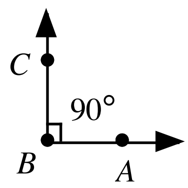
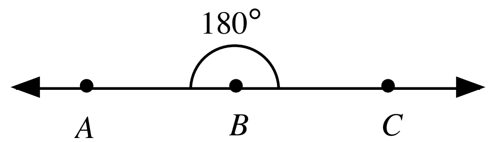
Section 1.2 Angles and Angle Measure

CLASSIFICATION OF ANGLES

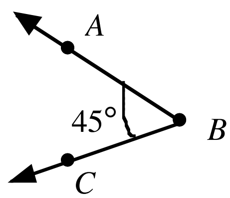
1. ***Right angles*** are angles which measure 90°.



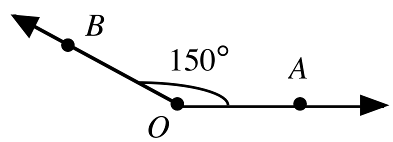
2. ***Straight angles*** are angles which measure 180°. Every line forms a straight angle.



3. Acute angles are angles which have a measure between 0° and 90°.

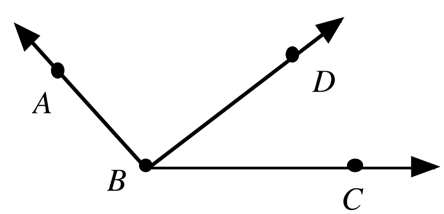


4. Obtuse angles are angles which measure greater than 90° and less than 180°.



ADJACENT ANGLES

5. ***Adjacent angles*** are any two angles that share a common vertex and a common side, forming an even larger angle; the shared side must be in the ***interior*** of the larger angle.



For example, all of the points on the ray *BD* (except the endpoint, *B*) are in the *interior* of the larger angle $\angle ABC$. In this case, we can say that $\angle ABD$ and $\angle DBC$ are adjacent angles. Together, they form the larger angle, $\angle ABC$.

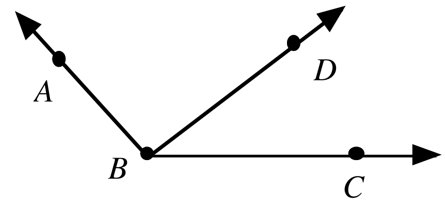
In fact, the measure of the larger angle is the sum of the measures of the two smaller angles:

$$m\angle ABC = m\angle ABD + m\angle DBC.$$

Likewise, the measure of one of the smaller angles is the difference between the measure of the largest angle and the measure of the other smaller angle:

$$m\angle DBC = m\angle ABC - m\angle ABD.$$

Example 1: Given the diagram below, find $m\angle ABC$ given the measures of $\angle ABD$ and $\angle DBC$.



a) $m\angle ABD = 80°$ and $m\angle DBC = 45°$

b) $m\angle ABD = 85°\ 39'\ 51''$ and $m\angle DBC = 51°\ 24'\ 26''$

Answer: Use $m\angle ABC = m\angle ABD + m\angle DBC$

a) $m\angle ABC = 80° + 45° = 125°$

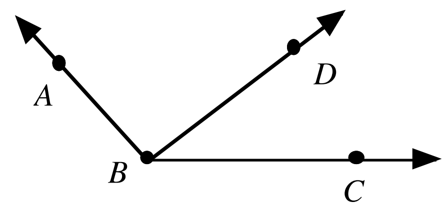
b) $m\angle ABC = 85°\ 39'\ 51'' + 51°\ 24'\ 26''$

$$\begin{array}{r} 85°\ 39'\ 51'' \\ +\ 51°\ 24'\ 26'' \\ \hline 136°\ 63'\ 77'' \end{array}$$

Adjust the seconds, then the minutes, to where they are both less than 60:

$136°\ 63'\ 77'' \Rightarrow 136°\ 64'\ 17'' \Rightarrow 137°\ 04'\ 17''$

Example 2: Given the diagram below, find $m\angle DBC$ given the measures of $\angle ABC$ and $\angle ABD$.



