Section 1.10 Translating between English and Algebra

Objectives
In this section, you will learn to:

- Identify the main operation in an expression.
- Translate between English and algebra.

To successfully complete this section, you need to understand:

- Translating operations to and from English (1.1)
- The order of operations (1.7)

INTRODUCTION

There was a time when there was no algebra and much of mathematics was written in phrases or sentences. Today, we still see mathematical problems in word form, and we call them "application problems," or "word problems," or "story problems." In those problems (Chapter 3), we convert written words into algebraic symbols with constants and variables.

In preparation for the work we will do in Chapter 3, this section discusses how to translate between English and algebra. To start, though, a new concept is introduced, the *main operation*.

THE MAIN OPERATION

Every algebraic or numerical expression has one operation that, in a sense, holds the whole expression together. This operation is referred to as the *main operation*. It is the main operation that lets us know if the expression is a sum, difference, product, quotient, power, or square root.

> In an expression, the **main operation** is, according to the order of operations, the last operation that is to be applied.

For example, in the expression $5 + 6 \cdot 3$, of the two operations present (addition and multiplication), the order of operations requires us to apply multiplication first and addition last. So, the main operation is *addition* (it is applied last), and we can think of $5 + 6 \cdot 3$ as a *sum*:

$5 + 6 \cdot 3$ is a sum of two parts: 5 and $6 \cdot 3$.

However, if the expression had been written, with parentheses, as $(5 + 6) \cdot 3$, then addition must be applied first—because of the grouping symbols—and multiplication is applied last, so the main operation is *multiplication*. This means that $(5 + 6) \cdot 3$ is a *product:*

$(5 + 6) \cdot 3$ is a product of two *factors*, $(5 + 6)$ and 3.

The expressions presented above illustrate two rules about the main operation:

> **1.** If an expression contains no grouping symbols, then the main operation will be the one with the lowest rank.
>
> **2.** If an expression does contain grouping symbols, then the main operation is the *outside* operation with the lowest rank..

Example 1: Identify the *main operation* in each expression and state whether it is a sum, difference, product, quotient, power, or square root. DO NOT EVALUATE THE EXPRESSIONS.

a) $6 + 12 \div 3$ b) $5^2 - 8$ c) $24 \div 6 \cdot 2$

d) $(9 - 5)^2$ e) $(6 + 12) \div 3$

Procedure: Identify the last operation to be done *if* you were to evaluate:

	Expression	**Main Operation**	**The expression is:**
a)	$6 + 12 \div 3$	addition	a sum
b)	$5^2 - 8$	subtraction	a difference
c)	$24 \div 6 \cdot 2$	multiplication	a product
d)	$(9 - 5)^2$	the power of 2 (square)	a power
e)	$(6 + 12) \div 3$	division	a quotient

You Try It 1 Identify the *main operation* in each expression and state whether it is a sum, difference, product, quotient, power, or square root. DO NOT EVALUATE THE EXPRESSIONS. Use example 1 as a guide.

	Expression	**Main Operation**	**The expression is:**
a)	$24 \div 6 + 2$	____________	____________
b)	$24 \div (6 + 2)$	____________	____________
c)	$(12 \div 2)^2$	____________	____________
d)	$9 \cdot \sqrt{25}$	____________	____________
e)	$\sqrt{3 \cdot 12}$	____________	____________
f)	$9 - 4^2$	____________	____________

Caution: It cannot be emphasized enough that the main operation of an expression is always the *last* to be applied, not the first.

TRANSLATING FROM ENGLISH TO ALGEBRA: ONE OPERATION

Variables are particularly helpful when they represent numbers whose values we do not yet know. If we must represent the sum of *a number* and 25, we can write this as $x + 25$. We recognize $x + 25$ as a sum, and we have used the x to represent *a number*.

Why would we need to consider writing expressions such as $x + 25$? Here is a simple example.

> Scott just turned 25 when his daughter Jennifer was born. In fact, Jennifer was born on Scott's 25th birthday. As Jennifer grew older, she began to understand how to calculate her dad's age. She realized that her dad's age was the sum of her age and 25.
>
> To think about how old her dad might be at various stages in her life, Jennifer represented her own age as J and was able to write an expression for her dad's age as $J + 25$. Then she thought,
>
> "When I'm 18 and graduate from high school, Dad will be $18 + 25 = 43$;
> when I'm 22 and graduate from college, Dad will be $22 + 25 = 47$;
> when I'm 30 and start a family of my own, Dad will be $30 + 25 = 55$;
> a good age to be a grandpa."

