# 1.2 Real Numbers

**Objectives**
In this section, you will learn to:

- Identify integers on the number line.
- Compare signed numbers.
- Find the absolute value of a number.
- Define *number*.
- Categorize numbers within the real number system.
- Identify irrational numbers.

## INTRODUCTION

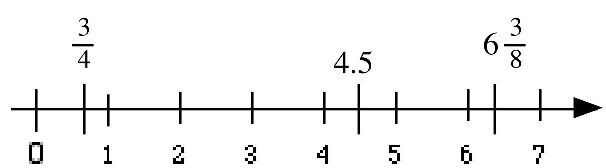
There was a TV show called “NUMB3RS” about a mathematical genius, Charlie Eppes (played by David Krumholtz), who uses mathematics to solve cases with his brother, FBI agent Don Eppes (played by Rob Morrow). Charlie uses mathematics in interesting ways to catch criminals, free hostages, and prevent evil deeds from occurring.

It’s true that Charlie uses very high level mathematics to help him solve crimes, but it’s surprising how much math we mere mortals use in our everyday lives. We use the mathematics of time to calculate how long it will take us to do a task or get from one place to another; we calculate how much we earn each week based on our wages and number of hours worked; and we use mathematics while shopping, paying bills, saving, and having fun using our hard-earned wages.

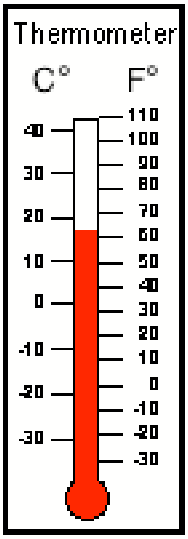
Throughout this text there are a variety of applications of mathematics to our real world, each of them with a numerical solution. To lay the foundation, we begin by learning about the numbers that make up those solutions.

## NUMBER LINES

We can represent numbers visually along a horizontal number line, including whole numbers (0, 1, 2, 3, ...), fractions $\left(\frac{3}{4}\right)$, decimals (4.5), and mixed numbers $\left(6\frac{3}{8}\right)$.




The number zero (0) has a special name on a number line; it is called the **origin**, which means the *beginning*. The arrowhead on the number line indicates the line goes on in that direction indefinitely.

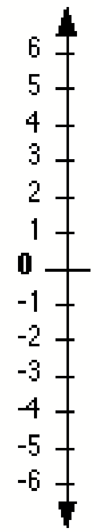


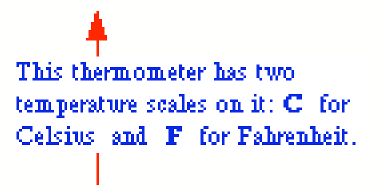

There are also vertical number lines; one example is a thermometer.

An outdoor thermometer (at left) includes numbers less than zero to indicate temperature below zero.

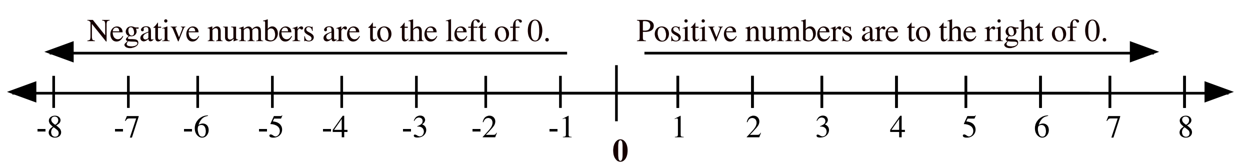
The numbers less than 0 are called **negative numbers**. Numbers greater than 0 are called **positive numbers**.

On a vertical number line (at right), 0 is still the origin, the positive numbers are above 0, and the negative numbers are below 0.




