Section 6.2 Long Division of Polynomials

INTRODUCTION

In Section 6.1 we learned to simplify a rational expression by factoring.

For example, $\dfrac{x^2 + 3x - 10}{x - 2} = \dfrac{(x + 5)(x - 2)}{(x - 2)} = \dfrac{(x + 5)}{1} = x + 5.$

However, if we try to simplify either $\dfrac{x^2 - 15x + 36}{x - 5}$ or $\dfrac{x^2 + x - 9}{x + 6}$ in this same manner, we won't get very far, because ...

$$\frac{x^2 - 15x + 36}{x - 5} = \frac{(x - 3)(x - 12)}{(x - 5)}$$ and there are no common factors to divide out;

and the numerator in $\dfrac{x^2 + x - 9}{x + 6}$ is not factorable.

Still, we can simplify them in a different sort of way ... using *long division*, and that is what this section is all about.

Before we can begin dividing polynomials, though, we need to be sure we have the proper preparation. What follows are some basic ideas with which you are already familiar, but they will help create the foundation used to develop our topic.

THE PREPARATION: TYPES OF FRACTIONS

Numerically, any fraction in which the value of the numerator is less than the value of the denominator is called a **proper fraction**, such as $\frac{3}{8}$, $\frac{1}{7}$ and $\frac{2}{5}$.

In algebra, a *proper fraction* is one in which the *degree* of the numerator is less than the *degree* of the denominator. Examples of proper algebraic fractions include:

$3x$	▯ degree is 1	$10x^3$	▯ degree is 3		5	▯ degree is 0
$8x^2$	▯ degree is 2 ,	$3x^4$	▯ degree is 4	and	$x + 3$	▯ degree is 1

In $\dfrac{10x^3}{3x^4}$, even though the numerator has a greater numerical value, it is the degree of the term that we look at.

Because the denominator, $3x^4$, has a degree of 4, and $10x^3$ has a degree of only 3, $\dfrac{10x^3}{3x^4}$ is a *proper* fraction.

Conversely, a numerical **improper fraction** has a numerator that is <u>greater than or equal to</u> the value of the denominator, such as $\frac{8}{3}$, $\frac{164}{5}$, $\frac{7}{1}$ and $\frac{5}{5}$. In algebra, this means that the *degree* of the numerator is greater than or equal to the *degree* of the denominator. Examples of *improper* algebraic fractions include:

$$\frac{3x^2}{8x}\ \begin{matrix}\leftarrow \text{degree is 2}\\ \leftarrow \text{degree is 1}\end{matrix}, \qquad \frac{12x^3}{5x^3}\ \begin{matrix}\leftarrow \text{degree is 3}\\ \leftarrow \text{degree is 3}\end{matrix} \qquad \text{and} \qquad \frac{x^2+3x-10}{x+4}\ \begin{matrix}\leftarrow \text{degree is 2}\\ \leftarrow \text{degree is 1}\end{matrix}$$

Some improper fractions can be simplified directly.

For example, $\frac{7}{1} = 7$, $\quad \frac{5}{5} = 1$, $\quad \frac{3x^3}{12x} = \frac{x^2}{4}$ $\quad$ and $\quad \frac{15x^3}{5x^3} = 3$.

Other improper fractions, especially those that are numerical only, can be rewritten as **mixed numbers**:

$$\frac{8}{3} = 2\tfrac{2}{3} \quad \text{and} \quad \frac{164}{5} = 32\tfrac{4}{5}.$$

The process for rewriting improper fractions as mixed numbers is a called **long division**:

Example 1: Divide using long division. Write the answer as a mixed number.

a) $\frac{8}{3}$ $\qquad$ b) $\frac{164}{5}$

Procedure: The numerator is called the dividend and the denominator is called the divisor.

a) $\frac{8}{3} = 8 \div 3$:

$$\begin{array}{rrl} \text{quotient} \longrightarrow & 2\,\tfrac{2}{3} & \\ \text{divisor} \longrightarrow & 3\,\overline{\big)\,8} & \longleftarrow \text{dividend} \\ & \underline{-\,6} & \\ & 2 & \longleftarrow \text{remainder} \end{array}$$

The result, $2\frac{2}{3}$ can also be written as $2 + \frac{2}{3}$.

b) $\frac{164}{5} = 164 \div 5$

$$\begin{array}{r} 32\,\tfrac{4}{5} \quad \text{or} \quad 32 + \tfrac{4}{5} \\ 5\,\overline{\big)\,164} \\ \underline{-\,15} \\ 14 \\ \underline{-\,10} \\ 4 \end{array}$$

Exercise 1 Divide using long division. Write the answer as a mixed number.

a) $\frac{91}{4}$

b) $\frac{475}{6}$

c) $\frac{1,382}{11}$

Just as $2\frac{2}{3}$, also written $2 + \frac{2}{3}$, is a mixed number, $x + 3 + \frac{5}{x-2}$ is a **mixed expression**. We will be working with mixed expressions shortly. Here is more preparation.

