Section 1.3 Adding Integers

Objectives
In this section, you will learn to:

- Represent integers as vectors.
- Add integers using either the number line or rules of addition.

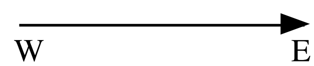
To successfully complete this section, you need to understand:

- The integer number line (1.2)
- Absolute value (1.2)

INTRODUCTION

A **vector** is a line segment of a certain length with an arrowhead at one end, indicating *direction*. One way we can use vectors is to represent the strength and direction of wind. The longer the vector, the stronger the wind it represents. The arrowhead indicates the direction the wind is blowing. For example,

The vector at right might represent a strong wind blowing from west to east:



This smaller vector might represent a more mild wind blowing from north to south:

VECTORS ALONG THE NUMBER LINE

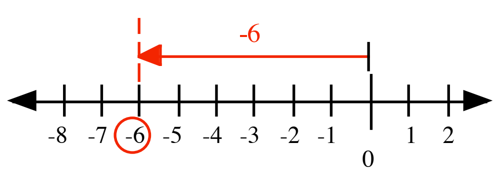
On a horizontal number line, negative numbers are always to the left of 0 and positive numbers are always to the right of 0. We can use this notion of *negative-left* and *positive-right* to further understand the new definition of *number*, introduced in Section 1.2:

Every non-zero number has both numerical value and direction. Zero has value but no direction.

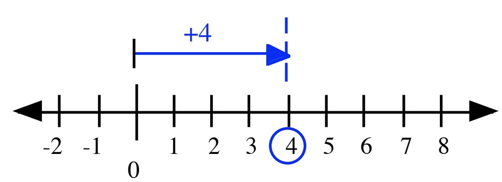
We can use vectors to represent numbers in this way. The length of the vector indicates the numerical value of the number and the arrow indicates whether it is positive or negative. A vector pointing left indicates a negative number, and a vector pointing right indicates a positive number.

For example,

Here is a vector that represents -6:



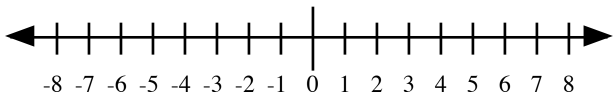
and here is a vector that represents +4:



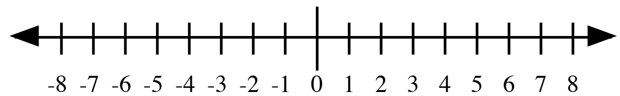
Note: As a vector, 0 has no length and no direction.

You Try It 1 Use a number line to represent the given number as a vector. For each of these, start the vector at the origin, 0.

a) -3

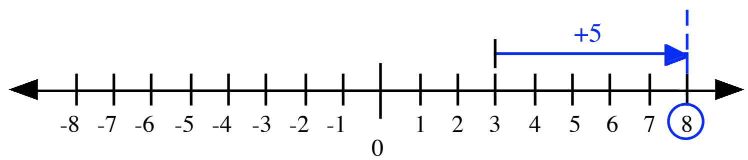


b) +7



A vector that represents a certain number, say +5, must always have the same length and direction. +5 will always have a length of 5 (its numerical value) and point to the right (because it is positive). However, a vector can have a starting point other than the origin.

If the vector representing +5 starts at +3 on the number line, it will still have a length of 5 and point to the right. It looks like this:



Notice that the vector starts at +3 and extends 5 units to the right. Notice, also, that it ends at 8. This is because 3 + 5 = 8.

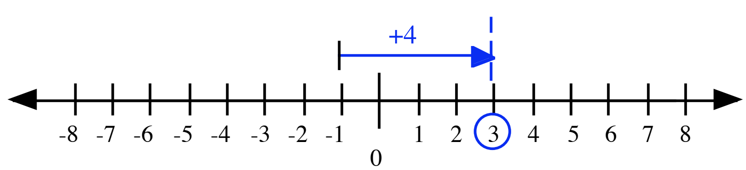
Example 1: Represent each number as a vector, starting at the given point. State where the vector stops.

a) Represent +4 starting at -1

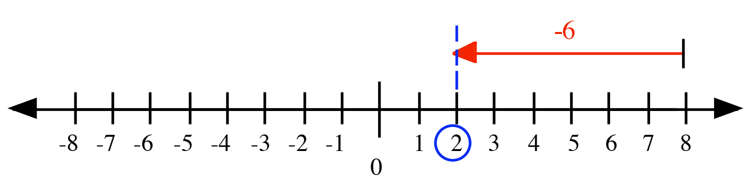
b) Represent -6 starting at 8

Answer:

a) Starting at -1, the vector for +4 has a length of 4 and points to the right. It stops at +3.