On a horizontal number line, positive numbers are to the right of 0 and negative numbers are to the left of 0. Even though 0 is in the middle now, it is still referred to as the *origin*.




| Together, positive and negative numbers are called **signed numbers**. |
|---|

| | |
|---|---|
| **Note:** | We sometimes write a positive number, such as 6, with a plus sign (+) before the number. The + is not necessary, but it gives emphasis to the fact that it is a positive number. So, +6 is the same as 6. In this case, the + indicates positive, not addition. |

Two signed numbers, one positive and one negative, with the same numeral are called **opposites**. They are on opposite sides of 0 and are the same distance from 0 on the number line.

For example, +2 and -2 are opposites. Each of these is the same distance from 0 on the number line: +2 is two units to the right of 0 and -2 is two units to the left of 0.

**Example 1:** Fill in the blanks.

a) The opposite of +9 is _______ .

b) The opposite of -32 is ______ .

c) _______ is the opposite of 12.

d) ______ is the opposite of -18.

e) -29 is the __________ of 29.

f) 25 is the _____________ of -25.

**Answer:** a) -9 b) 32 c) -12 d) 18 e) opposite f) opposite

**You Try It 1** Fill in the blanks. Use Example 1 as a guide.

a) The opposite of +6 is _____ .

b) The opposite of -9 is _____ .

c) _____ is the opposite of 13.

d) _____ is the opposite of -10.

e) -2 is the ________________ of 2.

f) +4 is the ________________ of -4.

The whole numbers and their opposites are called **integers**.

**Integers**

Each whole number and its opposite is an integer.

The list of integers can be written as ... -3, -2, -1, 0, 1, 2, 3, ...

0 (zero) is an integer but is neither positive nor negative.

**Think About It 1** What is the opposite of 0? Explain your answer.

______________________________________________________________

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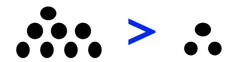
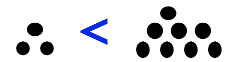
## COMPARING SIGNED NUMBERS

We know that 8 is greater than 3, and we can represent this as $8 > 3$. The symbol between the 8 and 3, >, is called the **greater than** symbol, and it is read "is greater than."




Similarly, 3 is less than 8, and we can represent this as $3 < 8$. The symbol between the 3 and 8, <, is called the **less than** symbol, and it is read "is less than."



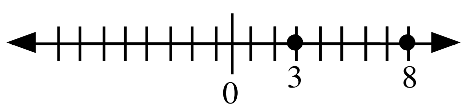

> **Note:** It is important to note the verb *is* when reading these inequality symbols. Later in Chapter 1, we will be asked to translate "3 less than a number." This expression does not contain the verb *is*, so it will have a different interpretation.

On a typical horizontal number line, *greater than* means "to the right of," and *less than* means "to the left of." So, 8 is greater than 3 because 8 is to the right of 3 on the number line.

Likewise, -5 is less than 2 because -5 is *to the left of* 2 on the number line.

$8 > 3$

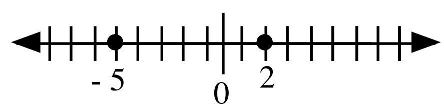
8 is greater than 3




8 is to the right of 3

$-5 < 2$

- 5 is less than 2

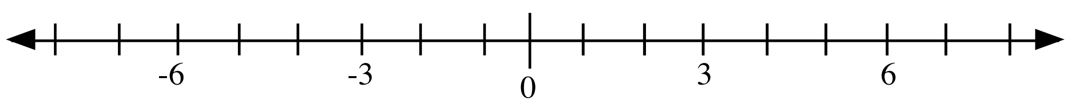



- 5 is to the left of 2

**Example 2:** Insert the correct symbol between each pair of numbers, either < (less than) or > (greater than).

a) 1 4  b) 3 -4  c) -8 -5  d) -1 -7

**Procedure:** Locate each number on the number line and decide whether the first number is to the left of or to the right of the second number.




**Answer:** a) $1 < 4$  b) $3 > -4$  c) $-8 < -5$  d) $-1 > -7$

**<u>You Try It 2</u>** Insert the correct symbol between each pair of numbers, either < (less than) or > (greater than). Use Example 2 as a guide.

