Section 1.5 Multiplying and Dividing Real Numbers

Objectives
In this section, you will learn to:

- Multiply signed numbers.
- Divide signed numbers.
- Multiply and divide signed fractions

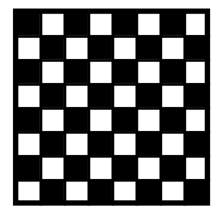
To successfully complete this section, you need to understand:

- Commutative Property of Multiplication (1.1)
- Associative Property of Multiplication (1.1)
- Adding integers (1.3)
- Reciprocal (R.3)
- Multiplying and dividing fractions (R.3)

INTRODUCTION

We now turn our attention to multiplication and division of real numbers. Just as we developed rules for the addition of real numbers, so shall we develop rules for their multiplication.

The rules for addition and rules for multiplication are different, just like rules for chess and checkers are different, and just like rules for soccer and football are different. These games may be played on the same checkerboard or on the same playing field, but you can't use the rules of one game to play the other.



Although the numbers will look the same, we can't use the rules of addition to evaluate an expression of multiplication.

THE PRODUCT OF A POSITIVE NUMBER AND A NEGATIVE NUMBER

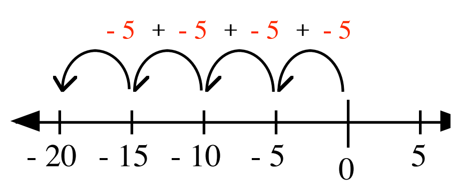
Multiplication of whole numbers is an abbreviation for repeated addition. For example, $3 \cdot 4 = 12$ because $3 \cdot 4$ means the sum of *three* 4's: $4 + 4 + 4 = 12$.

With this idea in mind, let's consider what happens when we add *two* -5's or $(-5) + (-5)$. We know that the answer is -10, but when we abbreviate this sum and turn it into multiplication, it looks like $2 \cdot (-5)$. This introduces us to the idea of multiplying signed numbers.

➤ **Positive x Negative:**

First: $2 \cdot (-5) = (-5) + (-5) = -10$.

Furthermore, $3 \cdot (-5) = (-5) + (-5) + (-5) = -15$.



We could add another (-5)—making it $4 \cdot (-5)$—and get -20, a number that is *more* negative, further to the left of 0 on the number line.

To further illustrate multiplying a positive number and a negative number, consider the progression of products in Think About It 1.

Think About It 1 Look carefully at this series of products. What pattern(s) do you see? Complete the final three products.

$3 \cdot 3 = 9$
$3 \cdot 2 = 6$
$3 \cdot 1 = 3$
$3 \cdot 0 = 0$
$3 \cdot (-1) =$ _____
$3 \cdot (-2) =$ _____
$3 \cdot (-3) =$ _____

____

____

The diagram of $4 \cdot (-5)$ on the number line, and Think About It 1 illustrate the first rule for multiplying signed numbers:

> The product of a positive number and a negative number is a negative number.
>
> Informally, we can write this as $(+) \cdot (-) = (-)$
>
> or pos · neg = neg

Applying this rule of multiplication suggests that $4 \cdot (-6) = -24$ and $10 \cdot (-8) = -80$. The Commutative Property of Multiplication allows us to write each product in a different order: $-6 \cdot 4 = -24$ and $-8 \cdot 10 = -80$.

So,

> The product of a negative number and a positive number is also a negative number.
>
> Informally, we can write this as $(-) \cdot (+) = (-)$
>
> or neg · pos = neg

This is true whether the numbers are integers, fractions, or decimals. We explore the product of signed fractions a little later in this section.

Example 1: Evaluate each product.

a) $-6 \cdot 3$ b) $7(-8)$ c) $-2(1.5)$

d) -0.5×0.8 e) $-1 \cdot 4$ f) $-9 \cdot 0$

Procedure: The product of a positive number and a negative number (in any order) is negative.