This example also demonstrates that an algebraic expression represents a number; the number represented by $J + 25$ is the age of Jennifer's dad.

In general, if we must express an unknown number, we can use any variable we choose. A commonly used variable is x.

Recall, from Section 1.1, that we translated from an English expression, such as "the sum of 8 and 15" into a numerical expression, 8 + 15. If one of the numbers is unknown, such as "the sum of *a number* and 15," then we use a variable to represent the unknown number.

Example 2: Translate each English expression into an algebraic expression.

a)	The sum of a number and 18.	b)	The difference of a number and 3.
c)	The product of 6 and a number.	d)	The quotient of 20 and a number.
e)	The square of a number.	f)	The square root of a number.

Procedure: Use any variable to represent the unknown number. (Here, x is used.)

Answer:

a)	$x + 18$	b)	$x - 3$ (but *not* $3 - x$)
c)	$6 \cdot x$ or $6x$	d)	$20 \div x$ or $\frac{20}{x}$ (but *not* $x \div 20$ or $\frac{x}{20}$)
e)	x^2	f)	$\sqrt{x}$

You Try It 2 Translate each English expression into an algebraic expression. Use Example 2 as a guide.

a) The product of 5 and a number.

b) The difference of 6 and a number.

c) The sum of 11 and a number.

d) The quotient of a number and 4.

e) The difference of a number and 9.

f) A number squared.

TRANSLATING FROM ALGEBRA TO ENGLISH: ONE OPERATION

The four basic operations—addition, subtraction, multiplication, and division—are called *binary operations* because they each must be applied to two numbers or terms (*bi-* means *two*), and their operators—plus, minus, times, and divided by—are always between two numbers or terms.

For this reason, the English form of an expression requires the word *and* between the two terms whenever one of the four basic operations is in the expression.

For example, $x + 9$ is the sum of $\underline{x}$ *and* $\underline{9}$.

This means that, when translating from algebra to English, if we write the word *sum* (or *difference* or *product* or *quotient*), then we must also write the word *and*.

Because exponents and radicals are applied to single numbers or terms, they are called *unary operations*, and their translations do not use the word *and*.

Now let's translate from algebra to English. Any variable in the algebraic expression should be translated as *a number* in English.

Example 3: Translate each algebraic expression into an English expression.

a) $x - 9$ b) y^3 c) $5 \cdot x$ d) $p \div 8$.

Procedure: Translate the variable as *a number*. Parts a), c), and d) all contain a binary operation and require the word *and* in the translation. Part b) contain a unary operation, so *and* is not used.

Answer:

a) The difference of a number *and* 9.

b) The cube of a number. (or, *A number cubed*.)

c) The product of 5 *and* a number.

d) The quotient of a number *and* 8.

You Try It 3 Translate each algebraic expression into an English expression. Use Example 3 as a guide.

a) $\sqrt{w}$

b) $m + 4$

c) $7 \cdot m$

d) $10 - y$

TRANSLATING FROM ALGEBRA TO ENGLISH: THE MAIN OPERATION

When an expression—English or algebraic—contains two operations, we rely on the main operation to give us an accurate translation. For example, the expression $5 + 2x$, contains both multiplication and addition. Would it be appropriate to call $5 + 2x$, a product or a sum?

Because addition has a lower rank than multiplication, it is applied last. Therefore, addition is the main operation. This means that the expression $5 + 2x$, is a sum, referring to its main operation:

The sum of 5 and $2x$.

Within this expression, is *sub-expression*, namely $2x$. $2x$ is a product, and multiplication is a *sub-operation* of the original expression, $5 + 2x$. This sub-expression is translated as <u>the product of 2 and a number.</u> This means the full English translation of $5 + 2x$ is

The sum of 5 and <u>the product of 2 and a number.</u>

Before we do a full translation of an expression containing two operations, let's practice translating only the main operation, demonstrated in Example 4.

Example 4: Translate each algebraic expression into an English expression. Translate only the main operation.

Procedure: Identify the expression by its main operation, then write its meaning in English. In parts b) and d), the main operation is a unary operation and the sub-expression is a quantity.

	Expression	Main operation	Answer (in English):
a)	$x^2 - 3$	Subtraction	The difference of x^2 and 3.
b)	$\sqrt{x+2}$	Radical	The square root of $(x + 2)$.
c)	$\sqrt{x} + 2$	Addition	The sum of $\sqrt{x}$ and 2.
d)	$(20 \div x)^4$	Power	The fourth power of $(20 \div x)$.
e)	$20 \div x^4$	Division	The quotient of 20 and x^4.

