1.2 Adding Integers

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Representing Numbers on the Number Line: The Vector

We call positive and negative numbers **signed numbers**. For example, - 7 has the negative sign in front and + 9 has the positive sign in front.

In Section 1.1 you were introduced to the new definition of number:

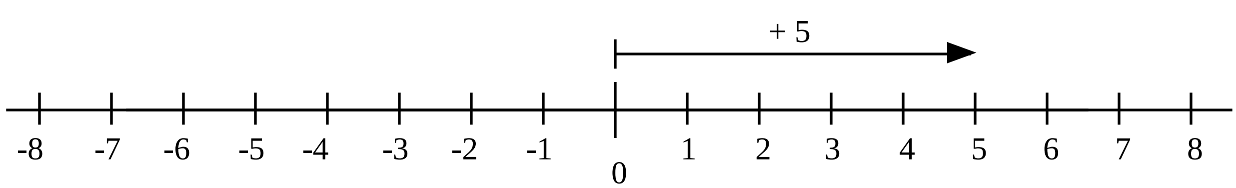
> Every number has both **value** and **direction**.

We can represent this new notion of number using a linear length—representing the numerical value of the number—and an arrow—representing the direction of the number. This linear length is called a **vector**, and though it has both value (length) and direction (left or right) it doesn't have a specific starting point. To represent a number's place on the number line with a vector, we'll need to start the vector at the origin, 0.

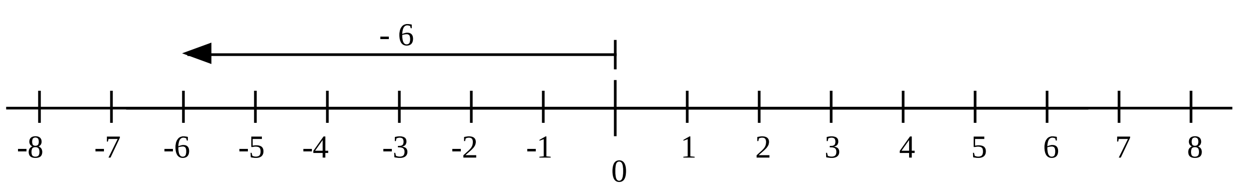
> **One special note:** 0 (zero), as a vector, has *no length* and *no direction*. In other words, 0 is neither positive nor negative.

Example 1: Represent each number as a vector.

a) the **vector** for + 5 has a <u>length of 5</u>, starting at 0, and points to the <u>right</u>:

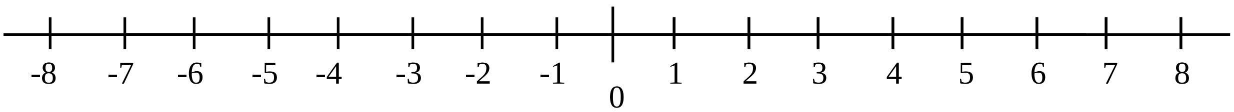


b) the **vector** for - 6 has a <u>length of 6</u>, starting at 0, and points to the <u>left</u>:

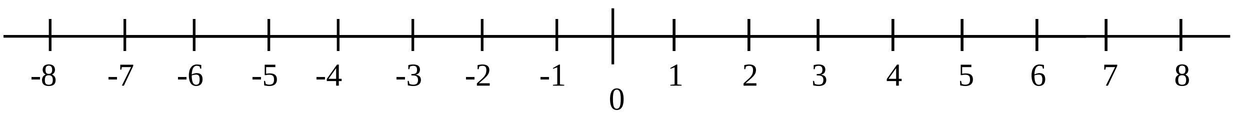


Exercise 1 Use the number line below to represent the given number as a **vector** with both a linear length and a direction. For each of these, start every length at the origin, 0.