a) $m\angle ABC = 140°$ and $m\angle ABD = 85°$

b) $m\angle ABC = 128°\ 39'\ 12''$ and $m\angle ABD = 53°\ 45'\ 31''$

Answer: Use $m\angle DBC = m\angle ABC - m\angle ABD$

a) $m\angle DBC = 140° - 85° = 55°$

b) $m\angle DBC = m\angle ABC - m\angle ABD$

$m\angle DBC = 128°\ 39'\ 12'' - 53°\ 45'\ 31''$

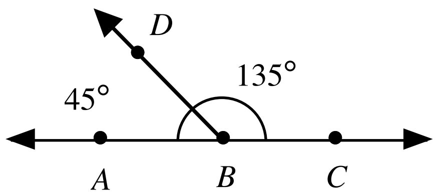
Notice that the minutes and seconds change as we borrow:

Make 1′ into 60″		*Make 1° into 60′*		*Now subtract*
$\begin{array}{r} 128°\ 39'\ 12'' \\ -\ 53°\ 45'\ 31'' \\ \hline \end{array}$	$\Rightarrow$	$\begin{array}{r} 128°\ 38'\ 72'' \\ -\ 53°\ 45'\ 31'' \\ \hline \end{array}$	$\Rightarrow$	$\begin{array}{r} 127°\ 98'\ 72'' \\ -\ 53°\ 45'\ 31'' \\ \hline \end{array}$

$\boldsymbol{m\angle DBC =}\ 74°\ 53'\ 41''$

SUPPLEMENTARY AND COMPLEMENTARY ANGLES

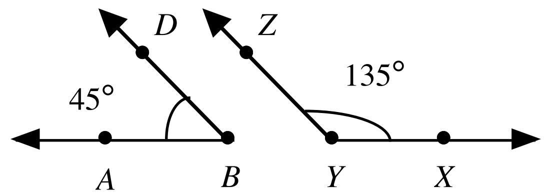
6. ***Supplementary angles*** are any two angles with measures that add to 180°. *Supplementary angles* can be adjacent, but they don't have to be.



Because 45° + 135° = 180°, we say that $\angle ABD$ and $\angle DBC$ are *supplementary*. We also say that $\angle DBC$ *is the supplement of* $\angle ABD$, and vice-versa.

$$m\angle ABD + m\angle DBC = 180°$$

and $$m\angle DBC = 180° - m\angle ABD.$$



$\angle XYZ$ is the supplement of $\angle$DBA.

$$m\angle ABD + m\angle XYZ = 180°$$

and $$m\angle XYZ = 180° - m\angle ABD.$$

Example 3: $\angle JFK$ and $\angle RHP$ are supplementary angles. Given $m\angle JFK$, find $m\angle RHP$.

a) $m\angle JFK = 140°$ b) $m\angle JFK = 54°\ 28'\ 15''$

Answer: Use $m\angle RHP = 180° - m\angle JFK$

a) $m\angle RHP = 180° - 140° = 40°$

b) $m\angle RHP = 180° - 54°\ 28'\ 15''$

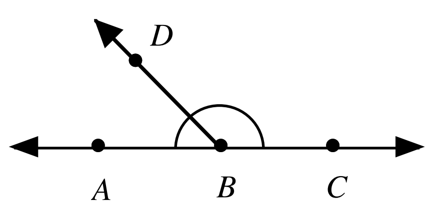
Make 1° into 60′ ⇨ *Make 1′ into 60″* ⇨ *Now subtract*

$$\begin{array}{r} 180°\ 00'\ 00'' \\ -\ 54°\ 28'\ 15'' \\ \hline \end{array} \Rightarrow \begin{array}{r} 179°\ 60'\ 00'' \\ -\ 54°\ 28'\ 15'' \\ \hline \end{array} \Rightarrow \begin{array}{r} 179°\ 59'\ 60'' \\ -\ 54°\ 28'\ 15'' \\ \hline \end{array}$$

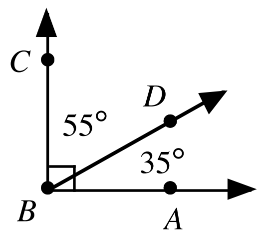
$$\boldsymbol{m\angle RHP} = 125°\ 31'\ 45''$$

Angles that are both adjacent and supplementary form a straight angle, a line:

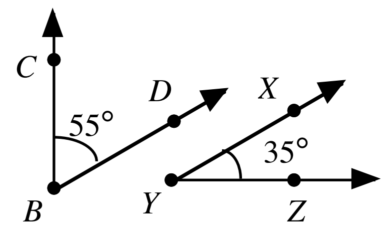
$$m\angle ABC = 180°$$



7. ***Complementary angles*** are any two angles with measures that add to 90°. *Complementary angles* can be adjacent, but they don′t have to be.