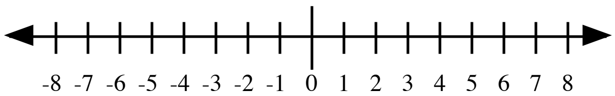


b) Starting at +8, the vector for -6 has a length of 6 and points to the left. It stops at +2.

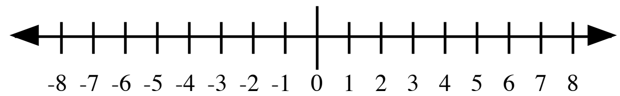


You Try It 2 Represent each number as a vector, starting at the given point. State where the vector stops. Use Example 1 as a guide.

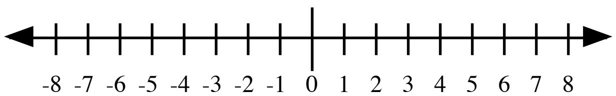
a) Represent -3 starting at +5



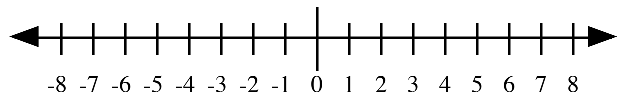
b) Represent +7 starting at -4



c) Represent 5 starting at -6

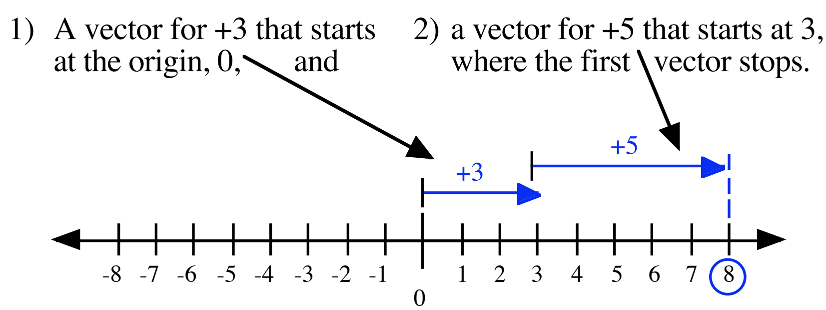


d) Represent -8 starting at +2



Adding Integers on the Number Line

We can use the number line to add numbers. For example, to add 3 + 5 we construct:

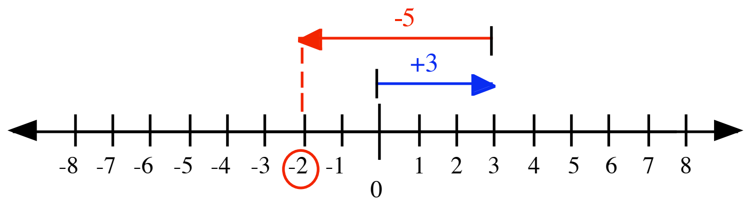


It doesn't matter if the numbers we are adding are positive or negative. Each number can be represented as a vector having a length and a direction. To add two integers, follow these guidelines:

Adding Two Integers on the Number Line
1) Draw a vector for the first number, starting from the origin, 0.
2) Draw a vector for the second number that starts where the first vector stopped.
3) The number at which the second vector stops is the sum.

We can add 3 and -5 by first drawing the vector for +3 (which starts at 0 and ends at 3). Then from 3 we draw the vector for -5.

Where the second vector stops, that is the sum: 3 + (-5) = -2.



Note: The sum of 3 and -5, written 3 + (-5), includes parentheses around the -5. The parentheses are used to separate the plus sign (+) from the negative sign in -5.

Example 2: Represent the sum by drawing two vectors. Use the guidelines for adding two numbers on the number line.