Answer: a) -18 b) -56 c) -3

d)	-0.4	e)	-4	f)	0	Any number times 0 is 0: $a \cdot 0 = 0$

You Try It 1 Evaluate each product. Use Example 1 as a guide.

a) $-7 \cdot 3$ b) $9 \cdot (-4)$ c) $0 \cdot (-8)$ d) -2×2.5 e) $3.8 \cdot (-1)$

THE PRODUCT OF TWO NEGATIVE NUMBERS

In Section 1.4 we were introduced to three meanings of the dash: as *minus, negative,* and *the opposite of.* A fourth meaning of the dash is "-1 times." We can see this fourth meaning of the dash in this example:

the opposite of 4 is -4
and **-1** · 4 is -4

This illustrates that *the opposite of a number* is the same as *-1 times* the number.

In other words, we can think of "multiplication by -1" as the same as "the opposite of."

By that same reasoning, if we multiply -1 by a negative number, we get the opposite of that negative number (a positive number) as a result:

$(-1) \cdot (-4) = +4$ (the opposite of -4).

We now have four meanings of the dash:

1. Minus $(6 - 2 = 4)$
2. Negative (-5 is "negative 5")
3. "the opposite of" (-8 is "the opposite of" 8)
4. "-1 times" (-4 is $-1 \cdot 4$)

This fourth meaning allows us to write any negative number as a product of -1 and a positive number. For example, -9 can be written as $-1 \cdot 9$ or as $9 \cdot (-1)$.

The fourth meaning of the dash also tells us that multiplying a number by -1 is the same as the opposite of the number. For example, $-1 \cdot 7 = -7$ (the opposite of 7) and $-1 \cdot (-5) = +5$ (the opposite of -5).

We use the different meanings of the dash to help us develop the rule for multiplying two negative numbers.

Think About It 2 What is the opposite of the opposite of 6? Explain your answer.

____

____

THE PRODUCT OF TWO NEGATIVE NUMBERS

Recall (Sec. 1.1) the Associative Property of Multiplication. It says that in a product, we can change the grouping of the factors without affecting the resulting product: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

Let's use the Associative Property with the fourth meaning of the dash. For example, consider the product (-5) · (-4).

$(-5) \cdot (-4)$	Use the fourth meaning of the dash to write the -5 as 5 · (-1), making this a product of three numbers.
$= 5 \cdot (-1) \cdot (-4)$	Use the Associative Property to group the last two factors.
$= 5 \cdot \underline{(-1) \cdot (-4)}$	Using the third meaning of the dash, (-1) · (-4) is *the opposite of* -4, which is +4.
$= 5 \cdot (+4)$	5 · (+4) = +20.
$= +20$	

Because 5 · (-1) · (-4) = +20, it must also be true that (-5) · (-4) = +20, and this suggests that the product of two negative numbers is positive.

> The product of two negative numbers is a positive number.
>
> Informally, we can write this as (–) · (–) = (+)
>
> or neg · neg = pos

Think About It 3 Look carefully at this series of products. What pattern(s) do you see? Complete the final three products.

3 · (-3) = -9
2 · (-3) = -6
1 · (-3) = -3
0 · (-3) = 0
-1 · (-3) = _____
-2 · (-3) = _____
-3 · (-3) = _____

____

____

Example 2: Evaluate each product.

a) $-10 \cdot (-6)$ b) $-5 \cdot (-5)$ c) $-0.3 \times (-12)$ d) $(-0.4) \times (-0.7)$

Procedure: The product of two negative numbers is a positive number.

Answer: a) +60 or 60 b) +25 or 25

c) 3.6 or +3.6 d) 0.28 or +0.28

You Try It 2 Evaluate each product. Use Example 2 as a guide.

a) $-2 \cdot (-8)$ b) $-0.5 \times (-9)$ c) $(-7) \cdot (-7)$ d) $-0.6 \times (-0.4)$

The next rule is actually a rule you've been using most of your life, although it is likely you have never had to think of it this way:

> The product of two positive numbers is a positive number.
>
> Informally, we can write this as $(+) \cdot (+) = (+)$
>
> or pos · pos = pos

Here is a summary of the product of two real numbers. The four rules presented in this section can be reduced to just two rules:

> **Multiplying Two Real Numbers:**
>
> 1) If the signs of the factors are different, the product will be negative.
> 2) If the signs of the factors are the same, the product will be positive.
>
> Also, if zero is a factor, the product will be 0: $a \cdot 0 = 0$ and $0 \cdot a = 0$.