We can say that $\angle ABD$ and $\angle DBC$ are *complementary*, or that *$\angle DBC$ is the complement of $\angle ABD$.*



Likewise, $\angle XYZ$ is the complement of $\angle DBC$.

Example 4: $\angle EFG$ and $\angle MPQ$ are complementary angles. Given $m\angle EFG$, find $m\angle MPQ$

a) $m\angle EFG = 53°$ b) $m\angle EFG = 54°\ 28'\ 15''$

Answer: Use $m\angle MPQ = 90° - m\angle EFG$

a) $m\angle MPQ = 90° - 53° = 37°$

b) $m\angle MPQ = 90° - 54°\ 28'\ 15''$

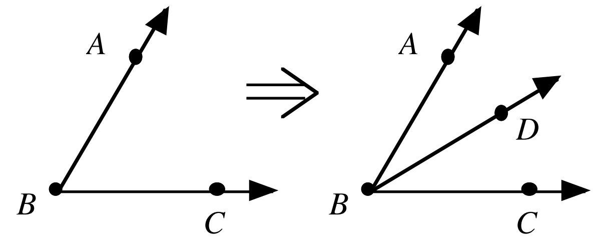
Make 1° into 60′		*Make 1′ into 60″*		*Now subtract*
90° 00′ 00″	⇨	89° 60′ 00″	⇨	89° 59′ 60″
– 54° 28′ 15″		– 54° 28′ 15″		– 54° 28′ 15″

$\boldsymbol{m\angle MPQ} = 35°\ 31'\ 45''$

ANGLE BISECTORS

An ***angle bisector*** splits an angle into two smaller, *congruent* angles.

Said more formally, an *angle bisector* is a ray that begins at the angle's vertex and passes through an interior point so that it forms two smaller, congruent angles.



Example 5: Ray BD bisects $\angle ABC$. Given the measure of $\angle ABC$, find the measure of $\angle DBC$. *Write answers in DMS form.*

a) $m\angle ABC = 48°$

b) $m\angle ABC = 53°$

c) $m\angle ABC = 65°\ 43'\ 10''$

d) $m\angle ABC = 72°\ 28'\ 31''$

Answer: Use $m\angle DBC = m\angle ABC \div 2$

a) $m\angle DBC = 48° \div 2 = 24°$

b) $m\angle DBC = 53° \div 2$. 53 is an odd number, but we can make it even by borrowing 1° from it and making $m\angle ABC = 52°\ 60'$. These are both divisible by 2. Here, division by 2 is written as a fraction:

$$m\angle DBC = \frac{53°}{2} = \frac{52°\ 60'}{2} = \frac{52°}{2} + \frac{60'}{2} = 26°\ 30'$$

c) $m\angle DBC = 65°\ 43'\ 10'' \div 2$. Both 65 and 43 are odd numbers, but we can make them even by borrowing 1 degree (or minute) from each to make them both even:

$$65°\ 43'\ 10'' = 64°\ 103'\ 10'' = 64°\ 102'\ 70''$$

We can now easily divide each of these by 2:

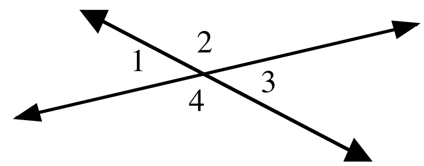
$$m\angle DBC = \frac{64°\ 102'\ 70''}{2} = 32°\ 51'\ 35''$$

d) The number of seconds is odd, 31, and there is nothing that will make it even. Because the other values are already even, we can divide as follows:

$$m\angle DBC = \frac{72°\ 28'\ 31''}{2} = 36°\ 14'\ 15.5''$$

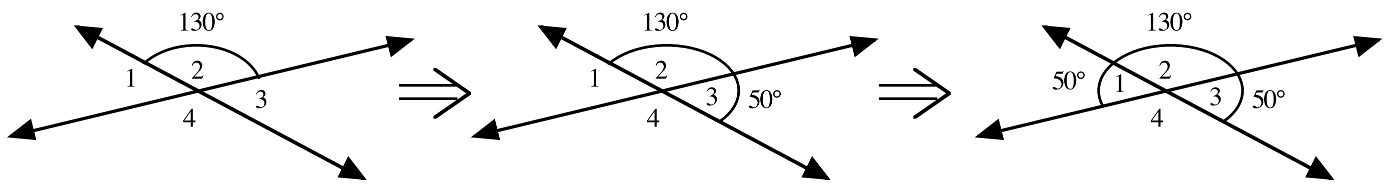
Vertical Angles

When two lines intersect in a plane, several pairs of angles are formed:



- Adjacent angles are supplementary (forming a straight line) and

- non-adjacent angles are ***vertical*** angles. It can easily be shown that **vertical angles are congruent to each other**. Consider this example:



Given: $m\angle 2 = 130°$

$\angle 3$ is supplementary to $\angle 2$ so $m\angle 3 = 180° - 130° = 50°$.

Likewise, $\angle 1$ is supplementary to $\angle 2$, so $m\angle 1$ is also 50°.

Therefore, $m\angle 1 = m\angle 3$, and $\angle 1 \cong \angle 3$.

It follows that $m\angle 4$ is also 130°. $\angle 4$ is vertical to $\angle 2$, and $\angle 4 \cong \angle 2$.

Example 6: In the diagram of intersecting lines, given $m\angle 1 = 27°$, find the measures of the other three angles.

Answer: $m\angle 3 = m\angle 1$ and $\angle 2$ and $\angle 4$ are both supplementary to $\angle 1$, so,

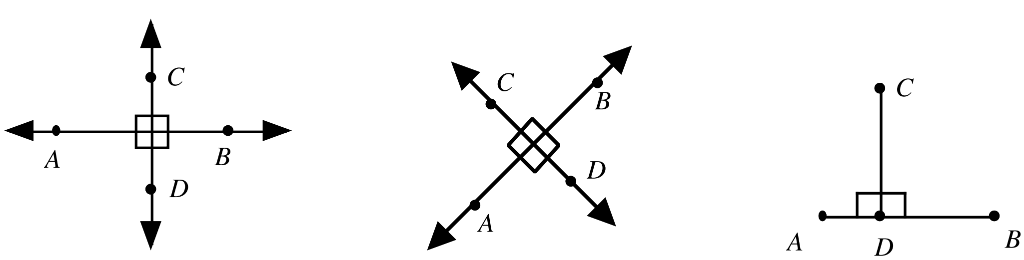
a) $m\angle 3 = 27°$

b) $m\angle 2 = 180° - 27° = 153°$

c) $m\angle 4 = 153°$.

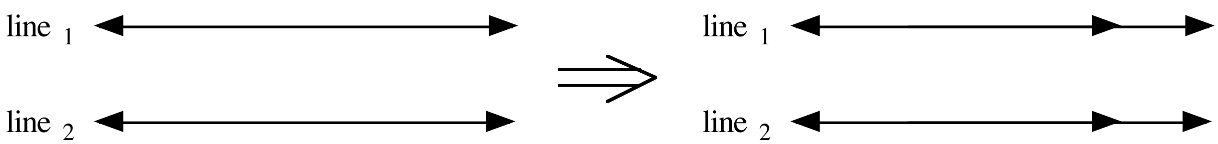
PERPENDICULAR LINES AND LINE SEGMENTS

Two lines in a plane that intersect to form four right angles are said to be ***perpendicular***. If two lines or line segments, $\overline{AB}$ and $\overline{CD}$, are perpendicular, then we may write $\overline{AB} \perp \overline{CD}$.

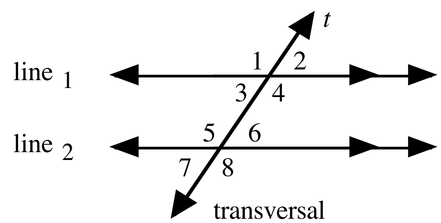


PARALLEL LINES AND TRANSVERSALS

Two lines, $line_1$ and $line_2$, in a plane are ***parallel*** if they never intersect. We sometimes use arrows going in the same direction to indicate that two lines (or line segments) are parallel to each other.



A line that intersects two (or more) parallel lines is called a ***transversal***. The transversal and the parallel lines form a total of eight angles.