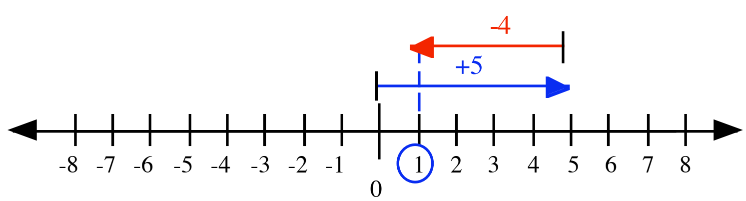
a) 5 + (-4) b) -6 + 8 c) 3 + (-7) d) -2 + (-5)

Procedure: Represent each number as a vector having length and direction.

Answer:

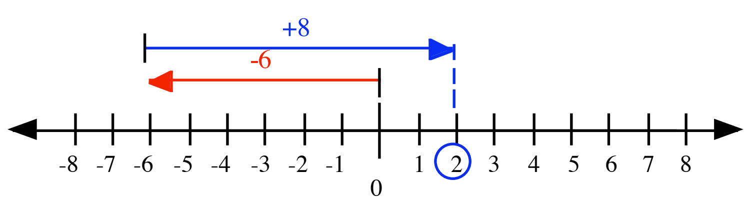
a) As vectors, 5 is 5 to the right, and -4 is 4 to the left.

5 + (-4) = 1



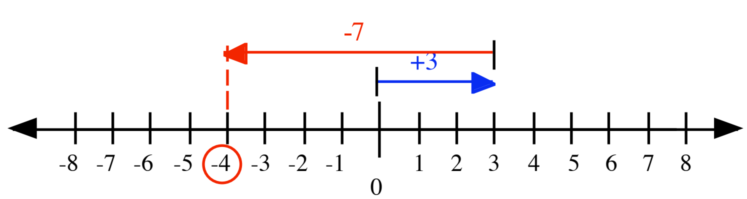
b) As vectors, -6 is 6 to the left, and 8 is 8 to the right.

-6 + 8 = 2



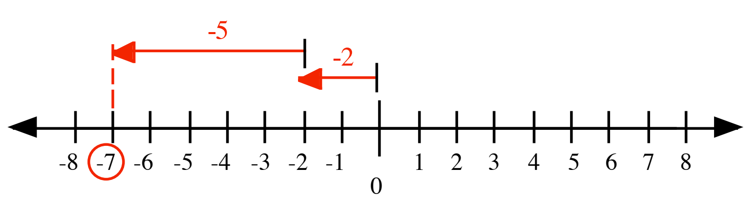
c) As vectors, 3 is 3 to the right, and -7 is 7 to the left.

3 + (-7) = -4



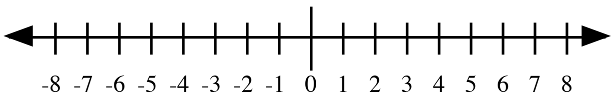
d) As vectors, -2 is 2 to the left, and -5 is 5 (more) to the left.

-2 + (-5) = -7

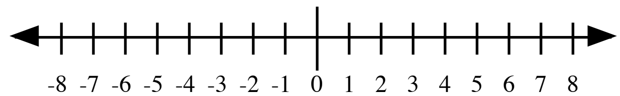


You Try It 3 Add. Use the number line to draw the vectors for each pair to evaluate the sum. Use Example 2 as a guide.

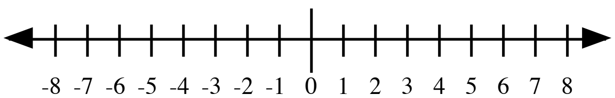
a) 8 + (-3)



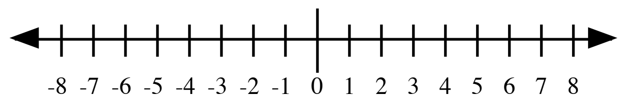
b) -6 + (-2)



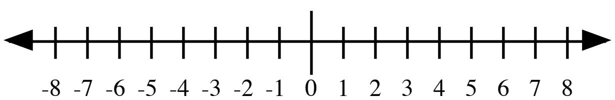
c) -3 + 6



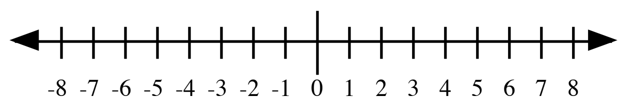
d) 2 + (-7)



e) -8 + 7



f) 6 + (-6)



As we have seen, the sum of two numbers can be found using the number line and vectors. For You Try It 4 you are encouraged to find the sums mentally by drawing the vector with your mind and not your pencil. Try to visualize the number line and the directions in which the vectors should go.

