

Section 0.8 Fractions, Decimals and Percents

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PLACE VALUE AND POWERS OF 10

We can place as many zeros as we wish “in front” of a whole number and it will not change the value of the whole number.

For example, 23 could be written as 023 or 0023 or even 00000023; it wouldn’t matter, they all have the same value as 23.

Basically, a number like **23** could be thought of as **2 tens** and **3 ones**:

$$(2 \cdot 10) + (3 \cdot 1) = 20 + 3 = \mathbf{23},$$

and a number like **023** could be thought of as **0 hundreds**, **2 tens** and **3 ones**:

$$(0 \cdot 100) + (2 \cdot 10) + (3 \cdot 1) = 0 + 20 + 3 = \mathbf{23} \text{ (still), and on and on.}$$

Just as we can place zeros at the beginning of a whole number, we can also place zeros at the “end” of any decimal: .5 is the same as .50 or .500 or even .500000. Why?

You probably know that .50 is “fifty hundredths,” which can be rewritten as $\frac{50}{100}$; similarly .5, “five tenths,” can be written as $\frac{5}{10}$. These fractions, though, are equivalent. This equivalence can be demonstrated in one of two ways:

1) we can **simplify** $\frac{50}{100}$ (though not completely) to get to $\frac{5}{10}$: $\frac{50}{100} = \frac{50 \div 10}{100 \div 10} = \frac{5}{10}$; or

2) we can **build up** $\frac{5}{10}$ to be $\frac{50}{100}$: $\frac{5}{10} \cdot \frac{10}{10} = \frac{50}{100}$.

We can continue to place zeros to the end of the decimal if we continue to multiply by $\frac{10}{10}$:

$$\frac{50}{100} \cdot \frac{10}{10} = \frac{500}{1,000} = .500 \text{ and } \frac{500}{1,000} \cdot \frac{10}{10} = \frac{5,000}{10,000} = .5000, \text{ etc.}$$

Please note: just as $\frac{5}{10}$ simplifies to $\frac{1}{2}$, for example, .5000 can simplify to .5.

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DECIMALS AND FRACTIONS, REWRITING ONE AS THE OTHER

Powers of 10 have an important property: the exponent (the power) indicates the number of zeros following the 1:

$$10^1 = 10 \text{ (one zero)}$$

$$10^2 = 10 \cdot 10 = 100 \text{ (two zeros)}$$

$$10^3 = 10 \cdot 10 \cdot 10 = 1,000 \text{ (three zeros)}$$

$$10^4 = 10 \cdot 10 \cdot 10 \cdot 10 = 10,000 \text{ (four zeros)}$$

and so on.

When a power of 10 is in the denominator of a fraction, the number of zeros indicates the number of *decimal places* the number has. Fractions such as $\frac{3}{10}$, $\frac{28}{100}$ and $\frac{47}{1,000}$ are called **decimal fractions** because their denominators are all powers of 10.

Decimals, themselves, are also sometimes called *decimal fractions* because each can be so readily written as a fraction with the denominator as a power of 10. Such decimals are **terminating decimals** because they are complete after a certain number of decimal places.

Example 1: Identify the number of decimal places, and then write each as a fraction with a power of 10 in the denominator. (Do not use exponents, and do not simplify the fraction.)

a) $0.2367 = \frac{2367}{10,000}$

four decimal places, the denominator is the 4th power of 10

b) $0.98 = \frac{98}{100}$

two decimal places, the denominator is the 2nd power of 10

c) $0.009 = \frac{9}{1,000}$

remember, 009 has the same value as 9

d) $2.7 = \frac{27}{10}$

e) $0.30 = 0.3 = \frac{3}{10}$

The 0 after the 3 is not necessary, so we first simplify it to *lowest decimal form*.