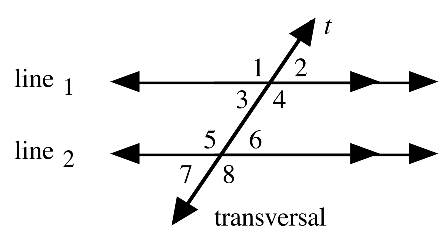
There are many pairs of congruent angles and many pairs of supplementary angles.

If the transversal is *not* perpendicular to the parallel lines, then four acute angles and four obtuse angles are formed. This leads us to the following:

(i) all of the acute angles are congruent to each other and all of the obtuse angles are congruent to each other;

(ii) each acute angle is supplementary to each obtuse angle.

To talk about these eight angles easily, we refer to them in this way: (On the blanks provided, list the angles that fit the description.)



(i) ***left*** side of transversal: 1, 3, 5, and ____

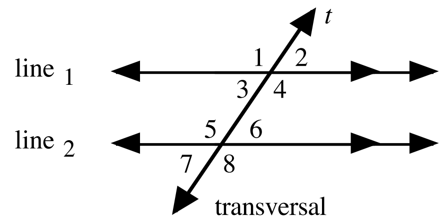
(ii) ***right*** side of transversal: ________________

(iii) ***directly above*** *a parallel line*: 1, 2, 5, and ____

(iv) ***directly below*** *a parallel line*: ________________

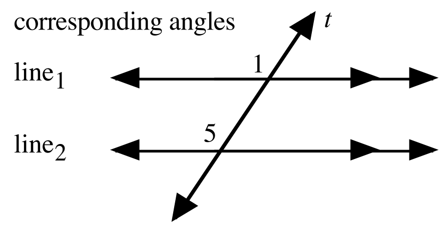
(v) ***interior angles***: 3, 4, 5, and ____

(vi) ***exterior angles***: ________________



In this setting, a pair of ***corresponding angles*** are angles that are on the same side of the transversal (left or right) and are either both above or both below the parallel lines.

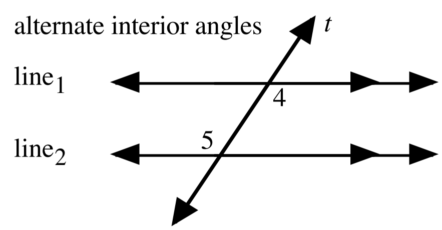
For example, $\angle 1$ and $\angle 5$ are a pair of corresponding angles because they are both to the *above-left* angles.



Corresponding angles are congruent.

Alternate interior angles are on opposite sides of the transversal (one on the left, one on the right) and are "between" the parallel lines: below line$_1$ and above line$_2$.

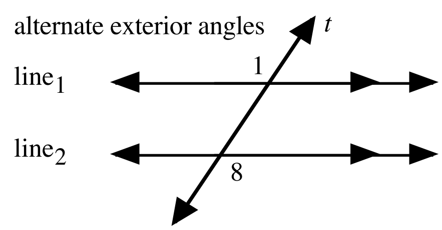
For example, $\angle 4$ and $\angle 5$ are alternate interior angles.



Alternate interior angles are congruent.

Alternate exterior angles are on opposite sides of the transversal (one on the left, one on the right) and are outside of the parallel lines: above line$_1$ and below line$_2$.

For example, $\angle 1$ and $\angle 8$ are alternate exterior angles.



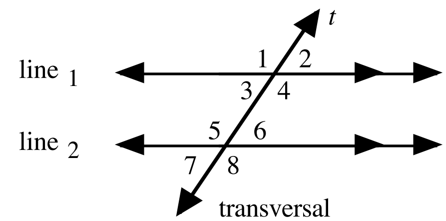
Alternate exterior angles are congruent.

As we already know, all of the vertical angles are congruent; for example $\angle 1 \cong \angle 4$ and $\angle 6 \cong \angle 7$.

Also, all of the adjacent angles form lines (straight angles), so each pair of adjacent angles is supplementary.

Section 1.2 Focus Exercises

For each pair of angles, determine if they have a special relationship (such as "alternate interior") and whether or not they are congruent.