You Try It 4 Add. If you need a number line, you may draw one yourself.

a) -13 + (-3)

b) 20 + (-11)

c) 6 + (-14)

d) -10 + 17

e) -12 + 7

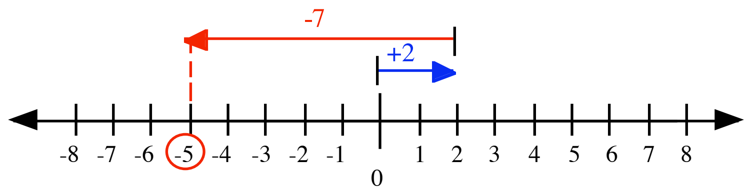
f) 16 + (-16)

ADDING INTEGERS USING THE COMMUTATIVE PROPERTY

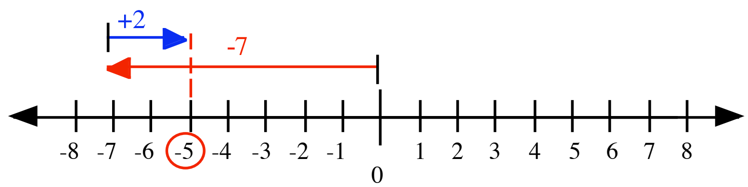
The Commutative Property of Addition works for integers as well as for whole numbers. Recall (Section 1.1) that the Commutative Property of Addition allows the sum 3 + 5 to be rewritten as 5 + 3. The same is true for integers.

For example, 2 + (-7) can be written as -7 + 2. They are both equal to -5:

$2 + (-7) = -5$



$-7 + 2 = -5$



Example 3: Rewrite each sum using the Commutative Property. Then, evaluate the original sum and the new sum.

a) 4 + (-1) b) -6 + 0 c) 3 + (-7) d) -5 + 5

Answer:

a) 4 + (-1) = -1 + 4
They both = +3.

b) -6 + 0 = 0 + (-6)
They both = -6.

c) 3 + (-7) = -7 + 3
They both = -4.

d) -5 + 5 = 5 + (-5)
They both = 0.

You Try It 5 Rewrite each sum using the Commutative Property. Then, evaluate the original sum and the new sum. Use Example 3 as a guide.

a) -10 + 3 = ______________

They both = ____________ .

b) -12 + (-4) = ______________

They both = ____________ .

c) 11 + (-8) = ______________

They both = __________ .

d) 0 + (-9) = ______________

They both = ___________ .

Rules for Adding Integers

By now you should have a sense of how to add integers on the number line. However, some integers are so large in value that placing them on a number line can be a bit difficult. For example, to evaluate the sum -23 + 48 would be a challenge on the number line.

We need to develop some rules for adding integers that have large values. These rules, though, must be consistent with our understanding of how to add small-valued integers. We'll develop the rules using smaller numbers and the number line.

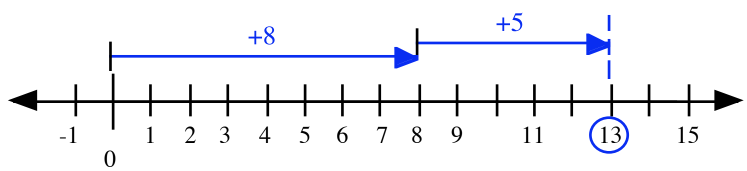
When the Signs Are the Same

The sum of two numbers with the same sign—*same direction*—is a number that has the same direction (same sign), and is either *more positive* or *more negative*.

For example,

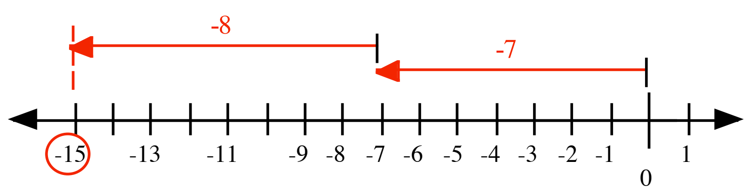
Two positive numbers, such as 8 and 5, add to a number that is *more positive*.

$$8 + 5 = 13$$



Two negative numbers, such as -7 and -8, add to a number that is *more negative*.

$$-7 + (-8) = -15$$



In each case we see the effect of adding two numbers with the same sign: the two vectors go in the same direction (both right or both left) creating a larger value in that direction. This means that we must add their numerical values and give the sum their common sign.