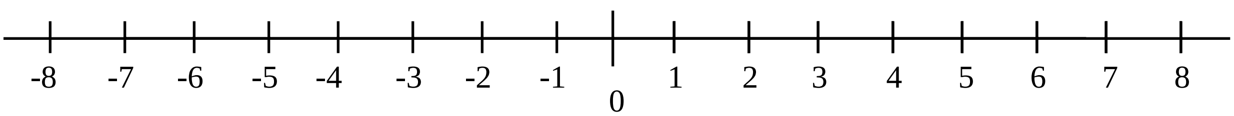
a) **- 3**



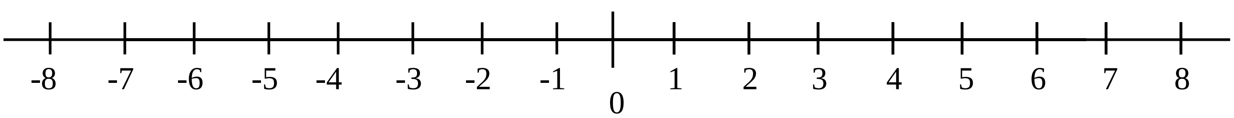
b) **+ 7**



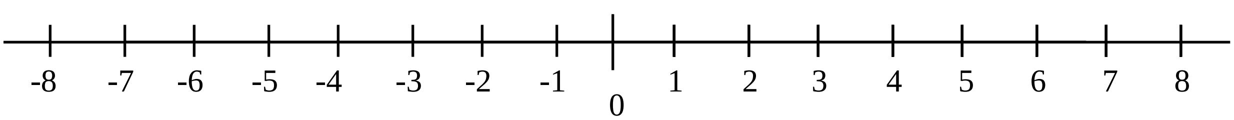
c) **+ 2**



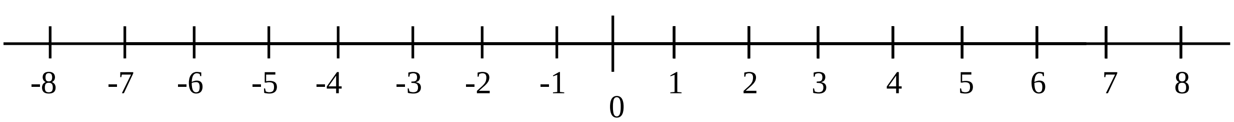
d) **- 1**



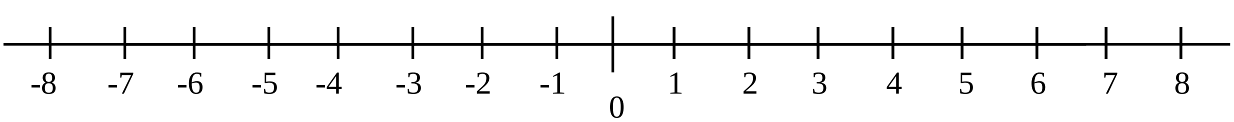
e) **- 8**



f) **- 4**

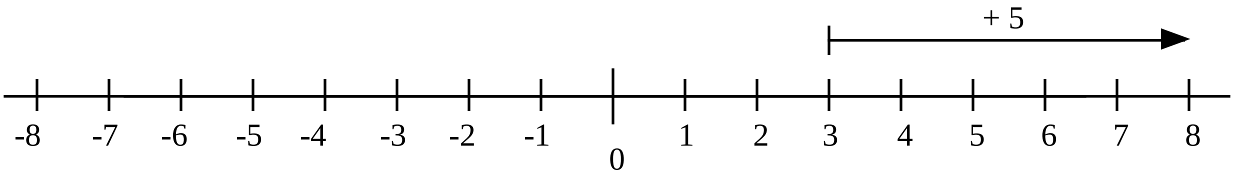


g) **+ 3**



A vector that represents a certain number, say + 5, must always have the same length and direction. For example, + 5 will *always* have a length of 5 (its numerical value) and point to the right (because it is positive). *However*, vectors are allowed to have different starting points.

For example, we know that the vector representing + 5 has a length of 5 and points to the right. If that vector starts at + 3 (on the number line) it will *still* have a length of 5 and point to the right. That would look like this:



Notice that the vector starts at + 3 and extends 5 spaces to the right. Notice, also, that <u>it ends at **8**</u>. You might guess that the reason it ends at 8 is because 3 + 5 = 8, and you'd be right!

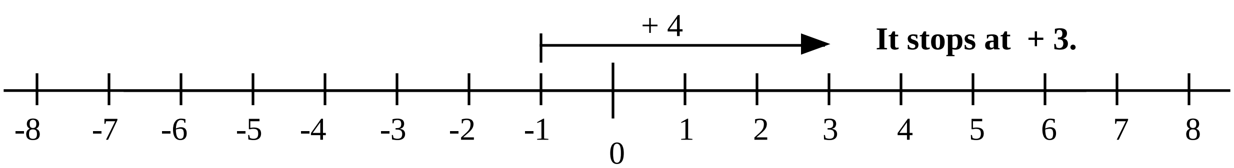
Example 2: Represent each number as a vector, starting at the given point. State where the vector stops.

a) Represent + 4 starting at - 1 b) Represent - 6 starting at + 8

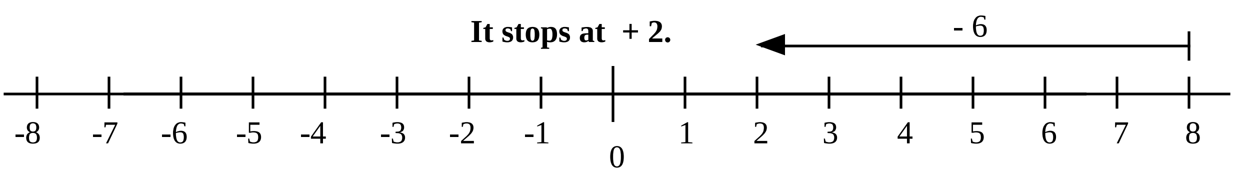
c) Represent + 3 starting at - 7 d) Represent - 2 starting at - 4

Answer Represent each number as a vector having length and direction (left or right).

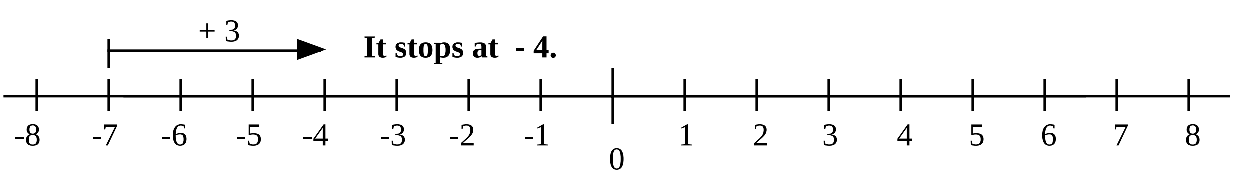
a) The **vector** for + 4 has a <u>length of 4</u> and points to the <u>right</u>; here it is, starting at - 1.



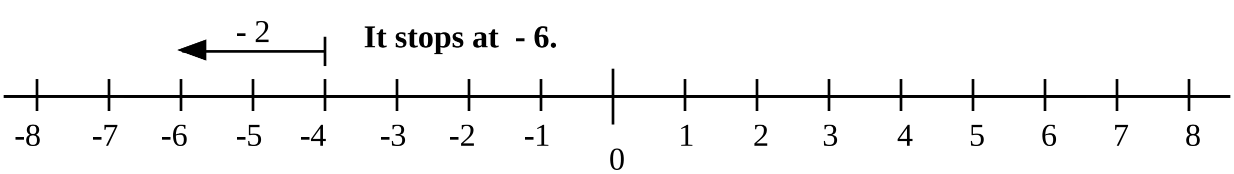
b) the **vector** for - 6 has a <u>length of 6</u> and points to the <u>left</u>; here it is, starting at + 8.



c) The **vector** for + 3 has a <u>length of 3</u> and points to the <u>right</u>; here it is, starting at - 7.