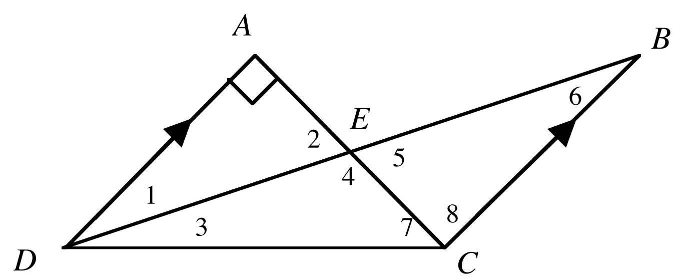


1. $\angle 1$ and $\angle 5$ **2.** $\angle 1$ and $\angle 8$

3. $\angle 1$ and $\angle 4$ **4.** $\angle 1$ and $\angle 2$

5. $\angle 2$ and $\angle 6$ **6.** $\angle 6$ and $\angle 3$

For #7 and 8, given the following diagram ($\overline{AC}$ and $\overline{BD}$ are line segments), find pairs of angles that are guaranteed to be:



** Hint: you might need to use letters to label an angle, such as ∠AEB or ∠ADC.*

7. congruent*

8. supplementary*

$\angle ABC$ and $\angle XYZ$ are complementary angles. Given the measure of $\angle ABC$, find $m\angle XYZ$.

9. $m\angle ABC = 60°$ **10.** $m\angle ABC = 35.8°$

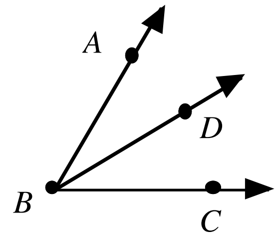
11. $m\angle ABC = 33°\ 47'$ **12.** $m\angle ABC = 82°\ 15'\ 36''$

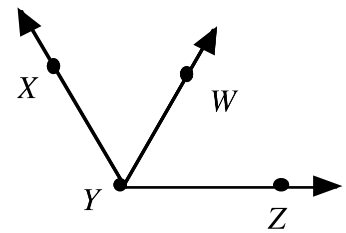
$\angle ABC$ and $\angle XYZ$ are supplementary angles. Given the measure of $\angle ABC$, find $m\angle XYZ$.

13. $m\angle ABC = 45°$ **14.** $m\angle ABC = 161.3°$

15. $m\angle ABC = 36°\ 15'$ **16.** $m\angle ABC = 102°\ 44'\ 08''$

For Exercise #17-24, use these diagrams of the acute and obtuse angles, $\angle ABC$ and $\angle XYZ$, and angle bisectors ray BD and ray YW.





Given either $m\angle ABC$ or $m\angle XYZ$, you are to find, respectively, either $m\angle ABD$ or $m\angle XYW$.

17. $m\angle ABC = 56°$

18. $m\angle XYZ = 148°$

19. $m\angle ABC = 61.9°$

20. $m\angle XYZ = 133.2°$

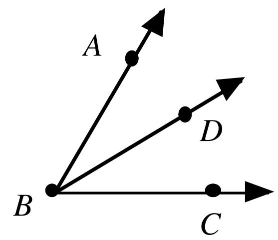
21. $m\angle ABC = 48°\ 32'\ 18''$

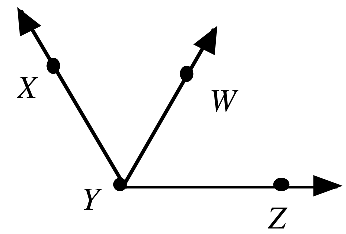
22. $m\angle XYZ = 156°\ 06'\ 42''$

23. $m\angle ABC = 37°\ 51'\ 02''$

24. $m\angle XYZ = 149°\ 17'\ 50''$

For Exercise #25-30, use these diagrams of the acute and obtuse angles, $\angle ABC$ and $\angle XYZ$, and angle bisectors ray BD and ray YW.





Given either $m\angle ABD$ or $m\angle XYW$, you are to find, respectively, either $m\angle ABC$ or $m\angle XYZ$.

25. $m\,\angle ABD = 27°$

26. $m\,\angle XYW = 67.8°$

27. $m\,\angle ABD = 22°\ 15'$

28. $m\,\angle XYW = 77°\ 05'\ 18''$

29. $m\,\angle ABD = 29°\ 36'\ 51''$

30. $m\,\angle XYW = 71°\ 55'\ 42''$