The Sum of Two Integers with the Same Sign

When finding the sum of two numbers of the same sign,

1) Determine the sign of the sum. It is the same as the sign of each number.

2) Add the numerical values.

Example 4: Add.

a) 12 + 26 b) -35 + (-52)

Procedure: In each sum, the numbers have the same sign, so we can use the guidelines for adding two integers with the same sign.

Answer:

a) 12 + 26

= 38

The signs are both positive, so the result is positive. Add: 12 + 26 = 38.

b) -35 + (-52)

= -87

The signs are both negative, so the result is *negative*. Add: 35 + 52 = 87.

You Try It 6 Add. Use Example 4 as a guide.

a) -15 + (-20) b) 27 + 41 c) -58 + (-32)

When the Signs Are Different

When adding two numbers of different signs (one positive and the other negative), we can use the Commutative Property and write the sum so that the larger-valued number is first. This is not a requirement, but it will help us develop the rule.

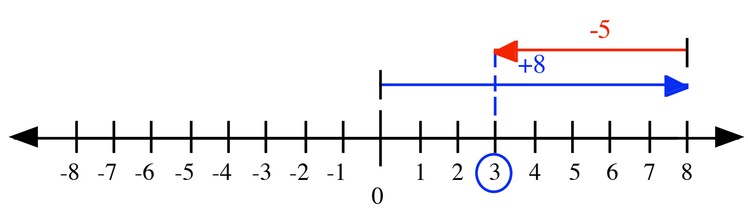
Note: The expression *larger-valued number* refers to the number with the larger numerical value.

For example, in the sum -5 + 8, the second number, 8, has the larger numerical value. If we choose, we can write the sum with the larger-valued number first: 8 + (-5).

Likewise, in the sum 3 + (-7), the second number, -7, has the larger numerical value. If we choose, we can write the sum with the larger-valued number first: -7 + 3.

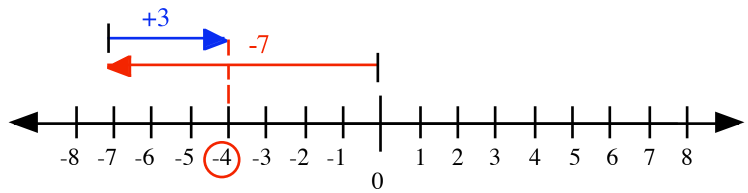
When written this way, the first vector will go in the direction of the larger-valued number. The second vector will go in the opposite direction, toward 0. However, because the second number has a smaller numerical value, the result will be on the same side of 0 as the first, larger-valued number.

For the sum 8 + (-5), the first vector, 8, stretches away from 0, to the right. The second vector, -5, reaches back toward 0 but not all the way to 0, so the resulting sum is positive:



8 + (-5) = 3

For the sum -7 + 3, the first vector, -7, stretches away from 0, to the left. The second vector, 3, reaches back toward 0 but not all the way to 0, so the resulting sum is negative:



-7 + 3 = -4

In each, we see the effect of adding numbers with different signs: the sum will have that same sign as the larger-valued number and the vectors go in opposite directions. This means that we must subtract their numerical values: larger numerical value – smaller numerical value.

Here are the guidelines for finding the sum of two numbers with different signs:

The Sum of Two Integers with Different Signs

When finding the sum of two numbers with different signs,

1) (Optional) Use the Commutative Property, if desired, to write the larger-valued number first.
2) The sign of the sum is the same as the sign of the larger-valued number.
3) Find the difference of (subtract) the two numerical values:
   larger numerical value – smaller numerical value.

Example 5: Add.

a) 29 + (-35) b) -37 + 49

Procedure: In each sum, the numbers have different signs, so we can use the guidelines for adding two integers with different signs.

Answer:

a) 29 + (-35) The signs are different. The larger-valued number, -35, is written second. We can write -35 first.

= -35 + 29 The result is negative. Subtract: 35 – 29 = 6, so the result is **-6.**

= -6

b) -37 + 49 The signs are different. The larger-valued number, 49, is written second. We can write 49 first.

