Section 1.9 Algebraic Expressions: The Distributive Property

Objectives
In this section, you will learn to:

- Apply the Distributive Property.

To successfully complete this section, you need to understand:

- The Associative Properties (1.1)
- The Commutative Properties (1.1)
- Multiplying real numbers (1.5)

INTRODUCTION

In Section 1.8, we added and subtracted terms, and we now turn our attention to multiplication. In this section we learn how to multiply a term by an integer, a skill that will prove helpful in the coming chapters. We also take another look at the Distributive Property, this time with variables.

MULTIPLYING A TERM BY AN INTEGER

There is occasion when we must multiply a term by an integer, such as $2 \cdot (7x)$ or $-5 \cdot (3y^2)$. Because multiplication is the only operation present, we can apply the Associative Property and regroup the multiplication. For example,

$$\begin{aligned} &2 \cdot (7x) \\ = \ &(2 \cdot 7)x \\ = \ &14x \end{aligned} \qquad\qquad \begin{aligned} &-5 \cdot (3y^2) \\ = \ &(-5 \cdot 3)y^2 \\ = \ &-15y^2 \end{aligned}$$

Once we understand this procedure, it is common to simplify these expressions by multiplying the numbers directly: $2(7x) = 14x$ and $-5 \cdot (3y^2) = -15y^2$.

Note two things:

1. the multiplication symbol (raised dot) is not required in the written product, and
2. the variable factor is unaffected by the multiplication of the two numbers.

It's also possible that the integer is on the right side, as in this expression, $(-9k)(-4)$. Again, we can multiply the numbers directly, as shown at right:

$$\begin{aligned} &(-9k) \cdot (-4) \\ = \ &36k \end{aligned}$$

Example 1: Simplify each expression by multiplying.

a) $4(5m)$ b) $-3(8p)$ c) $-6(-w^2)$ d) $(y)(-7)$

Procedure: Multiply the numbers directly; the variable factors are unaffected. Also, in part c), the coefficient of w^2 is -1, and in part d), the coefficient of y is 1.

Answer: a) $20m$ b) $-24p$ c) $6w^2$ d) $-7y$

You Try It 1 Simplify each expression by multiplying. Use Example 1 as a guide.

a) $9(4c)$ b) $-6(5y^2)$ c) $-1(-12a)$

d) $-6(v)$ e) $(-r)(-4)$ f) $(-2b^2)6$

THE DISTRIBUTIVE PROPERTY

Recall, from Section 1.1, the Distributive Property of Multiplication over Addition. The number that is distributed is called the *multiplier*. The multiplier can be on the right side or the left side of the parentheses.

The Distributive Property of Multiplication over Addition

1. The multiplier on the left side:

$\boldsymbol{b}(c + d) = \boldsymbol{b}c + \boldsymbol{b}d$

2. The multiplier on the right side:

$(c + d)\boldsymbol{b} = c\boldsymbol{b} + d\boldsymbol{b}$

Note: The Distributive Property is often written without showing the multiplication sign. With the multiplication sign, the distributive Property is $b \cdot (c + d) = b \cdot c + b \cdot d$.

Let's see why the distributive property works the way it does. Consider the example **3**$(x + 6)$. Because multiplication is an abbreviation for repeated addition, this means

"The sum of **three** $(x + 6)$'s."

This can be written as $= (x + 6) + (x + 6) + (x + 6)$.

The <u>associative property</u> can be used to write that expression without parentheses:

$$= x + 6 + x + 6 + x + 6$$

We can then use the <u>commutative property</u> to reorganize the numbers:

$$= x + x + x + 6 + 6 + 6$$

Let's group the x's separately from the 6's: $= (x + x + x) + (6 + 6 + 6)$

Each grouping has *three* of something: = **three** x's + **three** 6's

Abbreviating each, we get: $= \mathbf{3}(x) + \mathbf{3}(6)$

So, from the beginning: $\mathbf{3}(x + 6) = \mathbf{3}x + \mathbf{3}(6)$

$$= 3x + 18$$

Behold, the distributive property.

Example 2: For each expression, apply the distributive property.

a) $5(7y + 3)$ b) $(-3x^2 + 9x)2$

Procedure: Distribute the multiplier to each term in the quantity, then multiply the terms.

Answer:

a) $5(7y + 3)$ The multiplier is 5. Distribute "5 times" to each term in the quantity.

$= 5(7y) + 5(3)$ Multiply the terms.

$= 35y + 15$

b) $(-3x^2 + 9x)2$ The multiplier is 2. Distribute "times 2" to each term in the quantity.

$= (-3x^2)2 + (9x)2$ Multiply the terms.