Example 3: Multiply.

a) $-8 \cdot 6$ b) $3 \cdot 9$ c) $-4 \cdot (-2)$ d) $5 \cdot (-7)$ e) $-1 \cdot 0$

Procedure: Use the rules of multiplying two signed numbers.

Answer:		**Rule**
a)	$-8 \cdot 6 = -48$	The signs are different; the product is negative.
b)	$3 \cdot 9 = +27$ or 27	The signs are the same; the product is positive.
c)	$-4 \cdot (-2) = +8$ or 8	The signs are the same; the product is positive.
d)	$5 \cdot (-7) = -35$	The signs are different; the product is negative.
e)	$-1 \cdot 0 = 0$	One of the numbers is 0, so the product is 0.

You Try It 3 Find the product. Use Example 3 as a guide.

a) $-7 \cdot (-6)$ b) $-8 \cdot 7$ c) $2 \cdot (-6)$ d) $0 \cdot (-9)$ e) $8 \cdot 4$

THE PRODUCT OF MORE THAN TWO REAL NUMBERS

What if we were to multiply more than two signed numbers? No problem, the Associative Property and the Commutative Property allow us to multiply in any order we choose. What we find, though, is that

1) if the expression contains an odd number of negatives, then the end product is negative; and
2) if the expression contains an even number of negatives (or none at all), then the end product is positive.

Example 4: Find each product.

a) $2 \cdot 3 \cdot 4$ b) $-2 \cdot (-3) \cdot 4$

c) $-2 \cdot 3 \cdot 4$ d) $-2 \cdot (-3) \cdot (-4)$

Procedure: Notice the number of negatives in the expression. Use the Associative Property to group and multiply two numbers at a time. Notice whether the end product is positive or negative.

Answer:

a)	$2 \cdot 3 \cdot 4$	No negatives.	b)	$-2 \cdot (-3) \cdot 4$	Two negatives.
=	$(2 \cdot 3) \cdot 4$	↓	=	$[-2 \cdot (-3)] \cdot 4$	↓
=	$6 \cdot 4$	↓	=	$+6 \cdot 4$	↓
=	24	The end product is positive.	=	24	The end product is positive.
c)	$-2 \cdot 3 \cdot 4$	One negative.	d)	$-2 \cdot (-3) \cdot (-4)$	Three negatives.
=	$(-2 \cdot 3) \cdot 4$	↓	=	$[-2 \cdot (-3)] \cdot (-4)$	↓
=	$-6 \cdot 4$	↓	=	$+6 \cdot (-4)$	↓
=	-24	The end product is negative.	=	-24	The end product is negative.

So, three negative factors results in a negative product. What do you think will happen if there are four negative factors? Let's find out:

	$-1 \cdot (-2) \cdot (-3) \cdot (-4)$	There are two pairs of products, each with two negative factors.
=	$[-1 \cdot (-2)] \cdot [(-3) \cdot (-4)]$	Each pair results in a positive end product.
=	$(+2) \cdot (+12)$	
=	24	

Did you expect the answer to be positive? Notice that every *pair* of negative factors produces a positive number. We can actually make rules out of this:

Multiplying Two or More Signed Numbers

If there is no zero factor, then:

1) If there is an odd number of negative factors, the end product is negative; and

2) if there is an even number of negative factors, then the end product will be positive.