= 49 + (-37) The result is positive. Subtract: 49 – 37 = 12, so the result is **+12.**

= +12

You Try It 7 Add. Use Example 5 as a guide.

a) $18 + (-45)$ b) $-38 + 52$ c) $-19 + 35$ d) $-86 + 42$

In general, before evaluating the sum of two numbers, it is helpful to first determine whether the sum will be positive or negative. By writing the sum with the larger-valued number first, we get an immediate clue as to whether the result will be positive or negative: the result will have the same sign as the larger-valued number.

Think About It 1 Can the sum of two negative numbers ever be positive? Explain your answer or show an example that supports your answer.

____

____

Think About It 2 If two numbers have different signs but have the same numerical value, such as -3 and 3, then which sign should be given to their sum? Explain your answer.

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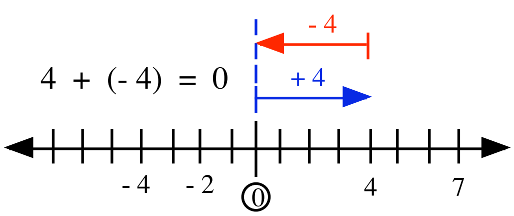
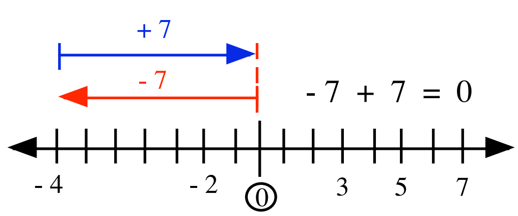
THE ADDITIVE INVERSE

A number and its opposite have vectors that are the same length but in opposite directions, so when they are added together, the sum is 0.





The sum of a number and its opposite is 0:

$a + (-a) = 0$ and $-a + a = 0$.

A more formal name for the opposite of a number is the **additive inverse.** The sum of a number and its additive inverse is always 0.

Example 6: Place the correct number in the blank space to create a sum of 0.

a) 6 + ___ = 0   b) -3 + ___ = 0   c) ___ + 5 = 0   d) ___ + (-7) = 0

Procedure: The sum of a number and its opposite is 0, so place the opposite of the sum's given number in the blank space,

Answer: a) -6   b) 3   c) -5   d) 7

You Try It 8 Place the correct number in the blank space to create a sum of 0. Use Example 6 as a guide.

a) 14 + ___ = 0   b) -8 + ___ = 0   c) ___ + (-2) = 0   d) ___ + 12 = 0

THE SUM OF MORE THAN TWO INTEGERS

A sum may contain more than two integers. If there is more than one positive integer, or more than one negative integer, we can use the Commutative Property and add the positive integers separately from the negative integers. We can then add the remaining two integers.

Example 7: Find the sum. 2 + (-3) + (-5) + 4

Procedure: This sum contains two positive numbers (2 and 4) and two negative numbers (-3 and -5). Use the Commutative Property to write the sum with the two positive numbers first.

Answer:

2 + (-3) + (-5) + 4	Rewrite the expression with the positive numbers first
= 2 + 4 + (-3) + (-5)	Add the positive numbers separately from the negative numbers.
= 6 + (-8)	This can be written as -8 +6.
= -2	

Note: The equal signs in the answer in Example 7 are called **continued equal signs**. They are used to show that the expression in one step is equivalent to the expression in the next step. Notice that the second and third equal signs are directly below the one above it. This helps us keep our work organized and easy to read.

You Try It 9 Find each sum. Use Example 7 as a guide. Use continued equal signs to show the progress from step to step.

a) $6 + (-4) + (-9) + 1$

b) $-5 + (-6) + 3 + (-7)$

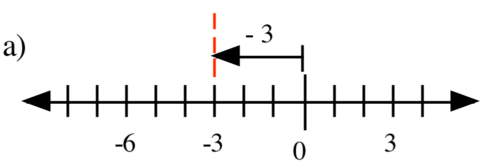
c) $-14 + 10 + (-3) + 9$

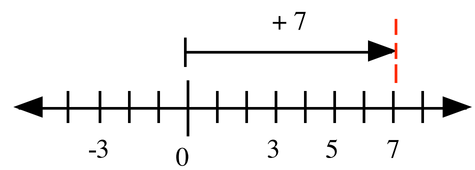
d) $20 + (-13) + (-6) + 4$

You Try It Answers

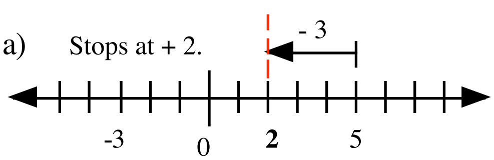
(Note: Positive answers may also be written with a plus sign in front.)