THE PREPARATION: SUBTRACTING POLYNOMIALS

You are already familiar with how to subtract polynomials: you must distribute the negative sign and add.

For example, subtract: $(7x + 8) - (4x - 2)$

$(7x + 8) + {}^{-}1(4x - 2)$	Subtracting means adding the opposite.
$= 7x + 8 - 4x + 2$	Distribute the - 1 to both terms;
$= 3x + 10$	Combine like terms.

If we were to set the same problem up vertically, the result will be the same *if done correctly*.

This second method allows you to distribute the negative and rewrite it as a sum; then add:

$$\begin{array}{r} 7x + 8 \\ -\ (4x - 2) \\ \hline \end{array} \longrightarrow \begin{array}{r} 7x + 8 \\ +\ {-4x} + 2 \\ \hline 3x + 10 \end{array} \longleftarrow$$

Notice that subtraction has been changed to "add the opposite."

(This may look like you have to write the problem twice. Actually, on your paper, this could all be done in one setting.)

Example 2: Subtract.

a)
$$\begin{array}{r} 3x + 5 \\ \underline{-\ (3x + 2)} \end{array}$$

b)
$$\begin{array}{r} -8x + 3 \\ \underline{-\ (-8x + 7)} \end{array}$$

c)
$$\begin{array}{r} 2x + 1 \\ \underline{-\ (2x - 4)} \end{array}$$

d)
$$\begin{array}{r} 4x - 3 \\ \underline{-\ (4x - 8)} \end{array}$$

Procedure: Keep in mind that ‘Subtracting’ means ‘adding the opposite.’ Distribute the negative sign through to the second quantity.

Answer: (Notice that, within each of these, the binomials have the same first term.)

a)
$$\begin{array}{r} 3x + 5 \\ \underline{+\ -3x - 2} \\ 3 \end{array}$$

b)
$$\begin{array}{r} -8x + 3 \\ \underline{+\ \ 8x - 7} \\ -4 \end{array}$$

c)
$$\begin{array}{r} 2x + 1 \\ \underline{+\ -2x + 4} \\ 5 \end{array}$$

d)
$$\begin{array}{r} 4x - 3 \\ \underline{+\ -4x + 8} \\ 5 \end{array}$$

Exercise 2 Subtract.

a)
$$\begin{array}{r} 4x + 8 \\ \underline{-\ (4x + 6)} \end{array}$$

b)
$$\begin{array}{r} -3x + 5 \\ \underline{-\ (-3x + 9)} \end{array}$$

c)
$$\begin{array}{r} 6x + 1 \\ \underline{-\ (6x - 9)} \end{array}$$

d)
$$\begin{array}{r} -5x - 16 \\ \underline{-\ (-5x - 10)} \end{array}$$

THE PREPARATION: MULTIPLICATION AND THE DISTRIBUTIVE PROPERTY

Multiply 8 ▯ 13. Here are four ways to approach the same problem. They all involve distribution.

a)
$$\begin{array}{r} 8 \\ \underline{\times\ 13} \\ 24 \\ \underline{+\ 80} \\ 104 \end{array}$$

b)
$$\begin{array}{r} 8 \\ \underline{\times\ (10 + 3)} \\ 24 \\ \underline{+\ 80} \\ 104 \end{array}$$

c)
$$\begin{array}{l} 8\,(10 + 3) \\ = 8 \times 10 + 8 \times 3 \\ = 80 + 24 \\ = 104 \end{array}$$

d)
$$\begin{array}{r} 8 \\ \underline{\times\ (10 + 3)} \\ 80 + 24 \\ = 104 \end{array}$$

For our work in dividing polynomials, the fourth way will be most valuable.

