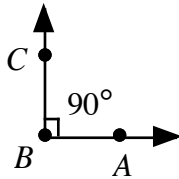


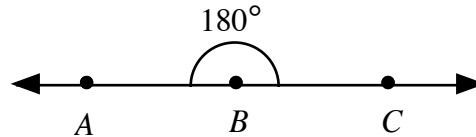
## Section 1.2 Angles and Angle Measure

### CLASSIFICATION OF ANGLES

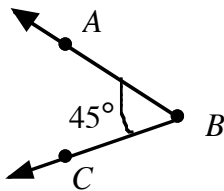
1. **Right angles** are angles which measure  $90^\circ$ .



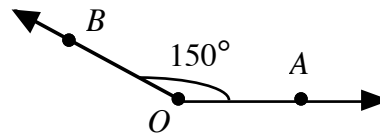
2. **Straight angles** are angles which measure  $180^\circ$ . Every line forms a straight angle.



3. Acute angles are angles which have a measure between  $0^\circ$  and  $90^\circ$ .



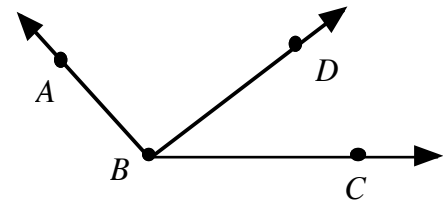
4. Obtuse angles are angles which measure greater than  $90^\circ$  and less than  $180^\circ$ .



### ADJACENT ANGLES

5. **Adjacent angles** are any two angles that share a common vertex and a common side, forming an even larger angle; the shared side must be in the *interior* of the larger angle.

For example, all of the points on the ray  $BD$  (except the endpoint,  $B$ ) are in the *interior* of the larger angle  $\angle ABC$ . In this case, we can say that  $\angle ABD$  and  $\angle DBC$  are adjacent angles. Together, they form the larger angle,  $\angle ABC$ .



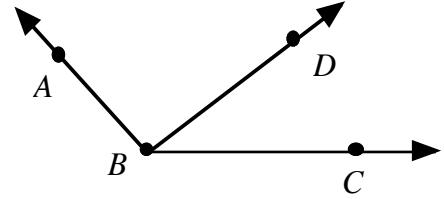
In fact, the measure of the larger angle is the sum of the measures of the two smaller angles:

$$m \angle ABC = m \angle ABD + m \angle DBC.$$

Likewise, the measure of one of the smaller angles is the difference between the measure of the largest angle and the measure of the other smaller angle:

$$m \angle DBC = m \angle ABC - m \angle ABD.$$

**Example 1:** Given the diagram below, find  $m\angle ABC$  given the measures of  $\angle ABD$  and  $\angle DBC$ .



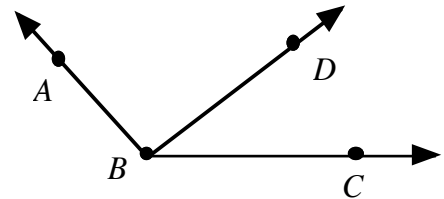
- a)  $m\angle ABD = 80^\circ$  and  $m\angle DBC = 45^\circ$   
 b)  $m\angle ABD = 85^\circ 39' 51''$  and  
 $m\angle DBC = 51^\circ 24' 26''$

**Answer:** Use  $m\angle ABC = m\angle ABD + m\angle DBC$

- a)  $m\angle ABC = 80^\circ + 45^\circ = 125^\circ$   
 b)  $m\angle ABC = 85^\circ 39' 51'' + 51^\circ 24' 26''$

$$\begin{array}{r} 85^\circ 39' 51'' \\ + 51^\circ 24' 26'' \\ \hline 136^\circ 63' 77'' \end{array} \quad \begin{array}{l} \text{Adjust the seconds, then the minutes, to where they} \\ \text{are both less than 60:} \end{array} \quad \begin{array}{l} \Rightarrow 136^\circ 64' 17'' \\ \Rightarrow 137^\circ 04' 17'' \end{array}$$

**Example 2:** Given the diagram below, find  $m\angle DBC$  given the measures of  $\angle ABC$  and  $\angle ABD$ .



- a)  $m\angle ABC = 140^\circ$  and  $m\angle ABD = 85^\circ$   
 b)  $m\angle ABC = 128^\circ 39' 12''$  and  
 $m\angle ABD = 53^\circ 45' 31''$