$= -6x^2 + 18x$

You Try It 2 For each expression, apply the distributive property. Use Example 2 as a guide.

a) $6(4x + 3)$ b) $(-6m^2 + 8)5$ c) $4(2y^4 + 6y^2)$

Technically, the distributive property is the product of a number (the multiplier) and a quantity (a sum of two or more terms). However, because we are familiar with negative numbers and how to multiply with them, we can slightly modify the distributive property to include subtraction.

For example, if we wish to distribute the multiplier 4 through the quantity $(5x - 3)$, the written product looks like this: $4(5x - 3)$. We can, however, think of the quantity $5x - 3$ as $5x + (-3)$, effectively changing the product to $4 \cdot (5x + (-3))$. Let's look at it one step at a time.

$4(5x - 3)$	Subtraction can be rewritten as a sum: "adding the opposite."
$= 4(5x + (-3))$	We can distribute the multiplier through to each term in the sum.
$= 4(5x) + 4(-3)$	Multiply: $4 \cdot (-3) = -12$
$= 20x + (-12)$	Rewrite the terms in a more simplified form; change *plus* (-12) to *minus* 12
$= 20x - 12$	

Actually, we don't need to go through all of that work if we remember that *the sign in front of a term belongs to that term.* For example, in the quantity $(5x - 3)$, the terms are $5x$ and -3.

So, in effect, when distributing the multiplier, 4, through to the sum $(5x - 3)$ we are multiplying 4 by both $+5x$ and -3. As we multiply 4 by $(+5x)$ we get $+20x$; and as we multiply 4 by (-3) we get -12. The $+20x$ shows up as just **20*x***, but the **-12** shows up as *minus* 12.

In other words,

$20x$ $\quad - 12$

$4 \cdot (5x - 3)$

$4(5x - 3)$	Treat $5x$ as $\underline{+5x}$ and *minus 3* as $\underline{-3}$.
$= 20x - 12$	$4(+5x) = +20x$; $4(-3) = -12$, or *minus 12*.

Notice that this requires fewer steps than before. This also means that we can distribute over subtraction as long as we understand that subtraction means the second term is negative.

Example 3: For each expression, apply the distributive property.

a) $9(4y^3 - 5)$ b) $2(4 - 6c)$ c) $(5w^2 - 2)8$

Procedure: Identify the multiplier and the signs of the terms in the quantity. Then, distributive and multiply in one step.

Answer: a) $9(4y^3 - 5)$ The multiplier is 9. The terms in the quantity are $4y^3$ and -5.

$= 36y^3 - 45$

b) $2(4 - 6c)$ The multiplier is 2. The terms in the quantity are 4 and $-6c$.

$= 8 - 12c$

c) $(5w^2 - 2)8$ The multiplier is 8. The terms in the quantity are $5w^2$ and -2.

$= 40w^2 - 16$

You Try It 3 For each expression, apply the distributive property. Use Example 3 as a guide.

a) $3(8x - 9v)$ b) $(-7x^2 - 4)3$ c) $2(2m^2 - 5k)$

THE DISTRIBUTIVE PROPERTY WITH A NEGATIVE MULTIPLIER

We can use the Distributive Property even when the multiplier is negative. We must be more careful when multiplying each term by a negative number, and we must continually remember that the sign in front of a number belongs to that number.

Consider, for example, the product of the multiplier -2 and the sum $(4x - 3)$. As a product, this looks like $-2(4x - 3)$:

$-8x$ $+6$

$\mathbf{-2} \cdot (4x - 3)$

$-2(4x - 3)$ Treat $4x$ as $+4x$ and -3 as -3.

$-2(+4x) = -8x$; $-2(-3) = +6$, or *plus 6*.

$= -8x + 6$

Let's practice using the distributive property:

Example 4: For each expression, apply the distributive property.

a) $-5(7x + 3)$ b) $-2(4c - 6)$ c) $-8(-5 + 2y)$ d) $-3(-6 - y)$

Procedure: Identify the multiplier and the signs of the terms in the quantity. Then, distributive and multiply in one step.

Answer: a) $-5(7x + 3)$ $-5\ (+7x) = -35x$; $-5(+3) = -15$, which shows up as *minus* 15.

$= -35x - 15$

b) $-2(4c - 6)$ $-2\ (+4c) = -8c$; $-2(-6) = +12$, which shows up as *plus* 12.

$= -8c + 12$

c) $-8(-5 + 2y)$ $-8\ (-5) = +40$; $-8\ (+2y) = -16y$, which shows up as *minus* $16y$.

$= 40 - 16y$

d) $-3(-6 - y)$ $-3\ (-6) = +18$; $-3\ (-y) = +3y$, which shows up as *plus* $3y$.