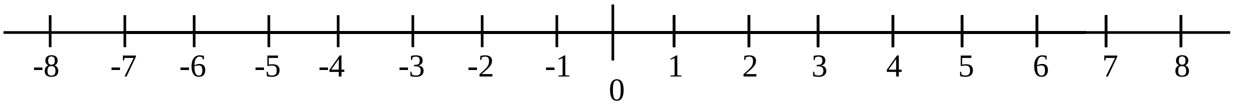


d) the **vector** for - 2 has a <u>length of 2</u> and points to the <u>left</u>; here it is, starting at - 4.

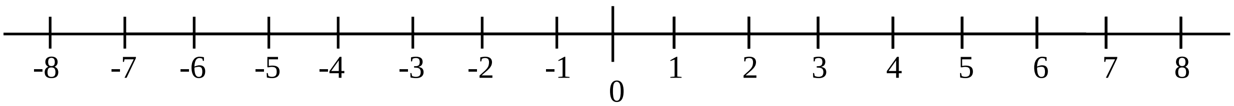


Exercise 2 Use the number line below to represent the given number as a **vector** with both a linear length and a direction. Start at the given value, and state where it stops.

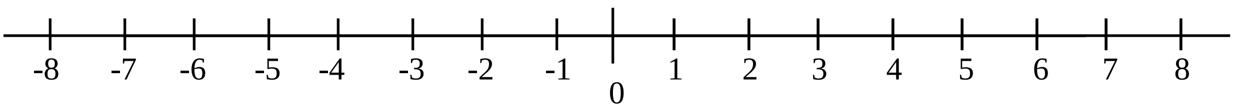
a) Represent **- 3**, starting at + 5.



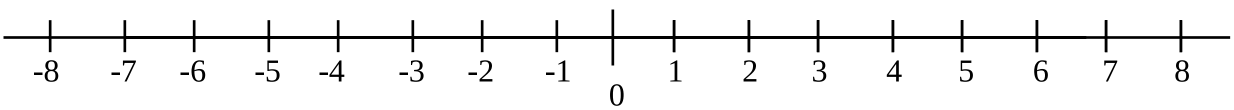
b) Represent **+ 7**, starting at - 4.



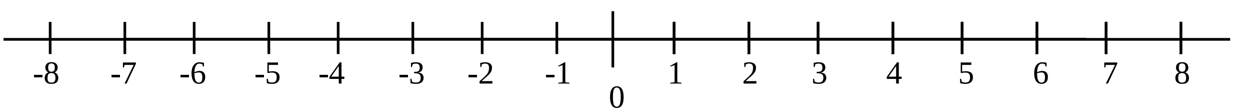
c) Represent **+ 2**, starting at - 3.



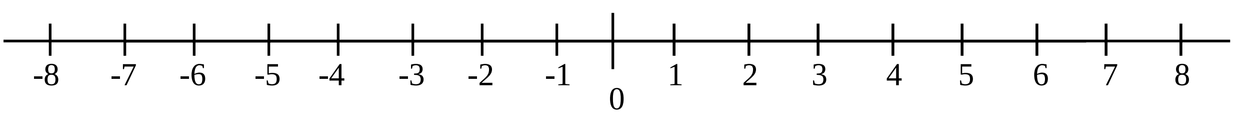
d) Represent **- 5**, starting at - 2.



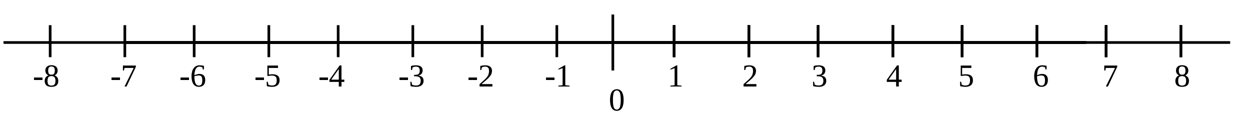
e) Represent **- 8**, starting at + 4.



f) Represent **- 4**, starting at - 3.



g) Represent **+ 3**, starting at + 5.



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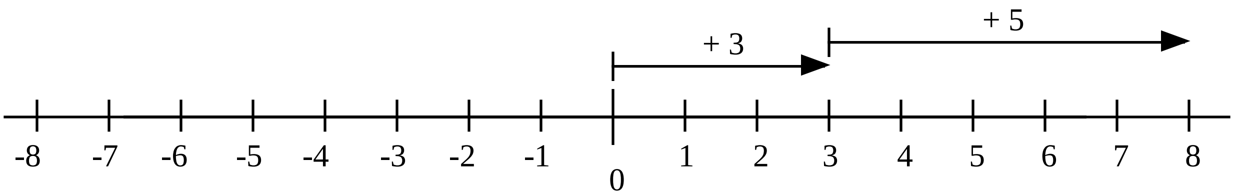
ADDING SIGNED NUMBERS ALONG THE NUMBER LINE

We can use the number line to add numbers. For example, to add 3 + 5 we construct:

1) a vector for 3 (really, + 3) that starts at the origin, 0

2) a vector for 5 (really, + 5) that *starts at 3* (where the first vector stopped).



The sum is + 8.

It doesn't matter if the numbers we are adding are positive or negative. Each number can be represented as a vector having a length and a direction. To add them, simply follow these guidelines.

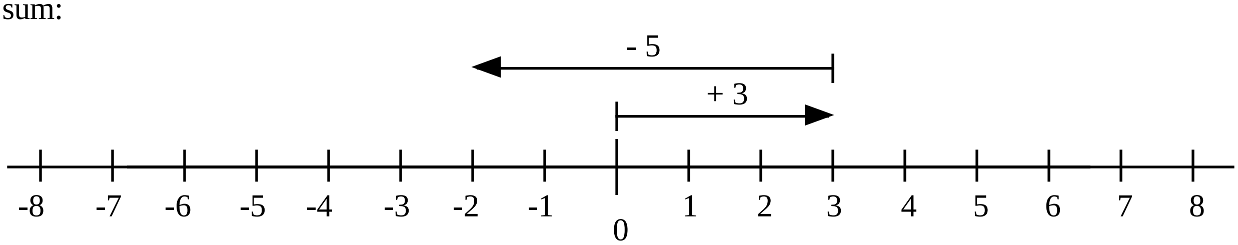
Adding two numbers using the number line.

