}$

36. $\begin{array}{r} 180° \; 00' \; 00'' \\ -\; 127° \; 26' \; 47'' \\ \hline \end{array}$