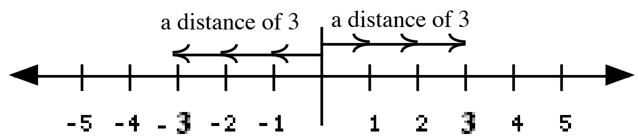
a) 9 4  b) -6 3  c) -2 -6  d) 4 -8  e) -9 -1

## ABSOLUTE VALUE

The **absolute value** of a number is its distance from 0 on the number line, and distance is always positive. So, the absolute value of a non-zero number is positive regardless of whether it is on the left side of 0 or the right side of 0.

The symbols we use to represent the absolute value of a number are called *absolute value bars* and look like this: | and | .

For example, both +3 and -3 are three units away from 0.




The *absolute value* of 3 is **3**: $|3| = 3$

The *absolute value* of -3 is also **3**: $|-3| = 3$

So, the absolute value of a positive number is positive, and the absolute value of a negative number is also positive.

| | |
|---|---|
| **Caution:** | Do not confuse "absolute value" with "the opposite." The absolute value of a positive number is *not* its opposite. |

What is the absolute value of 0? Because 0 is 0 (zero) units away from itself, $|0| = 0$.

| | | | |
|---|---|---|---|
| **Example 3:** | Evaluate each. | | |
| | a) $|9|$ | b) $|-7|$ | c) $|0|$ |
| **Procedure:** | The absolute value of a positive number is positive, and the absolute value of a negative number is also positive. | | |
| **Answer:** | a) 9 | b) 7 | c) 0 |

**You Try It 3** Evaluate each. Use Example 3 as a guide.

a) $|12|$ b) $|-5|$ c) $|+24|$ d) $|0|$ e) $|-1|$

**Think About It 2** Can the absolute value of a number ever be negative? Explain your answer.

______________________________________________________________________

______________________________________________________________________

**Think About It 3** Is the absolute value of every number a positive number? Explain your answer.

______________________________________________________________________

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### *NUMBER* DEFINED

The following definition of *number* is quite helpful in understanding how positive and negative numbers work together. Within this definition, *numerical value* is used; **numerical value** is another name for absolute value.

Every non-zero number has both a *numerical value* and a *direction*.

- A number's numerical value is its distance from 0 along the number line.
- A number's direction is either left or right, depending on its location—on the number line—in relation to 0 (zero).
- Negative numbers are to the left of 0, and positive numbers are to the right of 0.
- Zero has value but no direction.

**Example 4:** Identify the numerical value and direction of each number.

a) 6 The numerical value is 6 and the direction is to the right.

b) -8 The numerical value is 8 and the direction is to the left.

c) 0 The numerical value is 0 and it has no direction.

d) +9 The numerical value is 9 and the direction is to the right.

**You Try It 4** Identify the numerical value and direction of each number. Fill in the blanks. Use Example 4 as a guide.

a) 3 The numerical value is ______ , and the direction is ______________________________ .

b) -5 The ____________________ is _____ , and the direction is ________________________ .

c) +7 The ____________________ is _____ , and the ________________ is ______________ .

d) -4 The ____________________ is _____ , and the ________________ is ______________ .

e) +2 The ______________________________________________________________________ .

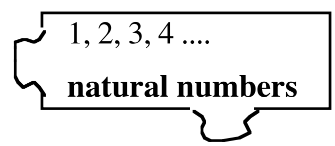
### THE REAL NUMBER SYSTEM

Every number that can be placed on the number line is a **real number**. Within the set of real numbers are many "subsets" (smaller sets of numbers within the larger set) that we can identify. The first four subsets fit into one another like puzzle pieces. These subsets are

**Natural numbers**

1, 2, 3, 4, 5, 6, 7, ...

The three dots, called an "ellipsis," indicate that the list's pattern continues indefinitely.

