# Section 1.6  The Factor Game

**Objectives**
In this section, you will learn to:

- Play the Factor Game.

To successfully complete this section,
you need to understand:

- Factor pairs (1.1)
- Adding integers (1.3)
- Multiplying real numbers (1.5)

## INTRODUCTION

Here is a game that will prove to be quite useful in Section 7.3, Factoring Trinomials.  At this point, though, it is a game that will strengthen your skills in adding and multiplying integers.

This game starts with two numbers, one is called the *product number* (product) and the other is called the *sum number* (sum).   You are asked to find a factor pair of the product number that will add to the sum number.

## THE FACTOR GAME

---

**The Factor Game**

We are given two numbers, a **product number** and a **sum number**.  We are to  find a factor pair (two factors) of the product number that add to the sum number.

The correct factor pair is called the **winning combination.**

---

**Special Notes:**   In a single Factor Game**,**

1.  it's possible that there is no winning combination, no factor pair that works, and

2.  if there is a winning combination, there will be only *one* winning combination.

To identify a possible factor pair combination, we can list some or all of the factor pairs of the product number and then identify which pair adds to the sum number.

It's easiest to understand the Factor Game through examples.  Look over each example carefully, and follow it through to get a full understanding.  To play the Factor Game efficiently, it is recommended that a factor pair table be created, like the ones shown in the examples, with the factor 1 in the first left position.

**Example 1:** Find the winning combination of the Factor Game with the given product and sum numbers. $\begin{cases} \text{Product} = 12 \\ \text{and sum} = 7 \end{cases}$

**Procedure:** Write out the factor pairs of the product, and see if any of them add to the sum.

**Answer:** Factor pairs of 12:

| | 12 | **Sum** | |
|---|---|---|---|
| 1 | 12 | 13 | ← Too large. |
| 2 | 6 | 8 | ← Closer, but not quite. |
| 3 | 4 | 7 | ← This is it! |

The winning combination is: _3 and 4_. *Check:* $3 \cdot 4 = 12$ ✓ and $3 + 4 = 7$ ✓

---

**Example 2:** Find the winning combination of the Factor Game with the given product and sum numbers. $\begin{cases} \text{Product} = 36 \\ \text{and sum} = 15 \end{cases}$

**Procedure:** Write out the factor pairs of the product, and see if any of them add to the sum. To play the Factor Game efficiently, it is recommended that *a factor pair table,* like the ones shown below, be created with the factor 1 in the first left position.

**Answer:** Factor pairs of 36:

| | 36 | **Sum** | |
|---|---|---|---|
| 1 | 36 | 37 | |
| 2 | 18 | 20 | |
| 3 | 12 | 15 | ← This is it! |
| 4 | 9 | 13 | Once you discover the correct factor pair, |
| 6 | 6 | 12 | it is not necessary to continue searching. |

The winning combination is: _3 and 12_. *Check:* $3 \cdot 12 = 36$ ✓ and $3 + 12 = 15$ ✓

---

It is possible that the factors in the winning combination are the same number. Also, as mentioned earlier, it's possible that there is no winning combination.

**Example 3:**   Find the winning combination to the Factor Game with the given product and sum numbers.

    a)   Product = 16          b)   Product = 30
       and sum = 8               and sum = 12

**Procedure:**   Write out the factor pairs of the product, and see if any of them add to the sum.

**Answer:**   a)   Factor pairs of 16:

| 16 | | **Sum** |
|---|---|---|
| 1 | 16 | 17 |
| 2 | 8 | 10 |
| 4 | 4 | 8  ← This is it! |

The winning combination is: 4 and 4.  *Check:*  $4 \cdot 4 = 16$ ✓ and $4 + 4 = 8$ ✓

   b)   Factor pairs of 30:

| 30 | | **Sum** |
|---|---|---|
| 1 | 30 | 31 |
| 2 | 15 | 17 |
| 3 | 10 | 13 |
| 5 | 6 | 11    None of these add to 12. |

There is no winning combination.

---

**You Try It 1**   Find the winning combination to the Factor Game with the given product and sum numbers.  Use Examples 1, 2, and 3 as guides.

a)   product = 24     b)   product = 30     c)   product = 36     d)   product = 15
     and sum = 10        and sum = 17        and sum = 12        and sum = 7

In Examples 1, 2, and 3, both the product number and the sum number are positive.  It's possible, though, to have one or both of them be negative.  Let's look at how the Factor Game works when the sum number is negative.