Example 3: Multiply, by distributing, the following. Use the method shown in (d) above.

a)
$$\begin{array}{r} 3 \\ \underline{\times\ (x + 5)} \end{array}$$

b)
$$\begin{array}{r} -2 \\ \underline{\times\ (3x - 4)} \end{array}$$

c)
$$\begin{array}{r} 6x \\ \underline{\times\ (2x - 1)} \end{array}$$

Procedure: Multiply mentally. The terms will be unlike terms, unable to be combined.

a)
$$\begin{array}{r} 3 \\ \underline{\times\ (x + 5)} \\ 3x + 15 \end{array}$$

b)
$$\begin{array}{r} -2 \\ \underline{\times\ (3x - 4)} \\ -6x + 8 \end{array}$$

c)
$$\begin{array}{r} 6x \\ \underline{\times\ (2x - 1)} \\ 12x^2 - 6x \end{array}$$

Exercise 3 Multiply, by distributing, the following. Use the method shown in (d) above.

a) $\begin{array}{r} 5 \\ \underline{\times\ (2x+4)} \end{array}$ b) $\begin{array}{r} -3 \\ \underline{\times\ (x-1)} \end{array}$ c) $\begin{array}{r} 2x \\ \underline{\times\ (3x-5)} \end{array}$ d) $\begin{array}{r} -4x \\ \underline{\times\ (5x+2)} \end{array}$

THE PREPARATION: MISSING TERMS IN A POLYNOMIAL

Some polynomial, when written in descending order, have all possible degrees represented. For example, the polynomial $2x^3 + 6x^2 - 4x + 9$ has all of the possible degrees represented: there are terms with degree 3, 2, 1 and 0 represented, nothing is missing.

Other polynomials have some missing terms. For example,

$2x^4 + 5x^2 - 4x + 9$ is missing an $\mathbf{x^3}$ term, and

$x^5 + 5x^3 - 4x^2 + 9$ is missing two terms: an $\mathbf{x^4}$ term and an $\mathbf{x}$ term.

Each of the missing terms could be put into place using a coefficient of 0 (zero):

$$2x^4 + 5x^2 - 4x + 9 \qquad\qquad x^5 + 5x^3 - 4x^2 + 9$$

$$= 2x^4 \mathbf{+ 0x^3} + 5x^2 - 4x + 9 \qquad\qquad = x^5 \mathbf{+ 0x^4} + 5x^3 - 4x^2 \mathbf{+ 0x} + 9$$

Example 4: Rewrite each polynomial in descending order with a complete set of terms.

a) $5x^3 - 6x - 2$ b) $3x^3 - 7$

Procedure: Identify the missing terms and write them in with a coefficient of 0.

a) $5x^3 - 6x - 2$ b) $3x^3 - 7$

$= 5x^3 + 0x^2 - 6x - 2$ $= 3x^3 + 0x^2 + 0x - 7$

Exercise 4 Rewrite each polynomial in descending order with a complete set of terms.

a) $2x^3 + x^2 - 9$ b) $1 - 4x^3 - 6x$ c) $8 - x^3$

Here is a recap of the preparation we've seen so far in this section:

TYPES OF FRACTIONS:

We looked at proper fractions, improper fractions and mixed numbers

SUBTRACTING POLYNOMIALS:

Remember to remove the parentheses and "add the opposite"

MULTIPLICATION AND THE DISTRIBUTIVE PROPERTY

We were able to multiply a little differently.

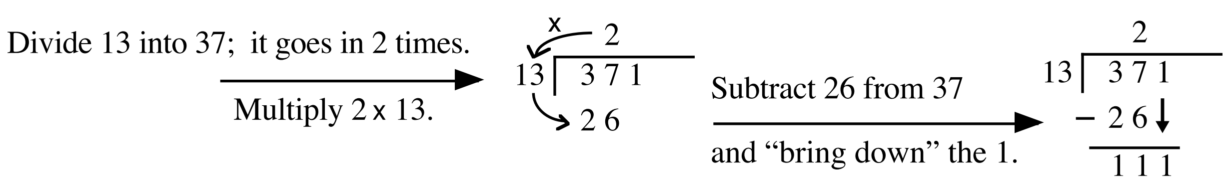
MISSING TERMS IN A POLYNOMIAL:

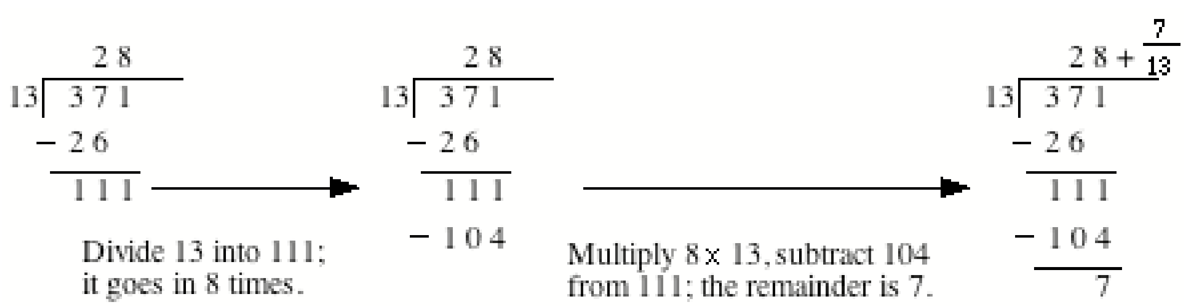
We learned to fill in all of the terms, in descending order, of a polynomial.

DIVIDING POLYNOMIALS

To best understand how to divide polynomials we need first look at dividing numbers using long division. What you'll eventually see is how place values (ones, tens and hundreds) play an important role.

Consider $371 \div 13$ using long division: $13\overline{)\,371\,}$:





Take a look back at the first step of dividing 13 into 371. When we begin the process, the first thing we do is divide 13 into 37. This is like the *improper* fraction $\frac{37}{13}$. When dividing, the whole number is **2** and there is a remainder, found by subtracting.

We’re going to take a different look at the same problem, 371 ÷ 13. This time we’re going to expand both numbers and treat 371 as $\underline{300 + 70 + 1}$ and 13 as $\underline{10 + 3}$.

(When written this way, the numbers start to look a little like a trinomial and a binomial.)

Throughout this next exercise, you’ll see many of the *preparation* ideas presented earlier in this section put to good use.

Divide: (300 + 70 + 1) ÷ (10 + 3). This time, instead of dividing 13 into 37, we’ll divide the first “term” of each: 300 ÷ 10 = 30

$$10 + 3\,\overline{\big)\,300 + 70 + 1} \longrightarrow \frac{300}{10} = \mathbf{30}$$

1 Divide 10 into 300.

2 Place 30 above 70 (they’re both “tens”)

$$\begin{array}{r} \mathbf{30} \\ 10 + 3\,\overline{\big)\,300 + 70 + 1} \\ 300 + 90 \end{array}$$

x

3 Next, multiply 30 by (10 + 3)

Next, we subtract (300 + 90) from what’s directly above it. Subtract by “adding the opposite.”

$$\begin{array}{r} \mathbf{30} \\ 10 + 3\,\overline{\big)\,300 + 70 + 1} \\ \underline{-\ (300 + 90)} \end{array} \xrightarrow[\text{becomes - 300 and + 90 becomes - 90.}]{\text{Add the opposite. Notice that 300}} \begin{array}{r} \mathbf{30} \\ 10 + 3\,\overline{\big)\,300 + 70 + 1} \\ \underline{+\ (-300 \overset{-}{\cancel{+}} 90)}\downarrow \\ -20 + 1 \end{array}$$

Repeat the process, this time dividing the new first term, - 20, by 10: - 20 ÷ 10 = **- 2**.
(Put this in the ones place.)

We’ll multiply and subtract (by adding the opposite).

$$\begin{array}{r} 30 - \mathbf{2} \\ 10 + 3\,\overline{\big)\,300 + 70 + 1} \\ \underline{+\ (-300 \overset{-}{\cancel{+}} 90)} \\ -20 + 1 \\ -(-20 - 6) \end{array}$$

x

Then, multiply **- 2** times (10 + 3)

$$\begin{array}{r} 30 - \mathbf{2} \\ 10 + 3\,\overline{\big)\,300 + 70 + 1} \\ \underline{+\ (-300 \overset{-}{\cancel{+}} 90)} \\ -20 + 1 \\ \underline{+\ (\overset{+}{\cancel{-}}20 \overset{+}{\cancel{-}}6)} \\ 7 \end{array}$$

Add the opposite. - 20 becomes + 20 and - 6 becomes + 6

$$\begin{array}{r} 30 - 2 + \frac{7}{13} \\ 10 + 3\,\overline{\big)\,300 + 70 + 1}\phantom{\frac{7}{13}} \\ \underline{+\ (-300 \overset{-}{\cancel{+}} 90)}\phantom{\,+1\frac{7}{13}} \\ -20 + 1\phantom{\frac{7}{13}} \\ \underline{+\ (\overset{+}{\cancel{-}}20 \overset{+}{\cancel{-}}6)}\phantom{\frac{7}{13}} \\ 7\phantom{\frac{7}{13}} \end{array}$$

remainder

Show the remainder above, in the answer.