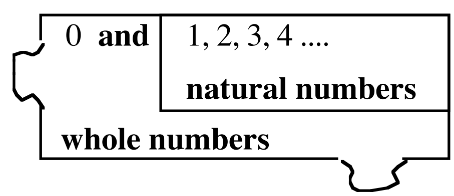



Natural numbers are also called *counting numbers*.

All of the natural numbers fit into the subset of *whole numbers:*

**Whole numbers**

0, 1, 2, 3, 4, 5, ...

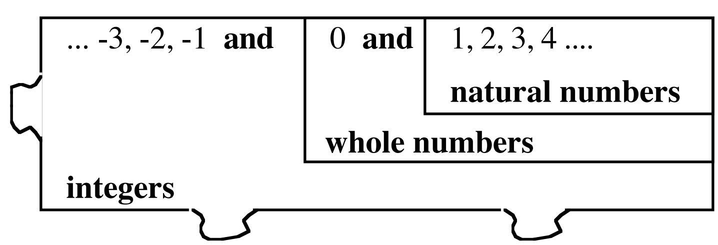



This list is just 0 included with the natural numbers.

All of the whole numbers fit into the subset of *integers:*

**Integers**

... -3, -2, -1, 0, 1, 2, 3, ...




The subset of integers includes all whole numbers and their opposites. "all positive and negative whole numbers."

**Note:** 0 (zero) is an integer but is neither positive nor negative.

All of the integers fit into the subset of *rational numbers:*

**Rational numbers**

A rational number is any number that can be written as a fraction, $\frac{a}{b}$ , $b \neq 0$, in which both the numerator, $a$, and denominator, $b$, are integers.

*fractions* **and** ... -3, -2, -1 **and** 0 **and** 1, 2, 3, 4 ....

$-\frac{2}{9}$ $\frac{1}{7}$ $\frac{6}{1}$ $\frac{3}{4}$

**natural numbers**

**whole numbers**

**integers**

*terminating decimals* *repeating decimals*

**Rational numbers** 5.09 and 0.025 $-0.\overline{3}$ and 2.161616....

**Types of rational numbers:**

**Integers**

Every integer is a rational number because it can be written as a fraction with a denominator of 1.

For example, $5 = \frac{5}{1}$ ; $-7 = \frac{-7}{1}$ ; and $0 = \frac{0}{1}$ .

**Note:** Zero (0) can be in the numerator of a fraction, $\frac{0}{5} = 0$, but never in the denominator:

$\frac{a}{0}$ is undefined, not a real number.

**Terminating Decimals**

Every decimal that has a specific number of decimal places is called a *terminating decimal*. Each terminating decimal can be written as a fraction with a denominator that is a power of 10, such as 100, 1,000, and 10,000.

For example, $0.75 = \frac{75}{100}$ ; $1.3 = \frac{13}{10}$ ; and $0.2908 = \frac{2{,}908}{10{,}000}$

**Repeating Decimals**

**Repeating decimals** never terminate and have a specific repeated block of digits called the **repetend.** Repeating decimals can be abbreviated by using a *bar* over the repetend. Any digits that precede the repetend is not included under the bar.

Here are a few examples of repeating decimals:

2.88888888888888... The repetend is 8. This number can be abbreviated as $2.\overline{8}$

0.56565656565656... The repetend is 56. This number can be abbreviated as $0.\overline{56}$

-4.2176363636363... The repetend is 63. This number can be abbreviated as $-4.217\overline{63}$

Below is a chart of the equivalent decimal values of some common fractions, all of which are rational numbers. Notice that all of the equivalent values are either terminating decimals or repeating decimals.