**Answer:** Use  $m\angle DBC = m\angle ABC - m\angle ABD$

- a)  $m\angle DBC = 140^\circ - 85^\circ = 55^\circ$   
 b)  $m\angle DBC = m\angle ABC - m\angle ABD$

$$m\angle DBC = 128^\circ 39' 12'' - 53^\circ 45' 31''$$

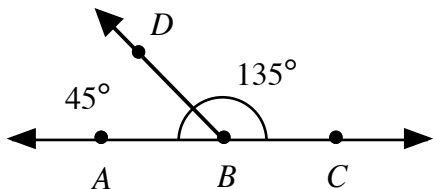
Notice that the minutes and seconds change as we borrow:

$$\begin{array}{r} \text{Make } 1' \text{ into } 60'' \\ 128^\circ 39' 12'' \\ - 53^\circ 45' 31'' \\ \hline \end{array} \quad \begin{array}{l} \Rightarrow \\ \text{Make } 1^\circ \text{ into } 60' \\ 128^\circ 38' 72'' \\ - 53^\circ 45' 31'' \\ \hline \end{array} \quad \begin{array}{l} \Rightarrow \\ \text{Now subtract} \\ 127^\circ 98' 72'' \\ - 53^\circ 45' 31'' \\ \hline \end{array}$$

$$m\angle DBC = 74^\circ 53' 41''$$

**SUPPLEMENTARY AND COMPLEMENTARY ANGLES**

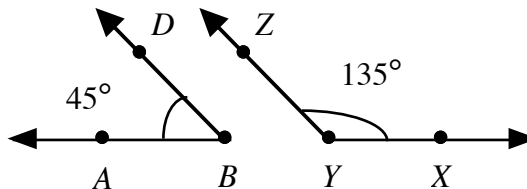
6. **Supplementary angles** are any two angles with measures that add to  $180^\circ$ . *Supplementary angles* can be adjacent, but they don't have to be.



Because  $45^\circ + 135^\circ = 180^\circ$ , we say that  $\angle ABD$  and  $\angle DBC$  are *supplementary*. We also say that  $\angle DBC$  is the *supplement* of  $\angle ABD$ , and vice-versa.

$$m \angle ABD + m \angle DBC = 180^\circ$$

and  $m \angle DBC = 180^\circ - m \angle ABD$ .



$\angle XYZ$  is the supplement of  $\angle DBA$ .

$$m \angle ABD + m \angle XYZ = 180^\circ$$

and  $m \angle XYZ = 180^\circ - m \angle ABD$ .

**Example 3:**  $\angle JFK$  and  $\angle RHP$  are supplementary angles. Given  $m \angle JFK$ , find  $m \angle RHP$ .

a)  $m \angle JFK = 140^\circ$

b)  $m \angle JFK = 54^\circ 28' 15''$

**Answer:** Use  $m \angle RHP = 180^\circ - m \angle JFK$

a)  $m \angle RHP = 180^\circ - 140^\circ = 40^\circ$

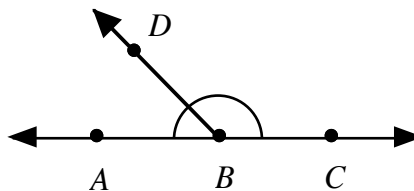
b)  $m \angle RHP = 180^\circ - 54^\circ 28' 15''$

$$\begin{array}{r}
 \text{Make } 1^\circ \text{ into } 60' \\
 180^\circ 00' 00'' \\
 - 54^\circ 28' 15'' \\
 \hline
 \end{array}
 \Rightarrow
 \begin{array}{r}
 \text{Make } 1' \text{ into } 60'' \\
 179^\circ 60' 00'' \\
 - 54^\circ 28' 15'' \\
 \hline
 \end{array}
 \Rightarrow
 \begin{array}{r}
 \text{Now subtract} \\
 179^\circ 59' 60'' \\
 - 54^\circ 28' 15'' \\
 \hline
 \end{array}$$

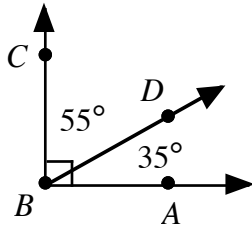
$m \angle RHP = 125^\circ 31' 45''$

Angles that are both adjacent and supplementary form a straight angle, a line:

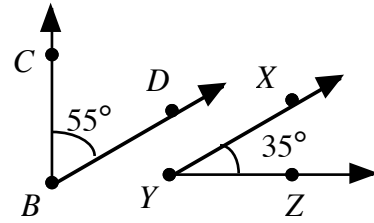
$$m \angle ABC = 180^\circ$$



7. **Complementary angles** are any two angles with measures that add to  $90^\circ$ . *Complementary angles* can be adjacent, but they don't have to be.