**THE FACTOR GAME AND A NEGATIVE SUM NUMBER**

You know that if the signs of two numbers are the same, then their product will be positive:

$$(+2) \cdot (+3) \ = \ +6 \qquad \text{and} \qquad (-2) \cdot (-3) \ = \ +6$$

It is also true that if the *product* of two factors is positive, then the *factors* must have the same sign, either both positive or both negative:

$$+6 \ = \ (+2) \cdot (+3) \qquad \text{and} \qquad +6 \ = \ (-2) \cdot (-3)$$

$$+6 \ = \ (+1) \cdot (+6) \qquad \text{and} \qquad +6 \ = \ (-1) \cdot (-6)$$

In the Factor Game, if the product number is positive, then the two numbers in the factor pair must have the same sign, either both positive or both negative.  It is the sign of the sum number that determines the signs of the factor pair.

For example, if the product number is +12 and the sum number is +7, the winning combination is +3 and +4, as we saw in Example 1.  However, if the product number is +12 but the sum number is -7, then the winning combination is -3 and -4.

---

| **Example 4:** | Find the winning combination of the Factor Game with the given product and sum numbers. |
|---|---|
| | a)  Product = +20 and sum = -12.  $\qquad$  b)  Product = +30 and sum = -11 |
| **Procedure:** | When the product is positive and the sum is negative, both factors in the factor pair must be negative. |

**Answer:**  a)  Factor pairs of +20:

| | +20 | Sum | |
|---|---|---|---|
| -1 | -20 | -21 | |
| -2 | -10 | -12 | ← This is it! |

The winning combination is: <u>-2 and -10</u>. *Check:*  $-2 \cdot (-10) \ = +20$ ✓  and  $-2 + (-10) = -12$ ✓

b)  Factor pairs of +30:

| | +30 | Sum | |
|---|---|---|---|
| -1 | -30 | -31 | |
| -2 | -15 | -17 | |
| -3 | -10 | -13 | |
| -5 | -6 | -11 | ← This is it! |

The winning combination is: <u>-5 and -6</u>. *Check:*  $-5 \cdot (-6) \ = +30$ ✓  and  $-5 + (-6) = -11$ ✓

---

<u>**You Try It 2**</u>      Find the winning combination of the Factor Game with the given product and sum numbers.  Use Example 4 as a guide.

a)    product $= +25$    b)    product $= +32$    c)    product $= +36$    d)    product $= +40$
      and sum $= -10$       and sum $= -12$       and sum $= -20$       and sum $= -14$

**THE FACTOR GAME AND A NEGATIVE PRODUCT NUMBER**

You know that if the signs of two numbers are different, such as +2 and -3 (or -2 and +3), then their product will be negative:

$$\left.\begin{array}{l} (-2) \cdot (+3) \\ (+2) \cdot (-3) \end{array}\right\} = -6$$

It is also true that if the product of two factors is *negative*, such as -8, then the two numbers in a factor pair must have *different* signs; one must be positive and the other must be negative, as shown in these four options:

$$-8 = \left\{\begin{array}{l} (-1) \cdot (+8) \\ (+1) \cdot (-8) \end{array}\right. \qquad \text{or} \qquad -8 = \left\{\begin{array}{l} (-2) \cdot (+4) \\ (+2) \cdot (-4) \end{array}\right.$$

Furthermore, the sum of two numbers with different signs could be either positive or negative; it is the number with the largest numerical value that determines the sign of the sum.

In the Factor Game, if the product number is negative, then the two numbers in the factor pair must have different signs.   The sign of the sum number indicates which sign the larger factor will have.

**Example 5:** Find the winning combination of the Factor Game with the given product and sum numbers.

a) Product = -12 and sum = +1.      b) Product = -30 and sum = -13

**Procedure:** When the product is negative, one of the factors must be positive and the other negative. The factor with the larger numerical value will have the same sign as the sum number.

**Answer:** a) The sum is positive, so the factor with the larger numerical value is positive and the factor with the smaller numerical value is negative.

Factor pairs of -12:

| -12 | | Sum |
|---|---|---|
| -1 | +12 | +11 |
| -2 | +6 | +4 |
| -3 | +4 | +1    ← This is it! |

The winning combination is: _-3 and +4_. Check: $-3 \cdot (+4) = -12$ ✓ and $-3 + (+4) = +1$ ✓

b) The sum is negative, so the factor with the larger numerical value is negative and the factor with the smaller numerical value is positive.