You Try It 4 Translate each algebraic expression into an English expression. Translate only the main operation. Use Example 4 as a guide.

	Expression	Main operation	In English
a)	$10 + x^2$	____________	________________________________
b)	$\sqrt{25 - x}$	____________	________________________________
c)	$x^2 \div 4$	____________	________________________________
d)	$6(4 - x)$	____________	________________________________
e)	$(x + 1)^5$	____________	________________________________
f)	$\sqrt{x} - 8$	____________	________________________________

TRANSLATING FROM ALGEBRA TO ENGLISH: TWO OPERATIONS

As mentioned, our task for Example 4 and You Try It 4 was to translate only the main operation. In this next example, we will translate both the main operation and the sub-operation. We have already seen $5 + 2x$ translated fully into

The sum of 5 and <u>the product of 2 and a number.</u>

The sub-expression is underlined to make it easier to read.

Let's look at some different example and see step-by-step diagrams to help us better understand how to translate from algebra to English. First consider the expression $3(x - 4)$. Of the two operations, multiplication and subtraction, multiplication is applied last and is the main operation. So, the expression is a product, and can be translated this way:

Step 1: Identify the main operation—multiplication—and write it first: "The product of ..." Because it is a binary operation, write the word *and*, leaving spaces for the two values that are being multiplied.

$3 \cdot (x - 4)$

The product of ______ and ______

Step 2: Place the two values being multiplied into the spaces around *and*.

$3 \cdot (x - 4)$

Step 3: Because there is a sub-expression, $(x - 4)$, we must now translate it. $(x - 4)$ is a difference:

The product of <u>3</u> and <u>$(x - 4)$</u>

the difference of a number and 4.

The full translation of $3(x - 4)$ is: The product of 3 and <u>the difference of a number and 4.</u>

When translating to an English form, we can show any sub-expression in parentheses. For example, once we recognize the main operation of $7x + 2$ as addition, we can write that it is "The sum of $(7x)$ and 2." The purpose of this is to recognize the sub-expression within the full expression. Here is a diagram for the translation of $7x + 2$:

Step 1: Identify the main operation—addition—and write it first: "The sum of ..." Because it is a binary operation, write the word *and*, leaving spaces for the two values that are being added. Also, place parentheses around the sub-expression, $7x$.

$(7x) + 2$

The sum of ______ and ______

Step 2: Place the two values being added into the spaces around *and*.

$(7x) + 2$

The sum of <u>$7x$</u> and <u>2</u>

Step 3: Because there is a sub-expression, $7x$, we must now translate it. $7x$ is a product:

the product of 7 and a number.

The full translation of $7x + 2$ is: The sum of <u>the product of 7 and a number</u> and 2.

If an operation is a unary operation, such as the square root or a power, then its translation does not use the word *and*. This is demonstrated in this next example. Translate $(x - 6)^2$ into English:

Step 1: Identify the main operation (square) and write it first. *Square* is <u>not</u> a binary operation, so the word *and* is not used. Leave space for the value that is being squared

$(x - 6)^2$

The square of ________

Step 2: Place the value being squared into the space after "The square of ..."

$(x - 6)^2$

The square of <u>$(x - 6)$</u>

Step 3: Because there is a sub-expression, $(x - 6)$, we must now translate it. $(x - 6)$ is a difference:

the difference of a number and 6.

The full translation of $(x - 6)^2$ is: The square of <u>the difference of a number and 6.</u>

Example 5: Translate each algebraic expression into an English expression.

a) $4 - x^2$ b) $(y + 9)^5$ c) $\sqrt{6w}$ d) $(x + 8) \div 2$

Procedure: Use the step-by-step approach in the examples above, and identify the main operation and the sub-expression.

	Main operation	**Sub-expression**	**In English):**
a)	Subtraction	x^2	The difference of 4 and x^2.
		Full answer:	The difference of 4 and <u>the square of a number</u>.
b)	Power	$(y + 9)$	The fifth power of $(y + 9)$.
		Full answer:	The fifth power of <u>the sum of a number and 9</u>.
c)	Square root	$6w$	The square root of $(6w)$.
		Full answer:	The square root of <u>the product of 6 and a number</u>.
d)	Division	$x + 8$	The quotient of $(x + 8)$ and 2.
		Full answer:	The quotient of <u>the sum of a number and 8</u> and 2.