Exercise 1: Write each as a fraction with a power of 10 in the denominator.

a) 0.8

b) 0.45

c) 0.319

d) 0.06

e) 0.0007

f) 3.14

We can easily rewrite a decimal fraction as a decimal by simply identifying the power of 10 in the denominator. Basically, the number of zeros in the denominator is used to determine the number of decimal places needed to the right of the decimal point. For example, a fraction like $\frac{34}{100}$ can be written as a decimal number with *two* decimal places; that is because the denominator, 100, has *two* zeros.

Sometimes, one of the digits is a zero. For example, $\frac{9}{100}$ could be thought of as $\frac{09}{100} = .09$.

Example 2: Rewrite each fraction as a decimal.

a) $\frac{17}{100}$ b) $\frac{3}{10}$ c) $\frac{58}{1,000}$ d) $\frac{189}{100}$

Procedure: The number of zeros in the denominator indicates the number of decimal places.

a) $\frac{17}{100} = .17$

b) $\frac{3}{10} = .3$

c) $\frac{58}{1,000} = .058$

d) $\frac{189}{100} = 1.89$

This numerator doesn't have enough numbers to fill out all three places in the decimal, so we need to think of it as $\frac{058}{1,000}$, which is 0.058.

This time we have more than two digits in the numerator, but the denominator suggests that we need only two of them—the last two—after the decimal; the first digit is the whole number.

Exercise 2: Rewrite each fraction as a decimal.

a) $\frac{26}{100}$ b) $\frac{3}{10}$ c) $\frac{194}{1,000}$

d) $\frac{634}{100}$ e) $\frac{23}{10}$ f) $\frac{5,402}{1,000}$

g) $\frac{6}{100}$ h) $\frac{14}{1,000}$ i) $\frac{7}{1,000}$

Is it possible to have a decimal in the numerator? Is it possible to have a fraction like $\frac{23.6}{100}$? The answer is “Yes!” If we have such a fraction, then it can be rewritten by multiplying both numerator and denominator by a power of 10.

Example 3: Rewrite each fraction as a decimal by first multiplying (by 1) to eliminate the decimal in the numerator. (The number of decimal places in the numerator indicates the power of 10 by which it should be multiplied.)

$$\text{a) } \frac{23.6}{100} = \frac{23.6}{100} \cdot \frac{10}{10} = \frac{236}{1,000} = .236 \quad \text{Multiply by } \frac{10}{10} \text{ because there is only one decimal place in the numerator.}$$

$$\text{b) } \frac{.008}{100} = \frac{.008}{100} \cdot \frac{1,000}{1,000} = \frac{8}{100,000} = .00008 \quad \text{Multiply by } \frac{1,000}{1,000} \text{ because there are three decimal place in the numerator.}$$

Exercise 3: Rewrite each fraction as a decimal by first multiplying (by 1) to eliminate the decimal in the numerator.

$$\text{a) } \frac{4.3}{10}$$

$$\text{b) } \frac{5.07}{10}$$

$$\text{c) } \frac{.035}{100}$$

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MORE TERMINATING DECIMALS

Decimal fractions (fractions in which the denominator is a power of 10) are not the only fractions that terminate. Other fractions, such as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{5}{8}$ and $\frac{6}{25}$ all result in terminating decimals. To demonstrate, we'll need to use long division.

First, we need to recognize that, for example, that $\frac{5}{8}$ is the same as $5 \div 8$. Second, we need to remember about using long division:

- (1) we can write $5 \div 8$ as $8 \overline{)5}$. Of course, 8 doesn't divide evenly into 5 so,
- (2) we need to consider that 5 can be written as 5.00000 (or as many zeros as we need);
- (3) the quotient (answer) will go above the long division bar, along with the decimal:

$$\begin{array}{r} .625 \\ 8 \overline{)5.00000} \\ \underline{-48} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Once the decimal is in place we can ignore it in the division process.