Factor pairs of -30:

| -30 | | Sum |
|---|---|---|
| +1 | -30 | -29 |
| +2 | -15 | -13    ← This is it! |

The winning combination is: _+2 and -15_. Check: $+2 \cdot (-15) = -30$ ✓ and $+2 + (-15) = -13$ ✓

---

**Think About It 1:** We know that $-3 + (-10) = -13$. In Example 5b), why isn't the winning combination _-3 and -10_?

_____

_____

---

**You Try It 3** Find the winning combination of the Factor Game with the given product and sum numbers. Use Example 5 as a guide.

a) product = -28 and sum = +3

b) product = -36 and sum = -5

c) product = -20 and sum = +12

d) product = -70 and sum = -3

# You Try It Answers

*Note: The numbers in a winning combination can be written in either order.*

**You Try It 1:**    a)  4 and 6    b)  2 and 15    c)  6 and 6    d)  No combination.

**You Try It 2:**    a)  -5 and -5    b)  -4 and -8    c)  -2 and -18    d)  -4 and -10

**You Try It 3:**    a)  -4 and +7    b)  +4 and -9    c)  No combination.    d)  +7 and -10

# Section 1.6 Exercises

*Think Again.*

**1.** If both the product number and the sum number are positive, under what circumstances is it possible for the sum number to be greater than a positive product number? Explain your answer or show an example that supports your answer.

**2.** There is one relatively small number for which the product number and the sum number are the same. What is this number?

*Focus Exercises.*

*Find the winning combination of the Factor Game with the given product and sum numbers.*

**3.** product = +10
and sum = +7

**4.** product = +8
and sum = +9

**5.** product = +30
and sum = +13

**6.** product = +18
and sum = +11

**7.** product = +24
and sum = +14

**8.** product = +20
and sum = +21

**9.** product = +21
and sum = +10

**10.** product = +18
and sum = +9

**11.** product = +24
and sum = +4

**12.** product = +50
and sum = +15

**13.** product = +30
and sum = +7

**14.** product = +45
and sum = +14

**15.** product = +6
and sum = -5

**16.** product = +8
and sum = -6

**17.** product = +28
and sum = -11

**18.** product = +16
and sum = -10

**19.** product = +36
and sum = -13

**20.** product = +18
and sum = -19

| | | |
|---|---|---|
| **21.** product = +10<br>and sum = -11 | **22.** product = +14<br>and sum = -15 | **23.** product = +24<br>and sum = -9 |
| **24.** product = +24<br>and sum = -25 | **25.** product = +48<br>and sum = -16 | **26.** product = +42<br>and sum = -23 |
| **27.** product = -20<br>and sum = +8 | **28.** product = -28<br>and sum = +12 | **29.** product = -30<br>and sum = +1 |
| **30.** product = -24<br>and sum = +5 | **31.** product = -30<br>and sum = +12 | **32.** product = -45<br>and sum = -4 |
| **33.** product = -42<br>and sum = +1 | **34.** product = -20<br>and sum = -9 | **35.** product = -25<br>and sum = -10 |
| **36.** product = -48<br>and sum = -13 | **37.** product = -60<br>and sum = -28 | **38.** product = -60<br>and sum = -7 |
| **39.** product = -60<br>and sum = -4 | **40.** product = -36<br>and sum = -5 | **41.** product = -36<br>and sum = 0 |
| **42.** product = -49<br>and sum = 0 | **43.** product = +16<br>and sum = 0 | **44.** product = +25<br>and sum = 0 |

*Think Outside the Box.*

*Create a Factor Game (product number and sum number) for a classmate that has the given feature.*

**45.** The product and sum numbers are both positive, and the winning combination contains two different numbers.

**46.** The product number is positive and the sum number is negative, and the winning combination contains two different numbers.

**47.** The winning combination contains two positive numbers that are the same.

**48.** The winning combination contains two negative numbers that are the same.

**49.** The product and sum numbers are both positive, but there is no winning combination.

**50.** The product and sum numbers are both negative, but there is no winning combination.

**51.** The product number is negative and the sum number is positive, and the winning combination contains two different numbers.

**52.** The product number is negative and the sum number is negative, and the winning combination contains two different numbers.

**53.** The winning combination contains two numbers that are opposites.