Example 5: Multiply. First decide whether the product is positive or negative, then multiply the numerical values of the factors.

a) (6) (-3) (-2)   b) (-5) (4) (10)

c) (-2) (-8) (-10)   d) (-4) (-3) (-5) (-1)

Answer:	Product of Factors	Number of Negatives	Result is	Product

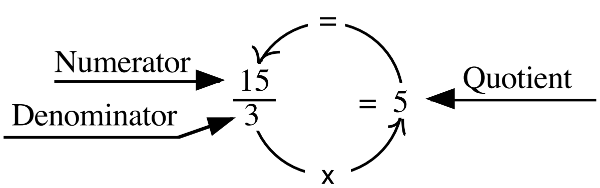
a)	(6) (-3) (-2)	Two (even)	Positive	(6) (-3) (-2) = +36
b)	(-5) (4) (10)	One (odd)	Negative	(-5) (4) (10) = -200
c)	(-2) (-8) (-10)	Three (odd)	Negative	(-2) (-8) (-10) = -160
d)	(-4) (-3) (-5) (-1)	Four (even)	Positive	(-4) (-3) (-5) (-1) = +60

You Try It 4 Multiply. First decide whether the product will be positive or negative, then multiply numerical values. Use Examples 4 and 5 as guides.

a) (2) (-4) (-5) b) (-4) (5) (3) (-2) c) (-3) (-1) (-6)

DIVIDING SIGNED NUMBERS

There is a circular connection between multiplication and division. In a fraction, such as $\frac{15}{3}$, the denominator times the quotient (answer) is the numerator:



Denominator	x	Quotient	=	Numerator
3	x	5	=	15

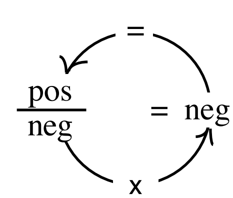
This circular relationship is also seen in standard division: $15 \div 3 = 5$ because $3 \times 5 = 15$.

This circular notion of multiplication and division works for negative numbers as well. We must rely on our knowledge of multiplying real numbers to understand the results of division.

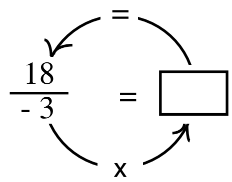
For example, we know that $-3 \times (-5) = +15$. Putting this set of numbers into the division process, we see that $\frac{15}{-3} = -5$ and $15 \div (-3) = -5$.

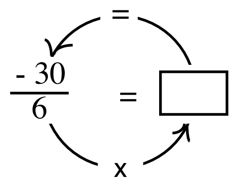
Put into words: Because Negative x Negative = Positive, it must be that

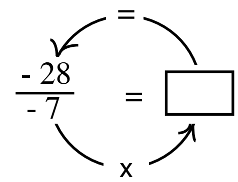
Positive ÷ Negative = Negative, or $\frac{\text{Positive}}{\text{Negative}}$ = Negative



You Try It 5 Use your understanding of the rules for multiplying signed numbers—and the circular relationship between multiplication and division—to fill in the missing quotient.

a) 

b) 

c) 

The rules of dividing signed numbers are exactly the same as those for multiplication:

Dividing Signed Numbers

In any division, either by fraction or standard form,

1) If there is an odd number of negative factors, the quotient will be negative; and

2) if there is an even number of negative factors, the quotient will be positive.

Note: We may never divide by 0. Zero may be a numerator but not a denominator.

Let's do some examples.

Example 6: Evaluate.

a) $\frac{-21}{3}$ b) $\frac{36}{-9}$ c) $-35 \div (-7)$ d) $32 \div 4$ e) $\frac{0}{-2}$ f) $\frac{-4}{0}$

Procedure: First decide whether the quotient is positive or negative (by the number of negatives), then divide the numerical values.

Answer:

	Quotient	Number of Negatives	Result is	Quotient
a)	$\frac{-21}{3}$	One (odd)	Negative	= -7
b)	$\frac{36}{-9}$	One (odd)	Negative	= -4
c)	$-35 \div (-7)$	Two (even)	Positive	= 5 or +5
d)	$32 \div 4$	Zero (even)	Positive	= 8 or +8

e) $\frac{0}{-2}$	One, but 0 is in the numerator.	0	= 0
f) $\frac{-4}{0}$	One, but 0 is in the denominator.	Undefined	None. Zero is in the denominator.