We'll continue to divide until we get a zero (0) remainder, indicating a terminating decimal.

$$\text{So, } \frac{5}{8} = .625.$$

Exercise 4:

Divide each of these fractions to get a terminating decimal.

a) $\frac{3}{4}$

b) $\frac{2}{5}$

c) $\frac{9}{20}$

d) $\frac{6}{25}$

As it turns out, if a fraction is completely simplified, and its denominator is composed only of factors of 2 and/or 5, then it will result in a terminating decimal. So, the numbers listed below would make good denominators for terminating decimals:

2	5	$10 = 2 \cdot 5$	
$4 = 2^2$	$25 = 5^2$	$20 = 2^2 \cdot 5$	$50 = 2 \cdot 5^2$
$8 = 2^3$	$125 = 5^3$	$40 = 2^3 \cdot 5$	$100 = 2^2 \cdot 5^2$
$16 = 2^4$	$125 = 5^3$	$80 = 2^4 \cdot 5$	$250 = 2 \cdot 5^3$

and so on.

Any other denominators, ones that have factors other than powers of 2 and/or 5, will result in what are called **repeating decimals**. Such denominators are, for example, 3, 6, 7, 9, 11, 12 and so on.

[Go to top](#)**REPEATING DECIMALS**

As mentioned, there are some denominators that will allow a fraction to be equivalent to a *terminating* decimal. But there are many more denominators that will cause the decimal equivalent to go on indefinitely, so that the decimal never terminates.

What you will notice, though, is that each of these fractions that *doesn't* terminate as a decimal has a set sequence of digits that repeat—the same digits in the very same order; of course, they are called **repeating decimals**. Sometimes it's only one digit that repeats, sometimes two digits, sometimes six or more digits that repeat.

Example 4: The following are repeating decimals. Each can be abbreviated by placing a bar over those digits that repeat. The ellipsis (...) is used to indicate that the pattern repeats indefinitely.

a) $4.55555555\dots = 4.\overline{5}$

b) $0.3838383838\dots = 0.\overline{38}$

c) $2.54716716716716\dots = 2.54\overline{716}$

It's okay to have other decimals before the repeating sequence.

Exercise 5:

Rewrite each repeating decimal into an abbreviated form.

a) $0.22222222... =$

b) $0.71717171... =$

c) $3.189189189189... =$

d) $6.0453535353... =$

As mentioned, some fractions become repeating decimals when divided out (using long division). We may place as many zeros as we wish, but if we never get a remainder of 0, then the result is a **repeating decimal**.

As you will see in this next example, the quotient continues in a certain repeating pattern. Once you see the pattern, you can stop dividing and place a bar over the repeating values.

For example, find the decimal equivalent of $\frac{3}{7}$.

$$\begin{array}{r}
 .4285714 \\
 7 \overline{) 3.00000000} \\
 \underline{- 28} \\
 20 \\
 \underline{- 14} \\
 60 \\
 \underline{- 56} \\
 40 \\
 \underline{- 35} \\
 50 \\
 \underline{- 49} \\
 10 \\
 \underline{- 7} \\
 30 \\
 \underline{- 28} \\
 2
 \end{array}$$

We'll keep dividing until we see a repeated remainder.

(Because the denominator is 7, this decimal will not terminate.)

← This is the number that started the whole process.

← This is the first repeated remainder.

Once we got a second 4 in the quotient (the result on top of the long division bar), that should be the clue that we're done dividing and we need to look closely for the repeated pattern. As it turns out, that second 4 wasn't necessary, as the repeated pattern is **428571**.

As a decimal, $\frac{3}{7} = .\overline{428571}$

Many denominators require fewer decimal places before you find the repeating pattern. For example, $\frac{1}{3} = .333333... = .\overline{3}$, $\frac{5}{11} = .454545... = .\overline{45}$ and $\frac{16}{37} = .432432... = .\overline{432}$

Some denominators, such as 12, have a factor of 2. This causes the pattern to be delayed before showing itself. For example, find the decimal equivalent of $\frac{5}{12}$.

$$\begin{array}{r}
 .4166 \\
 12 \overline{) 5.00000000} \\
 \underline{- 48} \\
 20 \\
 \underline{- 12} \\
 80 \\
 \underline{- 72} \\
 80 \\
 \underline{- 72} \\
 8
 \end{array}$$

We'll keep dividing until we see a repeated remainder.
(Because the denominator is 12, this decimal will not terminate.)