<u>You Try It 5</u> Translate each algebraic expression into an English expression. <u>Translate the full expression.</u> Use Example 5 as a guide.

a) $(4x)^2$ b) $\sqrt{m} - 5$ c) $5(w + 1)$ d) $8 - x \div 2$

	Main operation	**Sub-expression**	**In English:**
a)	__________	__________	________________________________
		Full answer:	____
b)	__________	__________	________________________________
		Full answer:	____
c)	__________	__________	________________________________
		Full answer:	____
d)	__________	__________	________________________________
		Full answer:	____

TRANSLATING FROM ENGLISH TO ALGEBRA: TWO OPERATIONS

As you might expect, translating to algebra from an English expression containing two operations also requires three steps, and they are the reverse of the steps presented above.

1. In the English expression, the main operation is written first. Prepare to translate by first circling the main operation and placing parentheses around the sub-expression.

2. Write the expression, still in English, with only the sub-expression translated into algebra. If the main operation is a binary operation, underline the word "and" at this time. (This helps us identify the two values of the binary operation.)

3. Translate the full expression into algebra.

Here are some examples of this type of translation. They are diagrammed using the three steps.

Example A: The product of 3 and the difference of a number and 4.

Step 1: The first operation mentioned is *product*. This is the main operation; circle it.

The sub-expression is "the difference of a number and 4." Place parentheses around that part of the expression.

The product of 3 and (the difference of a number and 4).

Step 2: Rewrite the expression in English, translating only the sub-expression.

The product of 3 <u>and</u> $(x - 4)$.

Step 3: The main operation is multiplication, a binary operation. The multiplication symbol must be placed between two values, 3 and $(x - 4)$.

$$3 \cdot (x - 4)$$

This expression could also be written as $3(x - 4)$.

Example B: The sum of the product of 5 and a number and 9.

Step 1: The first operation mentioned is *sum*. This is the main operation; circle it.

The sub-expression is “the product of 5 and a number.” Place parentheses around that part of the expression.

The sum of $\big($the product of 5 and a number$\big)$ and 9.

Step 2: Rewrite the expression in English, translating only the sub-expression.

The sum of $\big(5 \cdot x\big)$ <u>and</u> 9.

Step 3: The main operation is addition, a binary operation. The plus sign must be placed between two values, $(5 \cdot x)$ and 9. ($5 \cdot x$ can also be written as $5x$.)

$$5x + 9$$

Example C: The square root of the quotient of a number and 4.

Step 1: The first operation mentioned is *square root*. This is the main operation; circle it.

The sub-expression is “the quotient of a number and 5.” Place parentheses around that part of the expression.

The square root of $\big($the quotient of a number and 4$\big)$.

Step 2: Rewrite the expression in English, translating only the sub-expression.

The square root of $\big(x \div 4\big)$.

Step 3: The main operation is the square root, a unary operation. The radical must be placed completely around the value $(x \div 4)$.

$$\sqrt{x \div 4}$$

Let’s put this into practice.

You Try It 6 Translate each expression into algebra. Follow the guidelines of the examples above.

	English Expression	**Algebraic Expression**
a)	The product of 6 and the difference of 4 and a number.	____________________
b)	The quotient of the sum of 5 and a number and 6.	____________________
c)	The sum of 8 and the product of 5 and a number.	____________________
d)	The difference of 10 and the product of 9 and a number.	____________________
e)	The square root of the product of 25 and a number.	____________________
f)	The fifth power of the sum of a number and 1.	____________________

You Try It Answers

You Try It 1:
a) Addition; a sum
b) Division; a quotient
c) Square; a power
d) Multiplication; a product
e) Radical; a square root
f) Subtraction; a difference

You Try It 2:
a) $5 \cdot x$ or $5x$
b) $6 - x$
c) $11 + x$
d) $x \div 4$ or $\frac{x}{4}$
e) $x - 9$
f) x^2

You Try It 3:
a) The square root of a number .
b) The sum of a number and 4.
c) The product of 7 and a number.
d) The difference of 10 and a number.

You Try It 4:

a)	Addition	The sum of 10 and x^2.
b)	Radical	The square root of $(25 - x)$.
c)	Division	The quotient of x^2 and 4.
d)	Multiplication	The product of 6 and $(4 - x)$.
e)	Power	The fifth power of $(x + 1)$.
f)	Subtraction	The difference of $\sqrt{x}$ and 8.

You Try It 5:

a)	Power	$4x$	The square of <u>the product of 4 and a number</u>.
b)	Subtraction	$\sqrt{m}$	The difference of <u>the square root of a number</u> and 5.
c)	Multiplication	$(w + 1)$	The product of 5 and <u>the sum of a number and 1</u>.
d)	Subtraction	$x \div 2$	The difference of <u>the quotient of a number and 2</u>.