| Decimal Equivalents of Some Fractions | | | |
|---|---|---|---|
| $\frac{1}{2} = 0.5$ | $\frac{2}{5} = 0.4$ | $\frac{2}{7} = 0.\overline{285714}$ | $\frac{7}{8} = 0.875$ |
| $\frac{1}{3} = 0.\overline{3}$ | $\frac{3}{5} = 0.6$ | $\frac{3}{7} = 0.\overline{428571}$ | $\frac{1}{9} = 0.\overline{1}$ |
| $\frac{2}{3} = 0.\overline{6}$ | $\frac{4}{5} = 0.8$ | $\frac{4}{7} = 0.\overline{571428}$ | $\frac{2}{9} = 0.\overline{2}$ |
| $\frac{1}{4} = 0.25$ | $\frac{1}{6} = 0.1\overline{6}$ | $\frac{1}{8} = 0.125$ | $\frac{4}{9} = 0.\overline{4}$ |
| $\frac{3}{4} = 0.75$ | $\frac{5}{6} = 0.8\overline{3}$ | $\frac{3}{8} = 0.375$ | $\frac{5}{9} = 0.\overline{5}$ |
| $\frac{1}{5} = 0.2$ | $\frac{1}{7} = 0.\overline{142857}$ | $\frac{5}{8} = 0.625$ | $\frac{7}{9} = 0.\overline{7}$ |

**Think About it 4:** Based on the chart Decimal Equivalents of Some Fractions,

a) What is the decimal equivalent of $\frac{8}{9}$ ? ___________________

b) What is the decimal equivalent of $\frac{3}{9}$ ? ___________________

c) Does the decimal equivalent of $\frac{3}{9}$ appear somewhere else in the chart? Explain your answer.

_______________________________________________________________________________

**Example 5:** Justify why each is a rational number.

a) $\frac{-3}{5}$   b) 0.3333...   c) 0.167   d) 6

**Procedure:** Rational numbers can be any number that can be written as a fraction of integers, including terminating decimals and repeating decimals.

**Answer:** a) $\frac{-3}{5}$ is a rational number, because -3 and 5 are both integers.

b) 0.3333... is a rational number, because it is a repeating decimal.

c) 0.167 is a rational number, because it is a terminating decimal.

d) 6 is a rational number, because it can be expressed as the fraction $\frac{6}{1}$ .

**You Try It 5** Justify why each is a rational number. Use Example 9 as a guide.

a) -9 ________________________________________

b) 2.0815 ________________________________________

c) $3.6\overline{21}$ ________________________________________

d) $\frac{6}{-5}$ ________________________________________

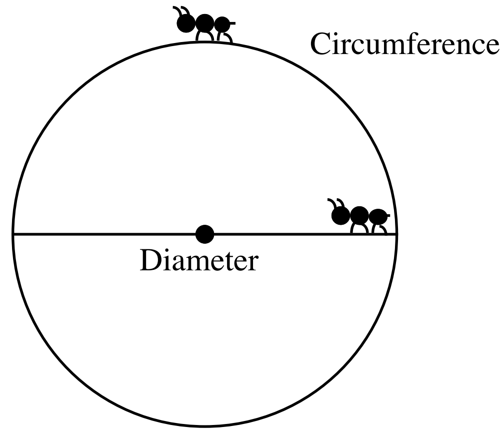
## IRRATIONAL NUMBERS

There is still one more subset of the real numbers to consider: *ir*-rational numbers, all of the numbers that cannot be made into a rational number. These are decimals that do not terminate and do not repeat (*non-terminating, non-repeating* decimals), such as 7.15642328493078425720039105... Because there is no pattern of repetition, the ellipsis at the end means that it continues *randomly* indefinitely.

One particular set of irrational numbers includes the square roots of non-perfect square numbers, such as $\sqrt{2}$ , $\sqrt{3}$ , $\sqrt{5}$ , $\sqrt{6}$ , $\sqrt{7}$ , $\sqrt{8}$ , and $\sqrt{10}$ , and so on. These are all irrational numbers.

For example, $\sqrt{2}$ is approximately 1.4142135623730950488016.... In these first twenty-two digits, nowhere does it repeat for more than two digits at a time. We don't know if there are other temporary repetitions, but by using higher mathematics, it can be shown that $\sqrt{2}$ is irrational.

Another particular irrational number is the number π (the Greek letter *pi*). It is the ratio of the circumference (the outer rim) of a circle to its diameter.




$$\pi = \frac{\text{Circumference}}{\text{Diameter}} = 3.1415926535898...$$

This ratio is a decimal that never terminates and never repeats any pattern, so it is irrational.