The answer, $30 - 2 + \frac{7}{13}$ *looks* different from $28\frac{7}{13}$, but they are the same (30 – 2 = 28), just in a different form.

We will now use the same procedure using algebra: dividing a polynomial by a binomial. The key is dividing "first term by first term".

Example 5: Divide $\dfrac{3x^2 + 7x + 1}{x + 3}$; also written as $(3x^2 + 7x + 1) \div (x + 3)$.

Procedure: Use the method outlined on the previous page. This is the same as the numerical example $(300 + 70 + 1) \div (10 + 3)$ with x replacing 10. $(300 = 3\cdot 100 = 3\cdot 10^2$ which becomes $3x^2)$

$$x + 3\,\overline{)\,3x^2 + 7x + 1}$$

① Divide x into $3x^2$.

$$\frac{3x^2}{x} = \mathbf{3x}$$

② Place 3x above 7x (they're like terms)

$$\begin{array}{r} \mathbf{3x} \\ x + 3\,\overline{)\,3x^2 + 7x + 1} \\ 3x^2 + 9x \end{array}$$

x

③ Next, multiply 3x by (x + 3)

$$\begin{array}{r} \mathbf{3x} \\ x + 3\,\overline{)\,3x^2 + 7x + 1} \\ \underline{-(3x^2 + 9x)} \end{array}$$

Add the opposite. Notice that $3x^2$ becomes $-3x^2$ and $+9x$ becomes $-9x$.

$$\begin{array}{r} \mathbf{3x} \\ x + 3\,\overline{)\,3x^2 + 7x + 1} \\ \underline{+(-3x^2 \cancel{+}\,^{-}9x)} \\ -2x + 1 \end{array}$$

x

$$\begin{array}{r} 3x - \mathbf{2} \\ x + 3\,\overline{)\,3x^2 + 7x + 1} \\ \underline{+(-3x^2 \cancel{+}\,^{-}9x)} \\ -2x + 1 \\ -(-2x - 6) \end{array}$$

Multiply **- 2** times (x + 3)

$$\begin{array}{r} 3x - \mathbf{2} \\ x + 3\,\overline{)\,3x^2 + 7x + 1} \\ \underline{+(-3x^2 \cancel{+}\,^{-}9x)} \\ -2x + 1 \\ \underline{+(\cancel{-}^{+}2x \cancel{-}^{+}6)} \\ 7 \end{array}$$

Add the opposite. - 2x becomes + 2x and - 6 becomes + 6

$$\begin{array}{r} 3x - 2 + \frac{7}{x+3} \\ x + 3\,\overline{)\,3x^2 + 7x + 1}\phantom{+\frac{7}{x+3}} \\ \underline{+(-3x^2 \cancel{+}\,^{-}9x)}\phantom{+\frac{7}{x+3}} \\ -2x + 1\phantom{+\frac{7}{x+3}} \\ \underline{+(\cancel{-}^{+}2x \cancel{-}^{+}6)}\phantom{+\frac{7}{x+3}} \\ 7\phantom{+\frac{7}{x+3}} \end{array}$$

remainder

Show the remainder above, in the answer.

Conclusion: $(3x^2 + 7x + 1) \div (x + 3) = \mathbf{3x - 2 + \frac{7}{x+3}}$

This very last step is what your work would normally look like.