We can say that  $\angle ABD$  and  $\angle DBC$  are *complementary*, or that  $\angle DBC$  is the *complement* of  $\angle ABD$ .



Likewise,  $\angle XYZ$  is the complement of  $\angle DBC$ .

**Example 4:**  $\angle EFG$  and  $\angle MPQ$  are complementary angles. Given  $m\angle EFG$ , find  $m\angle MPQ$

a)  $m\angle EFG = 53^\circ$

b)  $m\angle EFG = 54^\circ 28' 15''$

**Answer:** Use  $m\angle MPQ = 90^\circ - m\angle EFG$

a)  $m\angle MPQ = 90^\circ - 53^\circ = 37^\circ$

b)  $m\angle MPQ = 90^\circ - 54^\circ 28' 15''$

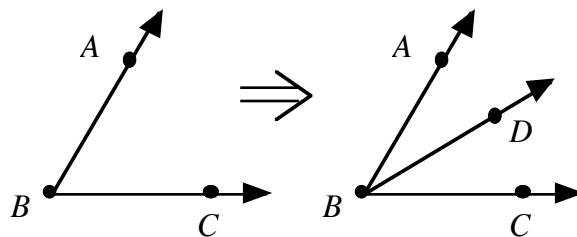
$$\begin{array}{r}
 \text{Make } 1^\circ \text{ into } 60' \\
 90^\circ 00' 00'' \\
 - 54^\circ 28' 15'' \\
 \hline
 \end{array}
 \Rightarrow
 \begin{array}{r}
 \text{Make } 1' \text{ into } 60'' \\
 89^\circ 60' 00'' \\
 - 54^\circ 28' 15'' \\
 \hline
 \end{array}
 \Rightarrow
 \begin{array}{r}
 \text{Now subtract} \\
 89^\circ 59' 60'' \\
 - 54^\circ 28' 15'' \\
 \hline
 \end{array}$$

$m\angle MPQ = 35^\circ 31' 45''$

**ANGLE BISECTORS**

An *angle bisector* splits an angle into two smaller, *congruent* angles.

Said more formally, an *angle bisector* is a ray that begins at the angle's vertex and passes through an interior point so that it forms two smaller, congruent angles.



**Example 5:** Ray  $BD$  bisects  $\angle ABC$ . Given the measure of  $\angle ABC$ , find the measure of  $\angle DBC$ . Write answers in DMS form.

- a)  $m\angle ABC = 48^\circ$
- b)  $m\angle ABC = 53^\circ$
- c)  $m\angle ABC = 65^\circ 43' 10''$
- d)  $m\angle ABC = 72^\circ 28' 31''$

**Answer:** Use  $m\angle DBC = m\angle ABC \div 2$

- a)  $m\angle DBC = 48^\circ \div 2 = 24^\circ$
- b)  $m\angle DBC = 53^\circ \div 2$ . 53 is an odd number, but we can make it even by borrowing  $1^\circ$  from it and making  $m\angle ABC = 52^\circ 60'$ . These are both divisible by 2:

$$m\angle DBC = 53^\circ \div 2 = (52^\circ 60') \div 2 = 26^\circ 30'$$

- c)  $m\angle DBC = 65^\circ 43' 10'' \div 2$ . Both 65 and 43 are odd numbers, but we can make them even by borrowing 1 degree (or minute) from each to make them both even:

$$65^\circ 43' 10'' = 64^\circ 103' 10'' = 64^\circ 102' 70''$$

We can now easily divide each of these by 2:

$$m\angle DBC = (64^\circ 102' 70'') \div 2 = 32^\circ 51' 35''.$$

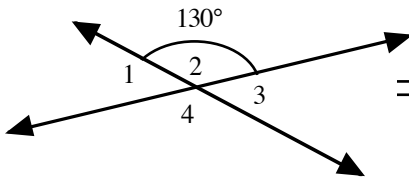
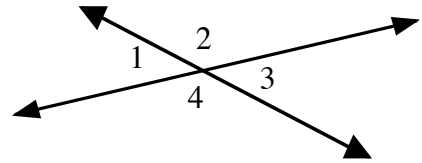
- d) The number of seconds is odd, 31, and there is nothing that will make it even. Because the other values are already even, we can divide as follows:

$$m\angle DBC = (72^\circ 28' 31'') \div 2 = 36^\circ 14' 15.5''.$$

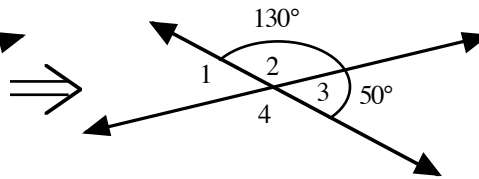
**VERTICAL ANGLES**

When two lines intersect in a plane, several pairs of angles are formed:

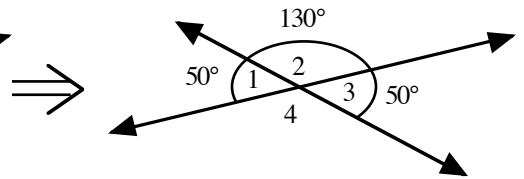
- Adjacent angles are supplementary (forming a straight line) and
- non-adjacent angles are *vertical* angles. It can easily be shown that **vertical angles are congruent to each other**:



Given:  $m\angle 2 = 130^\circ$



$\angle 3$  is supplementary to  $\angle 2$  so  
 $m\angle 3 = 180^\circ - 130^\circ = 50^\circ$ .



Likewise,  $\angle 1$  is supplementary to  $\angle 2$ , so  $m\angle 1$  is also  $50^\circ$ .

Therefore,  $m\angle 1 = m\angle 3$ , and  $\angle 1 \cong \angle 3$ .

It follows that  $m\angle 4$  is also  $130^\circ$ .  $\angle 4$  is vertical to  $\angle 2$ , and  $\angle 4 \cong \angle 2$ .

**Example 6:** In the diagram of intersecting lines, given  $m\angle 1 = 27^\circ$ , find the measures of the other three angles.

**Answer:**  $m\angle 3 = m\angle 1$  and  $\angle 2$  and  $\angle 4$  are both supplementary to  $\angle 1$ , so,

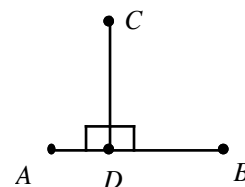
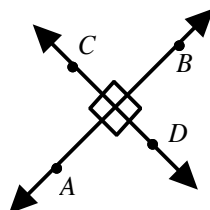
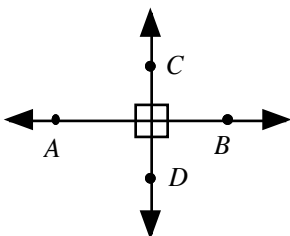
a)  $m\angle 3 = 27^\circ$

b)  $m\angle 2 = 180^\circ - 27^\circ = 153^\circ$

c)  $m\angle 4 = 153^\circ$ .

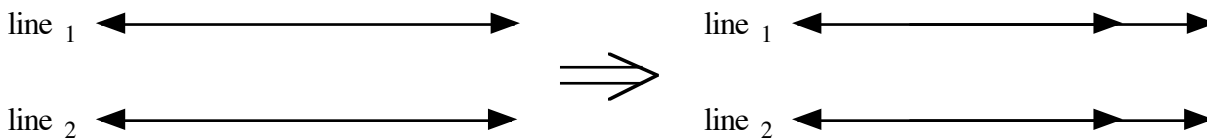
**PERPENDICULAR LINES AND LINE SEGMENTS**

Two lines in a plane that intersect to form four right angles are said to be *perpendicular*. If two lines or line segments,  $\overline{AB}$  and  $\overline{CD}$ , are perpendicular, then we may write  $\overline{AB} \perp \overline{CD}$ .



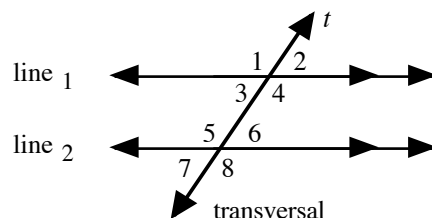
**PARALLEL LINES AND TRANSVERSALS**

Two lines in a plane, line<sub>1</sub> and line<sub>2</sub>, are *parallel* if they never intersect. We sometimes use arrows going in the same direction to indicate that two lines (or line segments) are parallel to each other.