You Try It 6 Evaluate. Use Example 6 as a guide.

a) $45 \div (-5)$

b) $\frac{-36}{-3}$

c) $-42 \div 0$

d) $-28 \div (-4)$

e) $\frac{0}{-8}$

f) $\frac{-54}{9}$

NEGATIVES IN A FRACTION

The rule for division of signed numbers leads to three ways that we can express negative fractions. Consider the following:

$$\frac{-8}{2} = -4 \qquad \frac{8}{-2} = -4 \qquad -\frac{8}{2} = -4$$

Because the result is -4 for each of these, it is safe for us to say that each fraction, as written, is equivalent to the other fractions:

$$\frac{-8}{2} = \frac{8}{-2} = -\frac{8}{2}$$

This shows that a negative fraction can have a single negative sign placed anywhere within the fraction:

Rule of Negatives in a Fraction

A *single* negative sign in a fraction may be placed anywhere within the fraction without changing the value of the fraction.

$$\frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$$

Typically, if a negative is in the denominator after evaluating, we use one more step to write the negative sign in either the numerator or in front of the fraction.

For example, $\frac{7}{-9}$ can be written as either $\frac{-7}{9}$ or as $-\frac{7}{9}$ (either in the numerator or in front of the fraction, but not both) .

However, if a fraction has two negatives, then it should first be simplified to a positive fraction. For example, the fraction $\frac{-6}{-8}$ has two negatives, making the whole fraction positive, and we should first write it as $\frac{6}{8}$ before simplifying it to $\frac{3}{4}$.

Think About It 4 If a single fraction has three negative signs, such as $-\frac{-9}{-12}$, will the fraction simplify to a positive or negative value? Explain your answer.

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____

Example 7: Simplify the fraction completely.

a) $\frac{10}{-15}$ b) $\frac{-12}{-20}$

Procedure: First decide whether the fraction is positive or negative, then simplify.

Answer: a) $\frac{10}{-15}$ A single negative. Write the negative in front of the fraction.

$= -\frac{10}{15}$ $\frac{10}{15}$ simplifies by a factor of 5 to $\frac{2}{3}$.

$= -\frac{2}{3}$

b) $\frac{-12}{-20}$ Two negatives means this fraction is positive.

$= \frac{12}{20}$ $\frac{12}{20}$ simplifies by a factor of 4 to $\frac{3}{5}$.

$= \frac{3}{5}$

You Try It 7 Simplify the fraction completely. Use Example 7 as a guide.

a) $\frac{-9}{-27}$ b) $-\frac{25}{-35}$ c) $\frac{-14}{42}$ d) $\frac{24}{-60}$

MULTIPLYING SIGNED FRACTIONS

When multiplying or dividing two fractions, there can be many negative signs within. Before starting the actual multiplication or division, it is best to determine whether the result will be positive or negative. An odd number of negatives gives a result that is negative, and an even number of negatives gives a result that is positive.

Example 8: Evaluate.

a) $\frac{-2}{3} \cdot \frac{5}{-4}$ b) $\frac{-4}{9} \cdot \frac{15}{2}$

Procedure: First decide whether the end result will be positive or negative. If the end result will be negative, be sure to show the negative sign at each step.

Answer: a) $\frac{-2}{3} \cdot \frac{5}{-4}$ Two negatives mean the end result is positive. To emphasize this, place a plus sign in front and eliminate all negative signs.

$= +\frac{2}{3} \cdot \frac{5}{4}$ Multiply and simplify. $\frac{2}{3} \cdot \frac{5}{4}$ simplifies by a factor of 2 to $\frac{1}{3} \cdot \frac{5}{2} = \frac{5}{6}$.

$= \frac{5}{6}$

b) $\frac{-4}{9} \cdot \frac{15}{2}$ One negative means the end result is negative. Place a negative sign in front of the first fraction and eliminate the other negative signs.