This shows every step compiled into one long-division problem.

$$\begin{array}{r} 3x - 2 + \frac{7}{x+3} \\ x + 3\,\overline{)\,3x^2 + 7x + 1}\phantom{+\frac{7}{x+3}} \\ \underline{+(-3x^2 \cancel{+}\,^{-}9x)}\phantom{+\frac{7}{x+3}} \\ -2x + 1\phantom{+\frac{7}{x+3}} \\ \underline{+(\cancel{-}^{+}2x \cancel{-}^{+}6)}\phantom{+\frac{7}{x+3}} \\ 7\phantom{+\frac{7}{x+3}} \end{array}$$

Example 6: Divide $(x^3 - 6x^2 + 7x + 1) \div (x - 4)$

Procedure: Use the same procedure as in Example 5. This time the division process is shown complete as you might write it. It's recommended that you do the step of changing the subtraction to *add the opposite*.

$$
\begin{array}{rl}
 & \overset{A}{x^2} \overset{B}{- 2x} \overset{C}{- 1} + \overset{D}{\dfrac{-3}{x-4}} \\
x - 4 \, \big) & x^3 - 6x^2 + 7x + 1 \\
A \quad - & \underline{(x^3 - 4x^2)} \\
 & \quad -2x^2 + 7x \\
B \quad - & \quad \underline{(-2x^2 + 8x)} \\
 & \qquad -x + 1 \\
C \quad - & \qquad \underline{(-x + 4)} \\
D & \qquad\quad -3
\end{array}
$$

A Divide x into x^3 to get x^2
Multiply $x^2 \cdot (x - 4)$ to get $x^3 - 4x^2$
and subtract. You may now change it to *add the opposite*.

B Divide x into $-2x^2$ to get $-2x$
Multiply $-2x \cdot (x - 4)$ to get $-2x^2 + 8x$
and subtract. You may now change it to *add the opposite*.

C Divide x into $-x$ to get -1.
Multiply and subtract (as above)

D -3 is the remainder and shows up in the answer as part of the fraction.

Because the remainder is negative, the answer could also be written with the reminader fraction subtracted. $\mathbf{x^2 - 2x - 1 - \dfrac{3}{x - 4}}$

Exercise 5 Divide each using long division.

a) $\dfrac{2x^2 - 5x + 7}{x - 3}$

b) $\dfrac{4y^2 + 18y - 3}{y + 5}$

c) $\dfrac{3m^3 - 7m^2 + 2m - 5}{m - 2}$

d) $\dfrac{p^3 + 6p^2 + 9p + 4}{p + 4}$

Before dividing, each polynomial must be put in descending order. Furthermore, if either polynomial has any missing terms, they must be written so that all terms are represented.

Example 7: Divide. Be sure to write the polynomial in descending order and include any missing terms. $\dfrac{5x^3 - 17x + 8}{x + 2}$

Procedure: The fraction can be written as $(5x^3 - 17x + 8) \div (x + 2)$

The dividend, $5x^3 - 17x + 8$, is missing a term. Before we can divide we must write it as $5x^3 + \mathbf{0x^2} - 17x + 8$.

$$
\begin{array}{rrrrl}
 & \text{A } 5x^2 & \text{B } -10x & \text{C } +3 & + \text{ D } \dfrac{2}{x+2} \\
x + 2 \,\big)\! & 5x^3 + 0x^2 & -17x & +8 & \\
\text{A} - & (5x^3 + 10x^2) & & & \\
\hline
 & -10x^2 & -17x & & \\
\text{B} - & (-10x^2 & -20x) & & \\
\hline
 & & 3x & +8 & \\
\text{C} & & -(3x & +6) & \\
\hline
 & & \text{D} & 2 &
\end{array}
$$

A Divide x into $5x^3$ to get $5x^2$
Multiply $5x^2 \cdot (x + 2)$ to get $5x^3 + 10x^2$
and subtract. You may now change it to *add the opposite*.

B Divide x into $-10x^2$ to get $-10x$
Multiply $-10x \cdot (x + 2)$ to get $-10x^2 - 20x$
and subtract. You may now change it to *add the opposite*.

C Divide x into 3x to get 3.
Multiply and subtract (as above)

D 2 is the remainder and shows up in the answer as part of the fraction.

$$\boxed{\text{So, } \frac{5x^3 - 17x + 8}{x + 2} = \mathbf{5x^2 - 10x + 3 + \frac{2}{x + 2}}}$$

Example 8: Divide. Be sure to write the polynomial in descending order and include any missing terms. $\dfrac{5x^2 - 25 + 2x^3}{2x - 3}$

Procedure: The fraction can also be written as $(5x^2 - 25 + 2x^3) \div (2x - 3)$

The dividend, $5x^2 - 25 + 2x^3$, is not in descending order; it is also missing a term. Before we can divide we must write it as $2x^3 + 5x^2 + 0x - 25$.