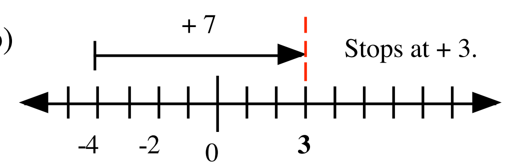
You Try It 1:

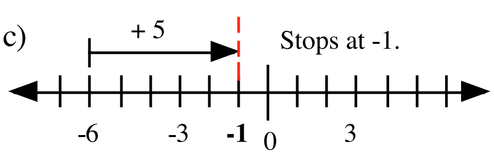
a) 

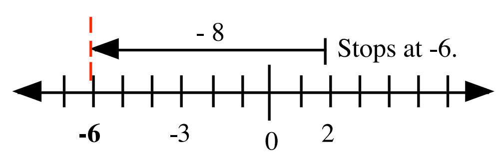
b) 

You Try It 2:

a) 

b) 

c) 

d) 

You Try It 3: a) 5 b) -8 c) 3 d) -5
e) -1 f) 0

You Try It 4: a) -16 b) 9 c) -8 d) 7
e) -5 f) 0

You Try It 5:	a)	3 + (-10); both = -7			b)	-4 + (-12); both = -16		
	c)	-8 + 11; both = 3			d)	-9 + 0; both = -9		
You Try It 6:	a)	-35	b)	68	c)	-90		
You Try It 7:	a)	-27	b)	14	c)	16	d)	-44
You Try It 8:	a)	-14	b)	8	c)	2	d)	-12
You Try It 9:	a)	-6	b)	-15	c)	2	d)	5

Section 1.3 Exercises

Think Again.

1. Can the sum of two negative numbers ever be positive? Explain your answer or show an example that supports your answer. *(Refer to Think About It 1)*

2. If a is a positive number, then under what circumstances will $-a + b$ be positive?

Focus Exercises.

Find the sum.

3.	9 + (-6)	**4.**	11 + (-4)	**5.**	4 + (-9)	**6.**	1 + (-6)
7.	-7 + (-2)	**8.**	-9 + (-4)	**9.**	2 + 8	**10.**	1 + 17
11.	-4 + 7	**12.**	-1 + 9	**13.**	-10 + 8	**14.**	-9 + 1
15.	-8 + 8	**16.**	-10 + 10	**17.**	9 + (-9)	**18.**	1 + (-1)
19.	0 + 1	**20.**	8 + 0	**21.**	0 + (-1)	**22.**	-2 + 0
23.	-1 + 7	**24.**	5 + (-8)	**25.**	1 + (-4)	**26.**	2 + (-2)
27.	+6 + (+12)	**28.**	-14 + (-2)	**29.**	+16 + (-2)	**30.**	-1 + (+2)
31.	11 + (-11)	**32.**	-12 + (-19)	**33.**	-4 + 5	**34.**	14 + 4
35.	55 + (-12)	**36.**	-55 + (-17)	**37.**	-41 + 56	**38.**	-24 + (-44)
39.	57 + (-21)	**40.**	-16 + 16				

41. $5 + (-9) + 6$

42. $9 + (-16) + 6$

43. $-4 + (-5) + 7$

44. $-6 + 9 + (-8)$

45. $-5 + (-2) + (-1)$

46. $-9 + (-6) + 10$

47. $2 + (-1) + 4 + (-15)$

48. $7 + (-2) + (-6) + 9$

49. $-9 + 6 + (-5) + 6$

50. $-6 + 9 + 0 + 2 + (-4) + (-2)$

Place the correct number in the blank space.

51. $9 + \_\_\_ = 0$

52. $1 + \_\_\_ = 0$

53. $-4 + \_\_\_ = 0$

54. $-12 + \_\_\_ = 0$

55. $\_\_\_ + 3 = 0$

56. $\_\_\_ + 11 = 0$

57. $\_\_\_ + (-1) = 0$

58. $\_\_\_ + (-15) = 0$

Think Outside the Box.

Evaluate.

59. $(-4 + 9) \cdot [11 + (-6)]$

60. $[80 + (-44)] \div (-16 + 28)$

61. $[8 + (-5)]^2$

62. $\sqrt{(-12 + 67)}$