← This is the first repeated remainder.

← Here it is again.

In the quotient, only the 6 repeats. So, as a decimal, $\frac{5}{12} = .4166666\dots = .41\overline{6}$

Exercise 6:

Divide each of these fractions to get a repeating decimal.

a) $\frac{5}{9}$

b) $\frac{4}{11}$

c) $\frac{12}{55}$

d) $\frac{13}{18}$

Every fraction with whole number terms is equivalent to a decimal that either terminates or repeats.

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WRITING REPEATING DECIMALS AS FRACTIONS

We have seen how to write terminating decimals as fractions, but how do we write repeating decimals as fractions? There is a simple method that will need to be explained later since it involves algebra with which you are not yet familiar. Also, we are limited—at this time—to the types of repeating decimals that we can rewrite: we'll only rewrite repeating decimals in which the first decimal place is the beginning of the repeated sequence of numbers.

The method is relatively easy; you simply need to count how many digits are in the repeated sequence. You are then able to write a fraction with the repeated sequence as the numerator and the denominator as a sequence of 9's. There will be as many 9's in the denominator as there are digits in the repeated sequence.

For example, $0.\overline{4}$ has *one* digit in the repeated sequence—the number 4—so it will have *one* 9—the number 9, itself—in the denominator and be rewritten as $\frac{4}{9}$. Similarly, $0.\overline{56}$ has *two* digits in the repeated sequence and will have *two* 9's in the denominator; so, $0.\overline{56} = \frac{56}{99}$.

Also, $\frac{4}{9}$ and $\frac{56}{99}$ cannot be simplified, but there are a number of repeated fractions that can. You must sometimes be persistent in the simplifying, though. Consider these examples:

Example 5: Rewrite each repeating decimal as a fraction; simplify the fraction completely.

a) $0.\overline{6}$ b) $0.\overline{72}$ c) $0.\overline{458}$

Procedure: a) $0.\overline{6} = \frac{6}{9} = \frac{2}{3}$; so $\frac{2}{3}$ is equivalent to $0.\overline{6}$.

b) $0.\overline{72} = \frac{72}{99} = \frac{8 \cdot 9}{11 \cdot 9} = \frac{8}{11}$

c) $0.\overline{458} = \frac{458}{999}$ this fraction will not simplify.

Exercise 7: Rewrite each repeating decimal as a fraction; simplify the fraction completely.

a) $0.\overline{5}$ b) $0.\overline{63}$ c) $0.\overline{081}$

Lastly,

Every decimal that either terminates or repeats can be written as a fraction

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PERCENT AND PERCENTAGE

Percent, as a word, can be thought of as two words “per cent.” **Per** means “divided by” and **cent** means “100.” Taken together, **percent** means “divided by 100.” 17 percent (written 17%), then, means “17 divided by 100,” or $\frac{17}{100}$.

A word that is often misused is *percentage*. It is frequently used (incorrectly) in place of “percent,” and though they are related, they do not have exactly the same meaning. **Percentage** is actually the number in front of the percent sign; so, in 17%, the *percentage* is 17.

Notice that percentage is, in this case, a whole number. By itself, it is not usually a fraction or a decimal number, though it could be.

Example 6: Identify the *percentage* in each of the following:

- a) 23% b) 125% c) 9% d) 4.7% e) $6\frac{1}{2}\%$

Procedure:

- a) In 23%, the percentage is 23;
- b) In 125%, the percentage is 125;
- c) In 9%, the percentage is 9;
- d) In 4.7%, the percentage is 4.7;
- e) In $6\frac{1}{2}\%$, the percentage is $6\frac{1}{2}$.