1) Draw (or think of) a vector for the first number, starting from the origin, 0.

2) Draw (or think of) a vector for the second number that starts where the first vector stopped.

3) The number at which the second vector stops is the sum.

For example, we can add + 3 and - 5, by first showing the vector for + 3 (which will start at 0); then, from + 3 we can draw the vector for - 5 (which points to the left). Where the second vector stops, that is the sum:



The sum is - 2.

By the way, the sum of + 3 and - 5 is written 3 + (- 5). Notice the parentheses around the - 5; in this case, the parentheses are acting as separators, separating the plus sign (+) from the negative sign in the number - 5.

We don't really need the parentheses; without them, it would look like this: 3 + - 5. This isn't so bad when it's printed out, but when written by hand, it can sometimes be a little confusing. So, we use the parentheses for clarity.

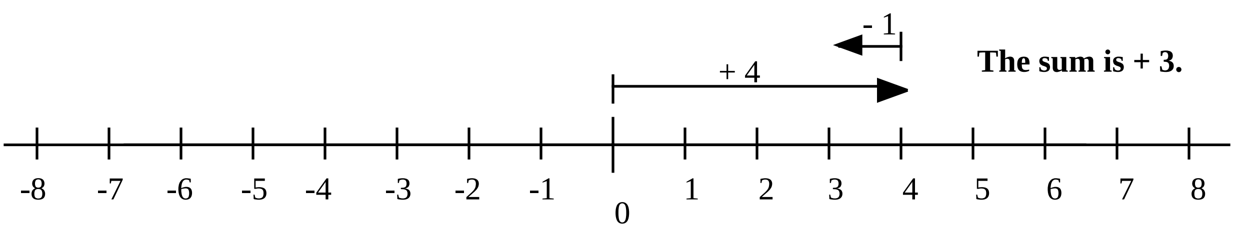
Example 3: Represent the sum by drawing two vectors. Use the guidelines for "adding two numbers using the number line."

a) 4 + (- 1) b) - 6 + 8

c) 3 + (- 7) d) - 5 + (- 2)

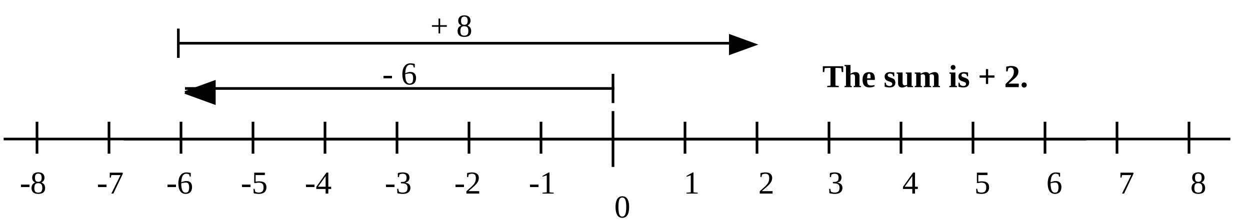
Answer Represent each number as a vector having length and direction (left or right).

a) The **vector** for + 4 has a length of 4 and points to the right; the vector - 1 has a length of 1 to the left.



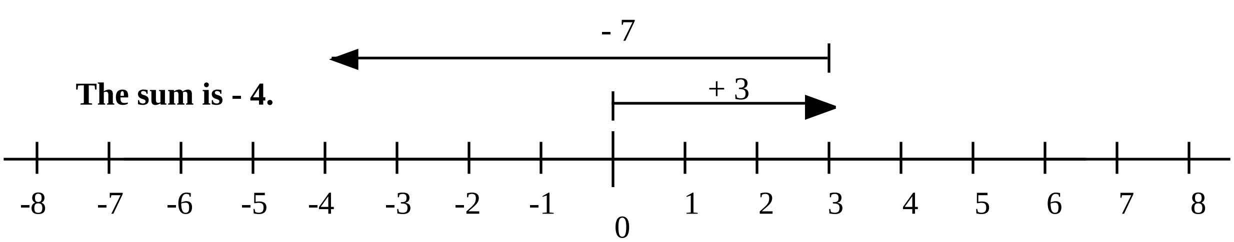
The sum is + 3.

b) - 6 has a length of 6 and points to the left; + 8 has a length of 8 and points to the right.



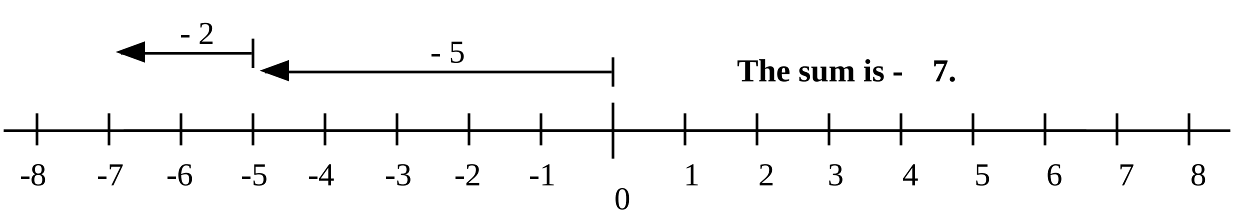
The sum is + 2.

c) + 3 is 3 to the right; - 7 is 7 to the left.



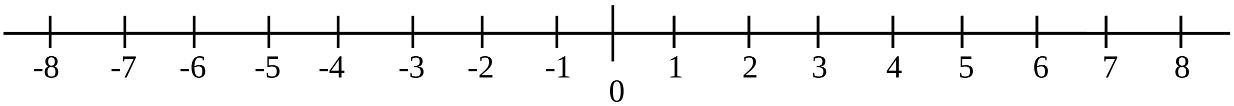
The sum is - 4.

d) - 5 is 5 to the left; - 2 is 2 to the left.