A line that intersects two (or more) parallel lines is called a *transversal*. The transversal and the parallel lines form a total of eight angles.

There are many pairs of congruent angles and many pairs of supplementary angles.

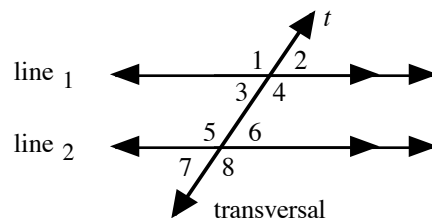


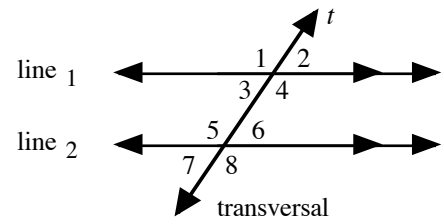
If the transversal is *not* perpendicular to the parallel lines, then four acute angles and four obtuse angles formed. This leads us to the following:

- (i) all of the acute angles are congruent to each other and all of the obtuse angles are congruent to each other;
- (ii) each acute angle is supplementary to each obtuse angle.

To talk about these eight angles easily, we refer to them in this way: (On the blanks provided, list the angles that fit the description.)

- (i) *left* side of transversal: 1, 3, 5, and \_\_\_\_\_
- (ii) *right* side of transversal: \_\_\_\_\_
- (iii) *directly above a parallel line*: 1, 2, 5, and \_\_\_\_\_
- (iv) *directly below a parallel line*: \_\_\_\_\_
- (v) *interior angles*: 3, 4, 5, and \_\_\_\_\_
- (vi) *exterior angles*: \_\_\_\_\_





In this setting, a pair of **corresponding angles** are angles that are on the same side of the transversal (left or right) and are either both above or both below the parallel lines.

For example,  $\angle 1$  and  $\angle 5$  are a pair of corresponding angles because they are both to the *above-left* angles.

Corresponding angles are congruent.

**Alternate interior angles** are on opposite sides of the transversal (one on the left, one on the right) and are “between” the parallel lines: below line<sub>1</sub> and above line<sub>2</sub>.

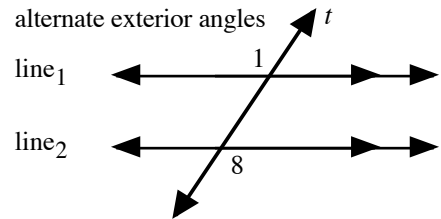
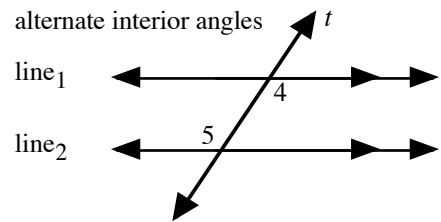
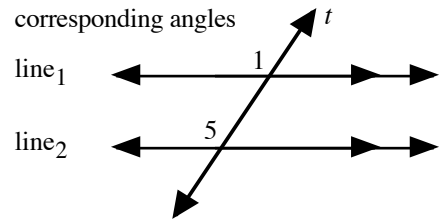
For example,  $\angle 4$  and  $\angle 5$  are alternate interior angles.

Alternate interior angles are congruent.

**Alternate exterior angles** are on opposite sides of the transversal (one on the left, one on the right) and are outside of the parallel lines: above line<sub>1</sub> and below line<sub>2</sub>.

For example,  $\angle 1$  and  $\angle 8$  are alternate exterior angles.

Alternate exterior angles are congruent.



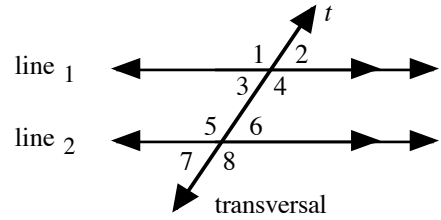
As we already know, all of the vertical angles are congruent; for example  $\angle 1 \cong \angle 4$  and  $\angle 6 \cong \angle 7$ .

Also, all of the adjacent angles form lines (straight angles), so each pair of adjacent angles is supplementary.