Exercise 8: Identify the *percentage* in each of the following. Write it out.

- a) 95% _____
- b) 1.25% _____
- c) 8% _____

Since the number that comes before the percent symbol (%) is called the percentage, then in interpreting, say, 17%, we can treat this as 17 per cent or 17 per 100. In rewriting a percent as a fraction, it is the percentage that is placed in the numerator, and 100 is placed in the denominator. Basically, in rewriting a percent as a fraction, the fraction becomes $\frac{\text{percentage}}{100}$.

Then, when the percent is written as a decimal fraction, it can be further written as a decimal.

Exercise 10: Rewrite each fraction as a percent.

a) $\frac{81}{100}$

b) $\frac{3}{100}$

c) $\frac{1.25}{100}$

d) $\frac{250}{100}$

If we wish to write a decimal as a percent, we can still use decimal fractions but only with a denominator of 100. That means that we must think in terms of *two* decimal places, even if the decimal number has only one decimal place or more than two.

Example 9: Rewrite each decimal as a percent by first rewriting the decimal as a fraction with a denominator of 100.

a) .58

b) .07

c) .4

d) .139

Procedure: First write the decimal as a fraction with 100 in the denominator

a) $.58 = \frac{58}{100} = 58\%$

b) $.07 = \frac{7}{100} = 7\%$

c) $.4 = .40 = \frac{40}{100} = 40\%$

d) $.139 = \frac{13.9}{100} = 13.9\%$

.4 must first be written so that it has two decimal places

.139 has too many decimal places; we need to consider the first two as the whole part of the percentage.

Exercise 11: Rewrite each decimal as a percent by *first* rewriting the decimal as a fraction with a denominator of 100.

a) .38

b) .9

c) .05

d) .206

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REWRITING DECIMALS AND PERCENTS DIRECTLY

It's possible to rewrite percents as decimals directly, without having to first write the fraction. A helpful way to think of rewriting back and forth between percents and decimals is with our understanding of dollars and cents.

We can express twenty-three cents in either one of two different ways: 23¢ and \$0.23. Notice that there is a direct relationship between 23¢ = \$0.23 and 23% = 0.23. The "cent" sign acts similar to the "percent" sign: removing it causes the value to be rewritten as a decimal.

It's not often that we need to consider decimal parts of a penny, but can you imagine how we might express twenty-three and a half cents? It could be written as 23.5¢ . This is a bit unusual from our daily contact with money, but actual sales tax is oftentimes many decimal places of a penny. Consider a sales tax rate of 7.5% on an item that has a retail price of \$11.95.

The tax is calculated by first rewriting 7.75% as a decimal, 0.075, then multiply that decimal by the retail price: $0.075 \times \$11.95 = \0.89625 or 89.625¢ .

Obviously, you can't pay the tax exactly as it is, but you can round it off to the nearest penny and pay that amount: 90¢ or \$0.90. (This tax is, of course, added to the retail price of \$11.95 to get a total amount of \$12.85.)

Rewriting decimals and percents:

1. Think "percents" and "cents;" think "decimals" and "dollars."
2. Removing the % is like removing the ¢: it must be rewritten as a decimal or dollar amount.

Notice the connection between the conversions from *cents to dollars* and *percent to decimals*:

Example 10: Rewrite each cent amount as a dollar amount:

- a) 18¢ b) 5¢ c) 147¢ d) 2.5¢

Procedure: a) 18¢ is \$0.18 b) 5¢ is \$0.05
c) 147¢ is \$1.47 d) 2.5¢ is \$0.025

Example 11: Rewrite each percent as a decimal:

- a) 18% b) 5% c) 147% d) 2.5%

Procedure: a) $18\% = 0.18$ b) $5\% = 0.05$
c) $147\% = 1.47$ d) $2.5\% = 0.025$

Exercise 12: Rewrite each percent as a decimal directly. (That's like writing pennies as dollars.)