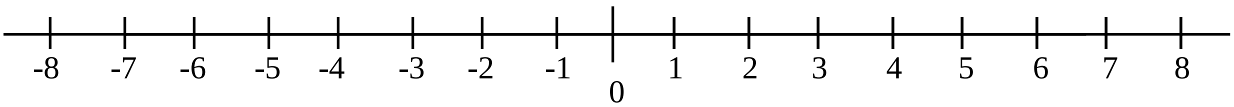


The sum is - 7.

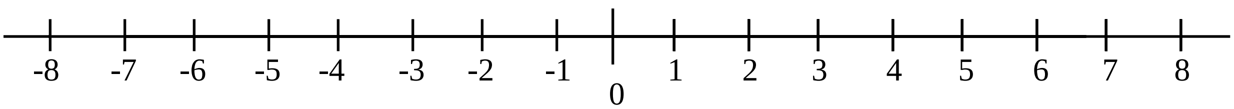
Exercise 3 Use the number line above each set to help you evaluate the following. You may use the same number line for two different sums.



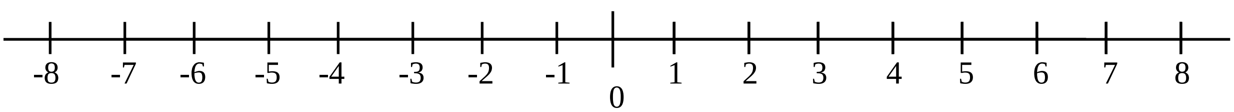
a) 3 + 4 = _____ b) - 6 + (- 2) = _____



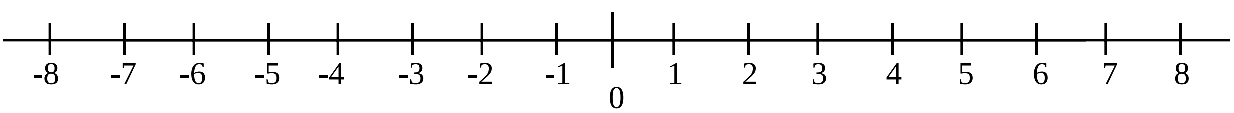
c) 8 + (- 3) = _____ d) 2 + (- 7) = _____



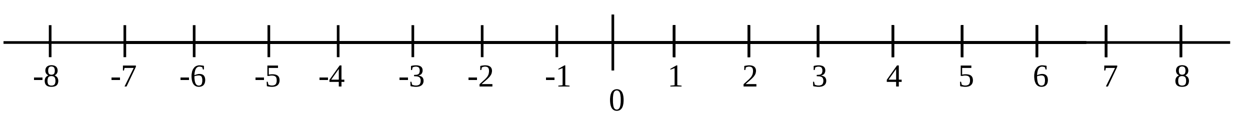
e) - 3 + (- 1) = _____ f) - 8 + 7 = _____



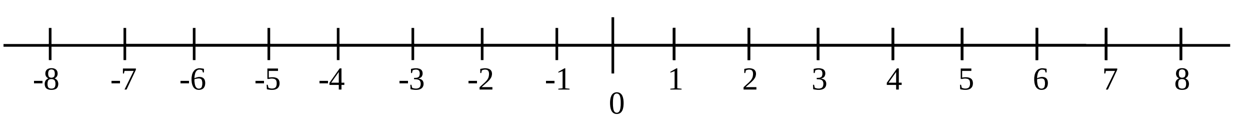
g) - 3 + 6 = _____ h) 6 + (- 6) = _____



i) - 4 + 4 = _____ j) 0 + (- 7) = _____



k) - 2 + 0 = _____ l) - 3 + 9 = _____



m) - 4 + (- 3) = _____ n) 5 + 2 = _____

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FINDING SUMS MENTALLY

As you have seen, and tried, the sum of two numbers can be found using the number line and drawing vectors. This time, find these sums mentally by drawing the vector with your mind and not your pencil. Try to visualize the number line and in which direction the vectors would go.

Exercise 4 Try to use a mental number line to evaluate the following.

a)	5 + 8 = _____	b)	- 10 + (- 6) = _____
c)	13 + (- 4) = _____	d)	2 + (- 10) = _____
e)	- 6 + (- 6) = _____	f)	- 12 + 7 = _____
g)	- 4 + 11 = _____	h)	10 + (- 10) = _____
i)	- 9 + 9 = _____	j)	0 + (- 15) = _____
k)	- 22 + 0 = _____	l)	- 13 + 9 = _____

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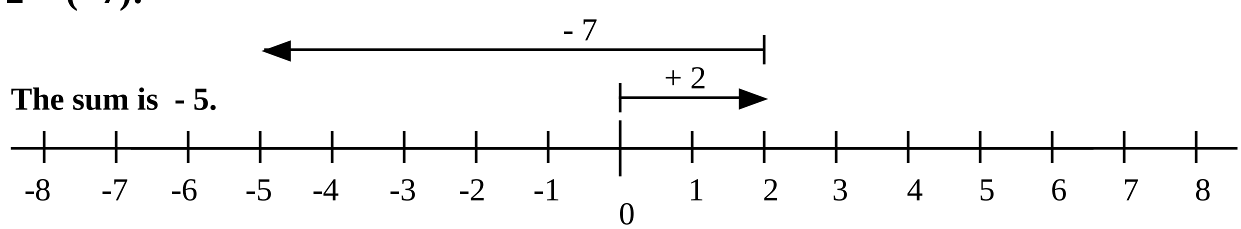
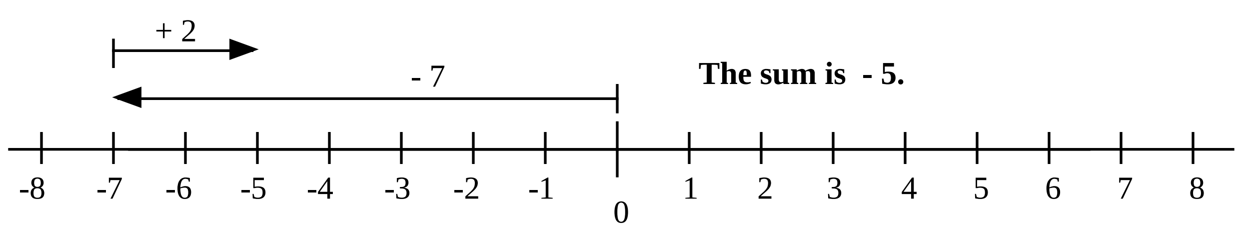
THE COMMUTATIVE PROPERTY