$$
\begin{array}{rl}
 & \mathbf{x^2 + 4x + 6 + \dfrac{-7}{2x-3}} \\
2x - 3 \,\big) & 2x^3 + 5x^2 + 0x - 25 \\
\text{A} & -\,\underline{(2x^3 - 3x^2)} \\
 & \quad 8x^2 + 0x \\
\text{B} & \quad -\,\underline{(8x^2 - 12x)} \\
 & \qquad 12x - 25 \\
\text{C} & \qquad -\,\underline{(12x - 18)} \\
\text{D} & \qquad\quad -7
\end{array}
$$

A. Divide $2x$ into $2x^3$ to get x^2
Multiply $x^2 \cdot (2x - 3)$ to get $2x^3 - 3x^2$
and subtract. You may now change it to *add the opposite*.

B. Divide $2x$ into $8x^2$ to get $4x$
Multiply $4x \cdot (2x - 3)$ to get $8x^2 - 12x$
and subtract. You may now change it to *add the opposite*.

C. Divide $2x$ into $12x$ to get 6.
Multiply and subtract (as above)

D. -7 is the remainder and shows up in the answer as part of the fraction.

Because the remainder is negative, the answer could also be written with the reminader fraction subtracted. $\mathbf{x^2 + 4x + 6 - \dfrac{7}{2x - 3}}$

$$\boxed{\text{So, } \frac{5x^2 - 25 + 2x^3}{2x - 3} = \mathbf{x^2 + 4x + 6 - \frac{7}{2x - 3}}}$$

Exercise 6 Divide each using long division. *Be sure to write each polynomial in descending order and to include any missing terms.*

a) $\dfrac{5 - 8x + 3x^2}{x - 2}$

b) $\dfrac{4x^3 + 10x^2 - 8x - 6}{x + 3}$

c) $\dfrac{2x^3 + 7x^2 + 10}{x + 4}$

d) $\dfrac{x^2 - 10x + 6x^3 + 1}{3x - 1}$

Answers to each Exercise

Section 6.2

Exercise 1: a) $22\frac{3}{4}$ b) $79\frac{1}{6}$ c) $125\frac{7}{11}$

Exercise 2: a) 2 b) - 4 c) 10 d) - 6

Exercise 3: a) $10x + 20$ b) $-3x + 3$ c) $6x^2 - 10x$ d) $-20x^2 - 8x$

Exercise 4: a) $2x^3 + x^2 + 0x - 9$ b) $-4x^3 + 0x^2 - 6x + 1$

c) $-x^3 + 0x^2 + 0x + 8$

Exercise 5: a) $2x + 1 + \dfrac{10}{x - 3}$ b) $4y - 2 + \dfrac{7}{y + 5}$

c) $3m^2 - m + \dfrac{-5}{m - 2}$ d) $p^2 + 2p + 1$

or $3m^2 - m - \dfrac{5}{m - 2}$

Exercise 6: a) $3x - 2 + \dfrac{1}{x - 2}$ b) $4x^2 - 2x - 2$

c) $2x^2 - x + 4 - \dfrac{6}{x + 4}$ d) $2x^2 + x - 3 - \dfrac{2}{3x - 1}$

Section 6.2 Focus Exercises

1. Divide each using long division.

a) $\dfrac{y^2 + 10y + 18}{y + 5}$

b) $\dfrac{3x^2 - 4x - 8}{x - 2}$

c) $(p^3 + 2p^2 - 5p - 1) \div (p + 3)$

d) $(3m^3 - 9m^2 - 14m + 8) \div (m - 4)$

2. Divide each using long division. *Be sure to write each polynomial in descending order*.

a) $(5 - 9x + 4x^2) \div (x - 1)$

b) $(3x^2 - 7 + 4x) \div (x + 2)$

c) $(6x + x^3 - 8 - 5x^2) \div (x - 4)$

d) $(14x^2 + 5x^3 - 4 - 6x) \div (x + 3)$

e) $(5x^3 + 12x^2 - x - 6) \div (5x + 2)$

f) $(x^2 - 7x + 10x^3 + 1) \div (2x - 1)$

3. Divide each using long division. *Be sure to write each polynomial in descending order*; also, be sure to include any missing terms.

a) $(2x^3 + 9x^2 + 21) \div (x + 5)$

b) $(12 - 11x^2 + 3x^3 - 9x) \div (x - 4)$

c) $(16 - 9x^2 + x^4) \div (x - 2)$

d) $(4x^3 - 5) \div (x + 1)$