- a) $64\% =$ b) $123\% =$ c) $1\% =$
- d) $7\% =$ e) $3.8\% =$ f) $0.5\% =$

Answers to each Exercise

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- Exercise 1** a) $\frac{8}{10}$ b) $\frac{45}{100}$ c) $\frac{319}{1,000}$ d) $\frac{6}{100}$
 e) $\frac{7}{10,000}$ f) $\frac{314}{100}$
- Exercise 2** a) .26 b) .3 c) .194 d) 6.34
 e) 2.3 f) 5.402 g) .06 h) .014
 i) .007
- Exercise 3** a) .43 b) .507 c) .00035
- Exercise 4** a) .75 b) .4 c) .45 d) .24
- Exercise 5** a) $0.\overline{2}$ b) $0.\overline{71}$ c) $3.\overline{189}$ d) $6.04\overline{53}$
- Exercise 6** a) $.555\dots = .\overline{5}$ b) $.36363636\dots = .\overline{36}$
 c) $.21811818\dots = .2\overline{18}$ d) $.72222\dots = .7\overline{2}$
- Exercise 7** a) $\frac{5}{9}$ b) $\frac{63}{99} = \frac{7}{11}$ c) $\frac{81}{999} = \frac{9}{111} = \frac{3}{37}$
- Exercise 8** a) In 95%, the percentage is 95. b) In 1.25%, the percentage is 1.25.
 c) In 8%, the percentage is 8.
- Exercise 9** a) $\frac{2}{100} = .02$ b) $\frac{41}{100} = .41$
 c) $\frac{3.28}{100} = \frac{328}{10,000} = .0328$ d) $\frac{135}{100} = 1.35$
- Exercise 10** a) 81% b) 3% c) 1.25% d) 250%
- Exercise 11** a) $\frac{38}{100} = 38\%$ b) $\frac{9}{10} = \frac{90}{100} = 90\%$
 c) $\frac{5}{100} = 5\%$ d) $\frac{20.6}{100} = 20.6\%$
- Exercise 12** a) .64 b) 1.23 c) .01 d) .07
 e) .038 f) .005
- Exercise 13** a) 81% b) 18% c) 7% d) 207%
 e) 13.4% f) 0.4%

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Section 0.8 Focus Exercises

1. Write each as a fraction with a power of 10 in the denominator.

a) 0.03

b) 1.6

c) 0.084

2. Rewrite each fraction as a decimal. For some, you may need to first multiply by 1 to eliminate the decimal in the numerator.

a) $\frac{61}{10}$

b) $\frac{7}{100}$

c) $\frac{209}{1,000}$

d) $\frac{108}{100}$

e) $\frac{9}{1,000}$

f) $\frac{56}{1,000}$

g) $\frac{2.7}{100}$

h) $\frac{0.6}{1,000}$

i) $\frac{0.04}{10}$

3. Divide each of these fractions to get a terminating decimal.

a) $\frac{3}{8}$

b) $\frac{7}{25}$

c) $\frac{11}{40}$

4. Divide each of these fractions to get a repeating decimal.

a) $\frac{7}{9}$

b) $\frac{9}{22}$

c) $\frac{4}{37}$

5. Rewrite each repeating decimal as a *fraction*; simplify the fraction completely.

a) $0.\overline{6}$

b) $0.\overline{54}$

c) $0.\overline{036}$

6. Rewrite each percent as a *decimal*. You may choose to first write the percent as a fraction with a denominator of 100.

a) 19%

b) 3%

c) 0.5%

d) 109%

7. Rewrite each fraction as a *percent*. You may choose to first write it as a fraction with a denominator of 100.

a) $\frac{2}{100}$

b) $\frac{9}{10}$

c) $\frac{3.6}{100}$

d) $\frac{193}{100}$

8. Rewrite each decimal as a percent. You may choose to first write the decimal as a fraction with a denominator of 100.

a) .58

b) .7

c) 1.04

d) .009

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