Here is a diagram for the set of irrational numbers.

This subset includes all non-terminating, non-repeating decimal numbers.

**Irrational numbers**

non-terminating, non-repeating decimals, such as:

- 2.7630842976215439002385799854226987...
- π (pi) π = 3.1415926535 8979323846 ...
- Square roots of non-perfect squares, such as $\sqrt{2}$ = 1.41421356237309504...

**Example 6:** For each decimal, decide if it is a rational or irrational number. State why.

a) 3.14 b) $\sqrt{5}$ c) 5.0851357400846219... d) 0.281281281...

**Procedure:** Terminating decimals and repeating decimals are rational numbers. Non-terminating, non-repeating decimals are irrational.

**Answer:**

a) Rational. It is a terminating decimal.

b) Irrational. It is the square root of a non-perfect square.

c) Irrational. It is a non-terminating, non-repeating decimal.

d) Rational. It is a repeating decimal.

**<u>You Try It 6</u>** For each decimal, decide if it is a rational or irrational number. State why. Use Example 6 as a guide.

a) $0.9\overline{7}$ b) 9.0164820359746... c) -5.1818 d) $\sqrt{8}$

Rational and irrational numbers are said to be *mutually exclusive*. This means that they are excluded from being a part of each other, so they have no numbers in common.

Together, rational and irrational numbers complete the real number system:

## The Real Number System

Begin here.

*fractions* **and** ... -3, -2, -1 **and** 0 **and** 1, 2, 3, 4 ....

$-\frac{2}{9}$ $\frac{1}{7}$

$\frac{6}{1}$ $\frac{3}{4}$

**natural numbers**

**whole numbers**

**integers**

*terminating decimals* *repeating decimals*

**Rational numbers** 5.09 and .025 $-0.\overline{3}$ and 2.161616....

**Irrational numbers**

non-terminating, non-repeating decimals, such as:

- 2.76308429762154390023857998542269 87...
- π (pi) π = 3.1415926535 8979323846 ...
- Square roots of non-perfect squares, such as
  $\sqrt{2}$ = 1.41421356237309504...

*but not*

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**Example 7:** Based on the discussion of the Real Number System, decide into which subset of real numbers each fits. Write all that apply. Choose from *natural, whole, integer, rational,* and *irrational.*

a) $-\frac{1}{2}$ b) $\sqrt{11}$ c) 6 d) 0.13 e) 0

**Procedure:** Use the Real Number System graphic to see into which category(ies) each number fits.

**Answer:** a) Rational b) Irrational c) Natural, whole, integer, and rational

d) Rational e) Whole, integer, and rational

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**You Try It 7** Decide into which subset of the real numbers each fits. Write all that apply. Choose from *natural, whole, integer, rational, and irrational.* Use Example 5 as a guide.

a) $1.\overline{43}$ b) $-\frac{4}{5}$ c) π d) -8 e) $-\sqrt{13}$ f) 168

# You Try It Answers:

**You Try It 1:** a) -6 b) 9 c) -13
d) 10 e) opposite f) opposite

**You Try It 2:** a) $9 > 4$ b) $-6 < 3$ c) $-2 > -6$ d) $4 > -8$ e) $-9 < -1$

**You Try It 3:** a) 12 b) 5 c) 24 d) 0 e) 1

**You Try It 4:** a) 3; to the right b) numerical value; 5; to the left
c) numerical value; 7; direction; to the right
d) numerical value; 4; direction; to the left
e) numerical value is 2 and the direction is to the right

**You Try It 5:** a) -9 is a rational number because it can be expressed as a fraction, $\frac{-9}{1}$ .
b) 2.0815 is a rational number because it is a terminating decimal.
c) $3.6\overline{21}$ is a rational number because it is a repeating decimal.
d) $\frac{6}{-5}$ is rational because 6 and -5 are both integers.