The Commutative Property of Addition works for integers as well as for whole numbers. You will recall that the Commutative Property of Addition allows the sum 3 + 5 to be rewritten as 5 + 3. The same is true for signed numbers. For example, 2 + (- 7) can be written as - 7 + 2. They are both equal to - 5:

2 + (- 7):



- 7 + 2:



Example 4: Rewrite each sum using the Commutative Property. Then, evaluate the original sum and the new sum.

a) 4 + (- 1)   b) - 6 + 8

c) 3 + (- 7)   d) - 5 + 5

Answer:

a) 4 + (- 1) = - 1 + 4
They both = + 3

b) - 6 + 8 = 8 + (- 6)
They both = + 2

c) 3 + (- 7) = - 7 + 3
They both = - 4

d) - 5 + 5 = 5 + (- 5)
They both = 0

Exercise 5 Rewrite each sum using the Commutative Property. Then, evaluate the original sum and the new sum.

a) - 10 + 3 = ________
They both = ______

b) - 12 + (- 4) = ________
They both = ______

c) 11 + (- 8) = ________
They both = ______

d) 4 + (- 9) = ________
They both = ______

e) - 4 + (- 11) = ________
They both = ______

f) - 9 + 15 = ________
They both = ______

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FINDING SUMS USING "AND"

You will shortly be introduced to a slightly different way to think of adding two integers. We'll not leave the number line, at least not mentally. You are encouraged to always consider the number line when adding two signed numbers.

First, though, we're going to use a special, yet quite simple, word to express addition; it is the word "and." We can think of the sum 3 + 5 as "3 and 5." This implies that we need to use the vectors for +3 and for +5 to come up with the sum.

The same is true for the sum - 6 + (- 4): think of this as "- 6 and - 4." Here we need the vectors for - 6 and for - 4. Also, the commutative Property says that it doesn't matter which of the two vectors we use first. With this in mind, we can add the numbers mentally.

Example 5: Write each sum using the word "and," then evaluate the sum. For positive numbers, write the plus sign (+) in front of them.

a) 7 + (- 3)   b) - 6 + 9

c) 5 + (- 8)   d) - 7 + 7

Answers:

a) 7 + (- 3) can be written as + 7 and - 3   The sum is + 4

b) - 6 + 9 can be written as - 6 and + 9   The sum is + 3

c) 5 + (- 8) can be written as + 5 and - 8   The sum is - 3

d) - 7 + 7 can be written as - 7 and + 7   The sum is 0

Exercise 6 Rewrite each sum using the word "and," then evaluate the sum. For positive numbers, write the plus sign (+) in front of them.

a) - 10 + 6 can be written as ____________________ The sum is __________

b) 7 + (- 9) can be written as ____________________ The sum is __________

c) - 1 + 5 can be written as ____________________ The sum is __________

d) - 5 + (- 7) can be written as ____________________ The sum is __________

e) - 6 + 5 can be written as ____________________ The sum is __________

f) 12 + (- 6) can be written as ____________________ The sum is __________

g) 8 + (- 8) can be written as ____________________ The sum is __________

h) - 3 + 3 can be written as ____________________ The sum is __________

You have been given a variety of techniques to add two signed numbers:

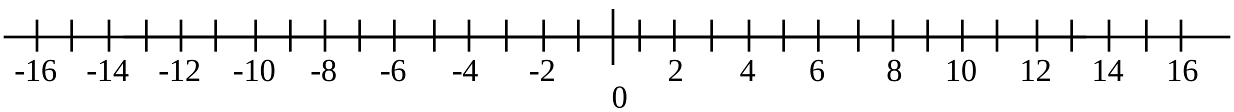
(1) You can think of numbers as vectors and use the number line to visually find the answer;

(2) you can *think* about vectors and the number line—without actually drawing the vectors—to find the answer;

(3) you can use the commutative property, if that is helpful;

(4) you can think of a sum using the word "and."

Exercise 7 Use the variety of techniques presented to add these signed numbers. A number line is provided here for your convenience.



a)	10 + 3 = _____	b)	- 10 + (- 3) = _____
c)	11 + 5 = _____	d)	- 11 + (- 5) = _____
e)	13 + 4 = _____	f)	- 13 + (- 4) = _____
g)	19 + 1 = _____	h)	- 19 + (- 1) = _____
i)	- 8 + 2 = _____	j)	7 + (- 4) = _____
k)	- 3 + 6 = _____	l)	2 + (- 5) = _____
m)	- 7 + 9 = _____	n)	9 + (- 6) = _____
o)	- 30 + 14 = _____	p)	53 + (- 33) = _____
q)	- 16 + 42 = _____	r)	28 + (- 51) = _____
s)	- 27 + 31 = _____	t)	36 + (- 61) = _____
u)	- 12 + (- 29) = _____	v)	- 46 + (- 18) = _____
w)	- 32 + (- 43) = _____	x)	- 53 + (- 37) = _____

At this point, if you feel confident that you know how to add signed numbers—and if your instructor allows—skip to the Focus Exercises. If, instead, you feel as though you need some more instruction, please keep reading and follow the **rules for adding signed numbers**.

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IDENTIFYING PATTERNS

By now you should have a sense of how to add integers (signed numbers) along the number line. However, some integers are large enough in value that placing them on a number line can be a bit difficult. For example, to evaluate the sum - 23 + 48 would be a challenge along the number line.