You Try It 6: a) $6(4 - x)$ b) $(5 + x) \div 6$ c) $8 + 5x$
d) $10 - 9x$ e) $\sqrt{25x}$ f) $(x + 1)^5$

Section 1.10 Exercises

Think Again.

For each expression, what is the main operation?

1. $\dfrac{4^2 - 2 \cdot 6}{9 + \sqrt{25}}$ **2.** $\sqrt{(10 - 8)(5 + 3)}$ **3.** $\left| \dfrac{8}{4} + \sqrt{36} - 9 \right|$ **4.** $\left(\dfrac{-2 \cdot 18}{9 - 5} \right)^2$

Focus Exercises.

Identify the main operation in each expression and state whether it is a sum, difference, product, quotient, power, or square root. Do not evaluate the expressions.

	Expression	Main Operation	The expression is a:
5.	$30 \div 5 \cdot 2$	________________	________________
6.	$15 - 6 \cdot 3$	________________	________________
7.	$(6 - 4)^2$	________________	________________
8.	$(15 - 6) \cdot 3$	________________	________________
9.	$18 \div (6 - 4)$	________________	________________
10.	$36 \div 3 + 3$	________________	________________
11.	$-24 \div \sqrt{9}$	________________	________________
12.	$\sqrt{9 + 16}$	________________	________________

Translate each English expression into an algebraic expression. Use any variable to represent the unknown number.

13. The difference of a number and 15.

14. The difference of 11 and a number.

15. The sum of 10 and a number.

16. The sum of a number and five.

17. The quotient of a number and 18.

18. The quotient of 16 and a number.

19. The square of a number.

20. The square root of a number.

21. The product of a number and 9.

22. The product of 7 and a number.

23. The square root of 49.

24. The square root of a number.

Translate each algebraic expression into an English expression.

25. $x - 8$	**26.** $5 - y$	**27.** $12 + v$	**38.** $w + 9$
29. $3 \cdot h$	**30.** $5x$	**31.** $20 \div y$	**32.** $m \div 6$
33. $\sqrt{k}$	**34.** x^2	**35.** m^5	**36.** $\sqrt{y}$

Translate each algebraic expression into an English expression. *<u>Translate only the main operation.</u>*

37. $x^2 + 6$	**38.** $3y - 10$	**39.** $6(m + 8)$	**40.** $9w^2$
41. $(3v)^2$	**42.** $\sqrt{25 - x}$	**43.** $(p + 4) \div 5$	**44.** $10 - \sqrt{x}$

Translate each algebraic expression into an English expression. *<u>Translate the full expression.</u>*

45. $10 - m^2$	**46.** $5x + 3$	**48.** $5(4 - w)$	**48.** $y^2 \div 4$
49. $(v + 1)^5$	**50.** $\sqrt{x - 5}$	**51.** $12 \div (p + 4)$	**52.** $\sqrt{r} - 8$

Translate each English expression into algebra.

53. The quotient of the square of a number and 4.

54. The quotient of 9 and the sum of a number and 4.

55. The sum of 3 and the quotient of a number and 6.

56. The difference of 5 and the product of a number and 9.

57. The difference of the square root of a number and 9.

58. The product of 6 and the sum of a number and 2.

59. The product of 5 and the square root of a number.

60. The square of the sum of a number and 6.

61. The square of the product of 8 and a number.

62. The sum of the product of 2 and a number and 5.

63. The difference of 2 and the sum of a number and 5.

64. The product of 3 and the square of a number.

65. The square root of the sum of a number and 2.

66. The sum of the square root of a number and 2.

67. The fourth power of the quotient of 20 and a number.

68. The quotient of 20 and the difference of a number and 8.

Think Outside the Box.

Translate each English expression into algebra.

69. The absolute value of the sum of a number and 9.

70. The sum the absolute value of a number and 6.

71. The product of the opposite of a number and 9.

72. The opposite of the difference of a number and 15.

Translate each English expression into a numerical expression and evaluate the expression.

73. The quotient of the square of 6 and 4.

74. The sum of 13 and the quotient of 18 and 6.

75. The difference of 5 and the product of 2 and 9.

76. The difference of the square root of 16 and 10.

77. The square of the difference of 4 and 7.

78. The difference of 2 and the sum of 6 and 5.

79. The square root of the sum of 13 and 12.

80. The quotient of 30 and the difference of 2 and 8.