$= -\frac{4}{9} \cdot \frac{15}{2}$ 9 and 15 simplify by a factor of 3; 4 and 2 simplify by a factor of 2.

$= -\frac{2}{3} \cdot \frac{5}{1}$ Multiply.

$= -\frac{10}{3}$

You Try It 8 Evaluate. Use Example 8 as a guide.

a) $\frac{-2}{3} \cdot \frac{-4}{5}$ b) $-\frac{15}{8} \cdot \frac{4}{3}$ c) $-\frac{10}{7} \cdot \left(-\frac{-14}{5}\right)$

THE RECIPROCAL OF A FRACTION

The *reciprocal* of a fraction, $\frac{a}{b}$, is the inverted fraction, $\frac{b}{a}$. For example, the reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$. If a fraction is negative, its reciprocal is also negative. For example, the reciprocal of $\frac{-6}{11}$ is $\frac{-11}{6}$.

Whether a fraction is positive or negative, the product of a fraction and its reciprocal is always 1. For example, $\frac{3}{8} \cdot \frac{8}{3} = \frac{24}{24} = 1$, and $\frac{-6}{11} \cdot \frac{-11}{6} = \frac{+66}{66} = 1$.

Example 9: Place the correct number in the blank space to create a product of 1.

a) $\frac{2}{5} \cdot$ ____ = 1 b) $\frac{-4}{7} \cdot$ ____ = 1 c) $\frac{1}{3} \cdot$ ____ = 1 d) $-9 \cdot$ ____ = 1

Procedure: In the blank space, place the reciprocal of the product's first number.

Answer: a) $\frac{5}{2}$ b) $\frac{-7}{4}$ c) $\frac{3}{1}$ or 3 d) $\frac{-1}{9}$

You Try It 9 Place the correct number in the blank space to create a product of 1. Use Example 9 as a guide.

a) $\frac{-3}{10} \cdot$ ____ = 1 b) $\frac{2}{9} \cdot$ ____ = 1 c) $6 \cdot$ ____ = 1 d) $\frac{-1}{2} \cdot$ ____ = 1

DIVIDING SIGNED FRACTIONS

Recall that division with fractions can be rewritten as multiplying the reciprocal of the second fraction. This is sometimes referred to as *invert and multiply*:

> To divide by a fraction, invert the second fraction and multiply.
>
> $$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Example 10: Evaluate.

a) $\frac{-1}{4} \div \frac{2}{-3}$ b) $\frac{3}{10} \div \left(-\frac{6}{25}\right)$

Procedure: First decide whether the end result will be positive or negative. If the end result will be negative, be sure to show the negative sign in front at each step.

Answer: a) $\frac{-1}{4} \div \frac{2}{-3}$ Two negatives mean the end result is positive. Change division to multiplication: $+\frac{1}{4} \cdot \frac{3}{2}$.

$= +\frac{1}{4} \cdot \frac{3}{2}$ Nothing simplifies, so multiply directly.

$= \frac{3}{8}$

b) $\frac{3}{10} \div \left(-\frac{6}{25}\right)$ One negative means the end result is negative. Change division to multiplication: $-\frac{3}{10} \cdot \frac{25}{6}$.

$= -\frac{3}{10} \cdot \frac{25}{6}$ 3 and 6 simplify by a factor of 3; 10 and 25 simplify by a factor of 5.

$= -\frac{1}{2} \cdot \frac{5}{2}$ Now multiply directly.

$= -\frac{5}{4}$

You Try It 10 Evaluate. Use Example 10 as a guide.

a) $-\frac{5}{6} \div \frac{-4}{3}$ b) $\frac{6}{12} \div \left(-\frac{7}{4}\right)$ c) $-\frac{20}{9} \div \left(-\frac{5}{6}\right)$

You Try It Answers

(Note: Positive answers may also be written with a plus sign in front.)