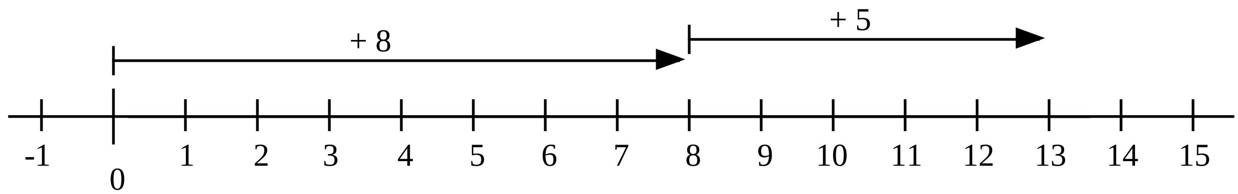
We need to develop some rules for adding signed numbers that have large values. These rules, though, will need to be consistent with our understanding of how to add small-valued integers. We'll develop the rules using smaller numbers and the number line.

Rule 1: The sum of two numbers with the same sign

When finding the sum of two numbers of the same sign, **add** their numerical values together and give the result that same sign.

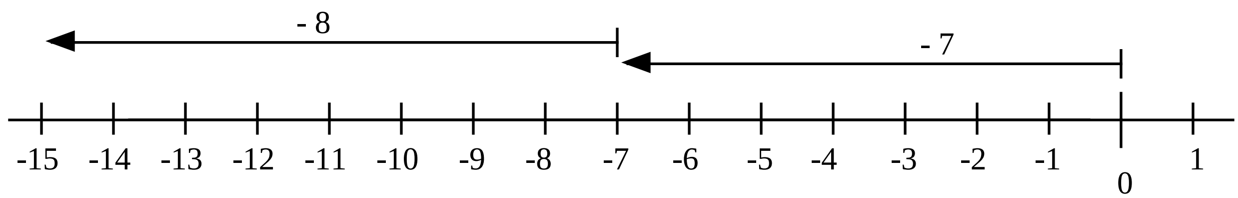
The sum of two numbers with the same sign—*same direction*—is a larger number that has the same direction (same sign).

The sum of two *positive* numbers is a larger *positive* number. That is because they have the same direction. So, when adding, for examples, + 8 + 5 (+ *8* and + *5*), the vectors take you further to the right, further away from 0 in the *positive* direction. The result is a *larger positive value*.



The sum is + 13.

The sum of two *negative* numbers is a larger *negative* number. Again, that is because they have the same direction. So, when adding, for examples, - 7 + (- 8) (- *7* and - *8*), the vectors take you further to the left, further away from 0 in the *negative* direction. The result is a *larger negative value*.



The sum is - 15.

Example 6: Use Rule 1 to find the sum of each of these.

a) + 7 + 4 b) + 6 + 9 c) + 12 + 26

d) - 5 + (- 8) e) - 7 + (- 3) f) - 35 + (- 52)

Answers:

a) + 7 + 4 = *+ 7 and + 4:* a larger positive number, namely + 11.

b) + 6 + 9 = *+ 6 and + 9* = + 15 (a larger positive number)

c) + 12 + 26 = *+ 12 and + 26* = + 38 (a larger positive number)

d) - 5 + (- 8) = *- 5 and - 8:* a larger *negative* number, namely - 13.

e) - 7 + (- 3) = *- 7 and - 3* = - 10 (a larger *negative* number)

f) - 35 + (- 52) = *- 35 and - 52* = - 87 (a larger *negative* number)

Exercise 8 Use Rule 1 to find each sum. You encouraged to write the sum using the word *and* as demonstrated in Example 6.

a) + 1 + 8 = _____

b) + 2 + 17 = _____

c) + 12 + 6 = _____

d) + 20 + 9 = _____

e) + 15 + 29 = _____

f) + 43 + 86 = _____

g) + 36 + 68 = _____

h) + 81 + 75 = _____

i) - 5 + (- 2) = _____

j) - 4 + (- 12) = _____

k) - 16 + (- 3) = _____

l) - 15 + (- 20) = _____

m) - 22 + (- 46) = _____

n) - 34 + (- 15) = _____

o) - 52 + (- 39) = _____

p) - 92 + (- 88) = _____

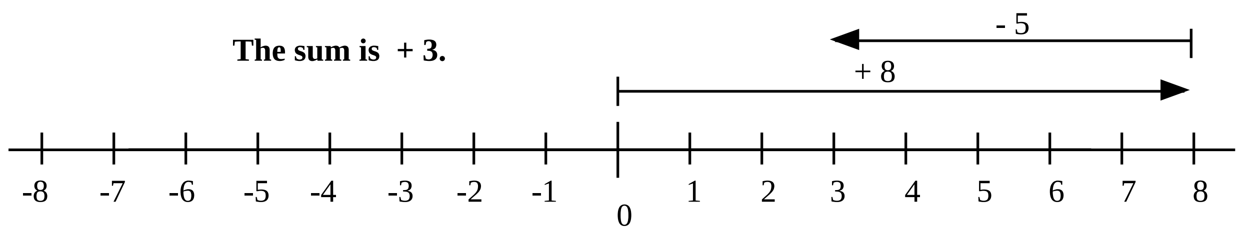
Rule 2: The sum of two numbers with opposite signs

When finding the sum of two numbers with opposite signs, **subtract** their numerical values and give the result the sign of the addend with the larger numerical value.

The sum of two numbers with opposite signs—*opposite directions*—is a number closer to 0; that resulting number has the same sign (same direction) as the addend with the larger numerical value.