$= 18 + 3y$

You Try It 4 For each expression, apply the distributive property. Use Example 4 as a guide.

a) $-5(6x + 3)$ b) $-2(8k^3 - 10)$ c) $-7(-4w^2 - 2w)$

You Try It Answers

You Try It 1:	a)	$36c$	b)	$-30y^2$	c)	$12a$
	d)	$-6v$	e)	$4r$	f)	$-12b^2$
You Try It 2:	a)	$24x + 18$	b)	$-30m^2 + 40$	c)	$8y^4 + 24y^2$
You Try It 3:	a)	$24x - 27v$	b)	$-21x^2 - 12$	c)	$4m^2 - 10k$
You Try It 4:	a)	$-30x - 15$	b)	$-16k^3 + 20$	c)	$28w^2 + 14w$

Section 1.9 Exercises

Think Again.

The distributive property can be used to multiply $8 \cdot 16$ by treating 16 as (10 + 6):

$$8 \cdot (10 + 6) = 8 \cdot 10 + 8 \cdot 6 = 80 + 48 = 128$$

We could also use subtraction and treat 16 as (20 – 4):

$$8 \cdot (20 - 4) = 8 \cdot 20 - 8 \cdot 4 = 160 - 32 = 128$$

1. Show how the *distributive* property can be used with subtraction to multiply $7 \cdot 29$.

2. Show how the *distributive* property can be used with subtraction to multiply $6 \cdot 39$.

3. Show how the *distributive* property can be used with subtraction to multiply $4 \cdot 58$.

4. Show how the *distributive* property can be used with subtraction to multiply $8 \cdot 78$.

Focus Exercises.

Simplify each expression by multiplying.

5.	$8(5w)$	**6.**	$4(3x^2)$	**7.**	$-2(10m)$	**8.**	$-6(6k)$
9.	$-7(-4r)$	**10.**	$-10(-9y^2)$	**11.**	$(-5x^2)9$	**12.**	$(-2h)3$
13.	$(5x)(-7)$	**14.**	$(-4d)(-4)$	**15.**	$(p)(-9)$	**16.**	$(d^3)(-4)$
17.	$-1(-x)$	**18.**	$(3w^2)(-1)$	**19.**	$(-4y)(1)$	**20.**	$(-2x)(-1)$

For each expression, apply the distributive property.

21. $2(8x + 4)$ **22.** $7(3m + 1)$ **23.** $4(7p^2 + 3)$ **24.** $9(3x^2 + 4y)$

25. $(5w + 8)3$ **26.** $(1 + 4v)10$ **27.** $(8r^2 + 11p)2$ **28.** $(a^2 + 3b)8$

29. $5(7x - 8)$ **30.** $6(9m^2 - 5)$ **31.** $8(1 - 2v)$ **32.** $(7 - 8x^2)3$

33. $5(-6p + 1)$ **34.** $2(-7w^2 + 5)$ **35.** $1(-4 + 9q)$ **36.** $1(-3 + 10x)$

37. $4(-2x - 10)$ **38.** $3(-2q^2 - 8)$ **39.** $9(-6 - 4h)$ **40.** $11(-6 - 3k^2)$

41. $-6(5x^2 + 2x)$ **42.** $-7(y^2 + 4y)$ **43.** $-10(2w - w^2)$ **44.** $-4(8p - 9p^2)$

45. $-5(-3w^2 - 7y)$ **46.** $-2(-5c^2 - 11d)$ **47.** $-1(7a^2 - 9b)$ **48.** $-1(4p^3 - 8q)$

49. $-1(x + 4)$ **50.** $-1(3y - 10)$ **51.** $-3(a + 1)$ **52.** $-5(m + 1)$

53. $-4(y^3 - 7y)$ **54.** $-2(9m^4 - 6m)$ **55.** $-1(-5k^3 - 6k)$ **56.** $-1(-p^2 + 3p)$

57. $-5(7m^3 + m)$ **58.** $-3(9v^3 + 6v)$ **59.** $-9(-x^5 - 3x^3)$ **60.** $-7(-y^6 - 2y^4)$

Think Outside the Box.

Multiply the terms. Use the meaning of the exponent to expand each, as necessary, and then multiply.

61. $4w(6w)$ **62.** $-2x(3x^2)$ **63.** $3m^3(-10m)$ **64.** $-7y^2(-5y^3)$

Apply the Distributive Property to each expression.

65. $5x(3x - 7)$ **66.** $-9v(1 - 2v)$ **67.** $6m^2(3m - 1)$ **68.** $-4x^2(-2x^2 + 5x)$