**You Try It 6:** a) Rational. It is a terminating decimal.
b) Irrational. It is a non-terminating, non-repeating decimal.
c) Rational. It is a terminating decimal.
d) Irrational. It is the square root of a non-perfect square.

**You Try It 7:** a) Rational b) Rational c) Irrational d) Integer, rational
e) Irrational f) Natural, whole, integer, rational

# Section 1.2 Exercises

***Think Again.***

**1.** Is $|-8|$ greater than or less than $|2|$? Explain your answer.

**2.** Is the absolute value of a number the same as the opposite of the number?

**3.** Refer to the chart of the Decimal Equivalents of Some Fractions. What is the fractional equivalent of $0.\overline{9}$ ? Show work to support your answer.

**4.** The number 2.02002000200002... has an interesting pattern of 0' and 2's. Is it a repeating decimal, as defined in this section? Is it a rational number? Explain your answers.

***Focus Exercises.***

*Identify the opposite of each number.*

| | | | |
|---|---|---|---|
| **5.** 8 | **6.** -16 | **7.** -18 | **8.** +23 |

*Insert the correct symbol between each pair of numbers, either $<$ (less than) or $>$ (greater than).*

| | | | |
|---|---|---|---|
| **9.** 0 5 | **10.** 0 -9 | **11.** -5 4 | **12.** -7 1 |
| **13.** -6 -3 | **14.** 13 0 | **15.** 1 -6 | **16.** -1 -9 |

*Find the absolute value.*

| | | | |
|---|---|---|---|
| **17.** $|-15|$ | **18.** $|+21|$ | **19.** $|-6.5|$ | **20.** $\left|\frac{3}{10}\right|$ |

*Identify the numerical value and direction of each number.*

| | | | |
|---|---|---|---|
| **21.** 6 | **22.** -12 | **23.** 0 | **24.** +10 |

*Decide into which subset of the real numbers each fits. Put a check mark in the box(es) that describe that number. Check all that apply. (Hint: Simplify first, wherever possible.)*

| | | Natural | Whole | Integer | Rational | Irrational |
|---|---|---|---|---|---|---|
| **25.** | -5 | ☐ | ☐ | ☐ | ☐ | ☐ |
| **26.** | 2.1654545454... | ☐ | ☐ | ☐ | ☐ | ☐ |
| **27.** | $-\frac{6}{7}$ | ☐ | ☐ | ☐ | ☐ | ☐ |
| **28.** | $\pi$ | ☐ | ☐ | ☐ | ☐ | ☐ |
| **29.** | $\frac{9}{3}$ | ☐ | ☐ | ☐ | ☐ | ☐ |
| **30.** | $\sqrt{6}$ | ☐ | ☐ | ☐ | ☐ | ☐ |

***Think Outside the Box.***

In a repeating decimal, the repetend can start immediately after the decimal point, as in 0.7272727272... ($0.\overline{72}$) or can start after one or more non-repeating digits, such as 0.587272727272... ($0.58\overline{72}$). For those repeating decimals that start immediately after the decimal point, we can identify its equivalent fraction using the following technique:

The numerator is the repetend, and the denominator is either 9, 99, 999 or more, depending on the number of digits in the repetend. For example,

a) $0.\overline{4}$ has one digit in the repetend, so its denominator is 9 (one 9): $0.\overline{4} = \frac{4}{9}$ ,

b) $0.\overline{72}$ has two digits in the repetend, so its denominator is 99 (two 9s): $0.\overline{72} = \frac{72}{99}$ ,

and so on.

Sometimes, the resulting fraction can be simplified. For example, $\frac{72}{99}$ can be simplified by a factor of 9: $\frac{72}{99} = \frac{72 \div 9}{99 \div 9} = \frac{8}{11}$

*Use this technique to find the fractional equivalent of each of these repeating decimals. Simplify.*

**31.** $0.\overline{6}$ **32.** $0.\overline{36}$ **33.** $0.\overline{12}$ **34.** $0.\overline{104}$