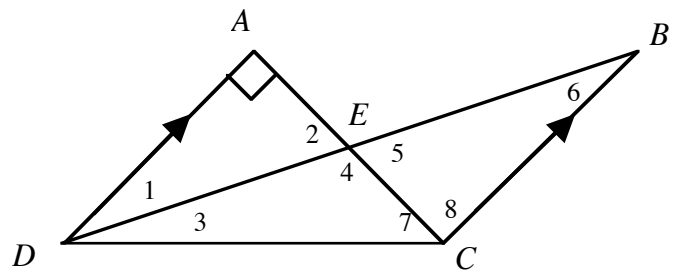
## Section 1.2 Focus Exercises

1. For each pair of angles, determine if they have a special relationship (such as “alternate interior”) and whether or not they are congruent.



- |                              |                              |                              |
|------------------------------|------------------------------|------------------------------|
| a) $\angle 1$ and $\angle 5$ | b) $\angle 1$ and $\angle 8$ |                              |
| c) $\angle 1$ and $\angle 4$ | d) $\angle 1$ and $\angle 2$ |                              |
| e) $\angle 2$ and $\angle 6$ | f) $\angle 6$ and $\angle 3$ | g) $\angle 2$ and $\angle 7$ |

2. Given the following diagram ( $\overline{AC}$  and  $\overline{BD}$  are line segments), find pairs of angles that are guaranteed to be:



*\* Hint: you might need to use letters to label an angle, such as  $\angle AEB$  or  $\angle ADC$ .*

- a) congruent\*
- b) supplementary\*

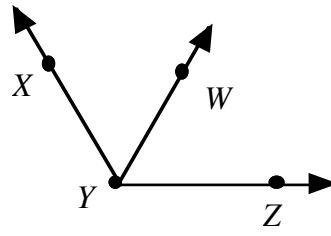
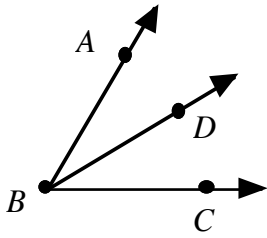
3.  $\angle ABC$  and  $\angle XYZ$  are complementary angles. Given the measure of  $\angle ABC$ , find  $m\angle XYZ$ .

- |                                 |                                      |
|---------------------------------|--------------------------------------|
| a) $m\angle ABC = 60^\circ$     | b) $m\angle ABC = 35.8^\circ$        |
| c) $m\angle ABC = 33^\circ 47'$ | d) $m\angle ABC = 82^\circ 15' 36''$ |

4.  $\angle ABC$  and  $\angle XYZ$  are supplementary angles. Given the measure of  $\angle ABC$ , find  $m\angle XYZ$ .

- |                                 |                                       |
|---------------------------------|---------------------------------------|
| a) $m\angle ABC = 45^\circ$     | b) $m\angle ABC = 161.3^\circ$        |
| c) $m\angle ABC = 36^\circ 15'$ | d) $m\angle ABC = 102^\circ 44' 08''$ |

For Exercise #5 and #6, use these diagrams of the acute and obtuse angles,  $\angle ABC$  and  $\angle XYZ$ , and angle bisectors ray  $BD$  and ray  $YW$ .



5. Given either  $m\angle ABC$  or  $m\angle XYZ$ , you are to find, respectively, either  $m\angle ABD$  or  $m\angle XYW$ .

a)  $m\angle ABC = 56^\circ$

b)  $m\angle XYZ = 148^\circ$

c)  $m\angle ABC = 61.9^\circ$

d)  $m\angle XYZ = 133.2^\circ$

e)  $m\angle ABC = 48^\circ 32' 18''$

f)  $m\angle XYZ = 156^\circ 06' 42''$

g)  $m\angle ABC = 37^\circ 51' 02''$

h)  $m\angle XYZ = 149^\circ 17' 50''$

6. Given either  $m\angle ABD$  or  $m\angle XYW$ , you are to find, respectively, either  $m\angle ABC$  or  $m\angle XYZ$ .

a)  $m\angle ABD = 27^\circ$

b)  $m\angle XYW = 67.8^\circ$

c)  $m\angle ABD = 22^\circ 15'$

d)  $m\angle XYW = 77^\circ 05' 18''$

e)  $m\angle ABD = 29^\circ 36' 51''$

f)  $m\angle XYW = 71^\circ 55' 42''$