You Try It 1:	a)	-21	b)	-36	c)	0	d)	-5	e)	-3.8
You Try It 2:	a)	16	b)	4.5	c)	49	d)	0.24		
You Try It 3:	a)	42	b)	-56	c)	-12	d)	0	e)	32
You Try It 4:	a)	40	b)	120	c)	-18				
You Try It 5:	a)	-6	b)	-5	c)	4				
You Try It 6:	a)	-9	b)	12	c)	Undefined				
	d)	7	e)	0	f)	-6				
You Try It 7:	a)	$\frac{1}{3}$	b)	$\frac{5}{7}$	c)	$-\frac{1}{3}$	d)	$-\frac{2}{5}$		
You Try It 8:	a)	$\frac{8}{15}$	b)	$-\frac{5}{2}$	c)	-4				
You Try It 9:	a)	$\frac{-10}{3}$	b)	$\frac{9}{2}$	c)	$\frac{1}{6}$	d)	-2		
You Try It 10:	a)	$\frac{5}{8}$	b)	$-\frac{2}{7}$	c)	$\frac{8}{3}$				

Section 1.5 Exercises

Think Again.

1. Predict whether the value of $(-6)^{17}$ will be positive or negative. Explain you answer.

2. If a product has three negative factors, four positive factors, and two factors of zero, is the end result positive or negative? Explain your answer or show an example that supports your answer.

3. In the division of two integers, if the quotient is -1, then what do you know about the dividend (numerator) and divisor (denominator)?

4. In the division of two integers, if the quotient is positive, then what do you know about the dividend (numerator) and divisor (denominator)?

Focus Exercises.

Evaluate.

5. $-2(-3)$

6. $0.5 \times (-5)$

7. $9 \cdot 10$

8. $(-3)(-3)$

9. $0 \cdot (-9)$

10. $8\,(-3)$

11. -0.4×3.5

12. $-4 \cdot 15$

13. $-3 \cdot (-9)$

14. $-7 \cdot 4$

15. $6 \cdot (-1)$

16. $-15 \cdot 2$

17. $10 \cdot (-18)$

18. 1.3×0.7

19. $-0.8 \times (-3)$

20. $-3 \cdot 2 \cdot (-4)$

21. $9 \cdot (-5) \cdot 7$

22. $-1 \cdot (-6) \cdot (-9)$

23. $-3 \cdot 2 \cdot (-4) \cdot 5$

24. $-5 \cdot (-1) \cdot 0 \cdot (-3)$

25. $-3 \cdot (-1) \cdot (-4) \cdot (-2)$

26. $-32 \div 8$

27. $-56 \div (-4)$

28. $63 \div (-9)$

29. $-48 \div (-12)$

30. $-90 \div (-6)$

31. $\frac{40}{-8}$

32. $\frac{-90}{5}$

33. $\frac{-32}{4}$

34. $\frac{0}{-7}$

35. $\frac{-92}{2}$

36. $\frac{-90}{10}$

37. $\frac{-49}{0}$

38. $\frac{-12}{-3}$

39. $\frac{-35}{-7}$

40. $\frac{-32}{-4}$

41. $-\frac{5}{14} \cdot \frac{-4}{15}$

42. $\frac{8}{9} \cdot \left(-\frac{7}{12}\right)$

43. $-\frac{-8}{5} \cdot \left(-\frac{3}{4}\right)$

44. $\frac{9}{16} \div \left(-\frac{12}{32}\right)$

45. $\frac{-2}{9} \div \frac{8}{-3}$

46. $\frac{-10}{-3} \div \frac{-5}{9}$

Think Outside the Box.

Evaluate.

47. $(-1)^{24}$

48. $[-7 - (-12)] \cdot (-11 + 5)$

49. $\frac{-20 - 12}{-6 + 2}$

50. $-\frac{1}{9} \cdot \frac{15}{4} \cdot \frac{-6}{5}$

51. $\frac{-7}{10} \div \frac{14}{-5} \cdot \frac{-2}{3}$

52. $-\frac{10}{9} \div \frac{4}{3} \div \frac{-5}{2}$