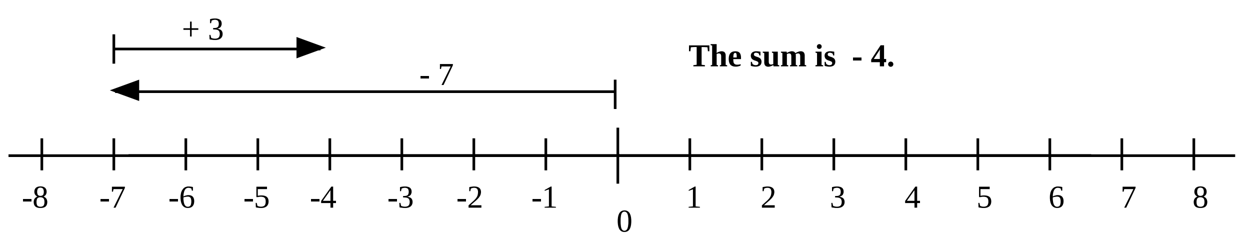
For example, if we are to find the sum of + 8 + (- 5), we can draw two vectors, one "stretching" away from 0 (zero) to the right, and the other bringing it back toward zero (heading left), but not all the way back to 0.

The sum is + 3.



Another example: if we are to find the sum of - 7 + 3, we can draw two vectors, one "stretching" away from 0 (zero) to the left, and the other bringing it back toward zero (heading right), but not all the way back to 0.

The sum is - 4.



The *commutative property of addition* allows us to write a sum in any order. For the sake of understanding this rule, we could rewrite any sum so that the larger valued number is written first.

So here are the guidelines for finding the sum of two numbers with **opposite signs**:

(1) Rewrite the addends so that the number with the larger value is first

(2) The sign of the first number will be the sign of the result, so you could write the sign of that number in front of your answer (even if it is positive); in other words, decide whether the answer will be positive or negative before writing the full answer.

(3) Subtract: larger value – smaller value to find the resulting value .

Example 7: Find the sum of each of these. Notice that the addends have different signs.

a) + 7 + (- 5)	b) + 6 + (- 8)	c) + 12 + (- 38)
d) - 5 + 4	e) - 7 + 19	f) - 35 + 29

Answers: Follow the guidelines of Rule 2 to find the sum.

a) + 7 + (- 5) = + 2

The larger-valued number is already written first: *+ 7 and - 5*

The larger value is positive, so the result will be *positive* as well.

Their difference: 7 – 5 = 2, so the result is *+ 2*.

b) + 6 + (- 8) = - 2

This could be rewritten as - 8 + 6: *- 8 and + 6*

The larger value is negative, so the result will be *negative* as well.

Their difference: 8 – 6 = 2, so the result is *- 2*.

c) + 12 + (- 38) = - 26

This could be rewritten as - 38 + 12: *- 38 and + 12*

The larger value is negative, so the result will be *negative* as well.

Their difference: 38 – 12 = 26, so the result is *- 26*.

d) - 5 + 4 = - 1

The larger-valued number is already written first: *- 5 and + 4*

The larger value is negative, so the result will be *negative* as well.

Their difference: 5 – 4 = 1, so the result is *- 1*.

e) - 7 + 19 = + 12

This could be rewritten as 19 + (- 7): *+ 19 and - 7*

The larger value is positive, so the result will be positive as well.

Their difference: 19 – 7 = 12, so the result is *+ 12*.

f) - 35 + 29 = - 6

The larger-valued number is already written first: *- 35 and + 29*

The larger value is negative, so the result will be *negative* as well.

Their difference: 35 – 29 = 6, so the result is *- 6*.

Exercise 9 Use Rule 2—the sum of two numbers with opposite signs—and the guidelines to find each sum. You may show any work below each problem. You encouraged to write the sum using the word *and* as demonstrated in Example 7.

a) $-1 + 8 =$ ________ b) $-5 + 2 =$ ________

c) $+2 + (-7) =$ ________ d) $+9 + (-2) =$ ________

e) $-9 + 2 =$ ________ f) $-3 + 6 =$ ________

g) $+4 + (-10) =$ ________ h) $+7 + (-6) =$ ________

i) $-3 + 2 =$ ________ j) $-7 + 12 =$ ________

k) $+12 + (-17) =$ ________ l) $+34 + (-20) =$ ________

m) $+15 + (-29) =$ ________ n) $+86 + (-42) =$ ________

o) $-18 + 42 =$ ________ p) $-36 + 52 =$ ________

q) $-73 + 46 =$ ________ r) $-92 + 15 =$ ________

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Answers to each Exercise

Section 1.2

Exercise 1: (The answers to Exercise 1 are not given.)

Exercise 2: (The answers to Exercise 2 are not given.)

Exercise 3:

a) 7	b) - 8	c) + 5	d) - 5
e) - 4	f) - 1	g) + 3	h) 0
i) 0	j) - 7	k) - 2	l) + 6
m) - 7	n) + 7		

Exercise 4:

a) 13	b) - 16	c) + 9	d) - 8
e) - 12	f) - 5	g) + 7	h) 0
i) 0	j) - 15	k) - 22	l) - 4

Exercise 5:

a) 3 + (- 10) = - 7	b) - 4 + (- 12) = - 16
c) - 8 + 11 = + 3	d) - 9 + 4 = - 5
e) -11 + (- 4) = - 15	f) 15 + (- 9) = + 6

Exercise 6:

a) - 10 and + 6 the sum is - 4	b) + 7 and - 9 the sum is - 2
c) - 1 and + 5 the sum is + 4	d) - 5 and - 7 the sum is - 12
e) - 6 and + 5 the sum is - 1	f) + 12 and - 6 the sum is + 6
g) + 8 and - 8 the sum is 0	h) - 3 and + 3 the sum is 0

Exercise 7:

a) + 13	b) - 13	c) + 16	d) - 16
e) + 17	f) - 17	g) + 20	h) - 20
i) - 6	j) + 3	k) + 3	l) - 3
m) + 2	n) + 3	o) - 16	p) + 20
q) + 26	r) - 23	s) + 4	t) - 25
u) - 41	v) - 64	w) - 75	x) - 90

Exercise 8:	a) + 9	b) + 19	c) + 18	d) + 29
	e) + 44	f) + 129	g) + 104	h) + 156
	i) - 7	j) - 16	k) - 19	l) - 35
	m) - 68	n) - 49	o) - 91	p) - 180

Exercise 9:	a) + 7	b) - 3	c) - 5	d) + 7
	e) - 7	f) + 3	g) - 6	h) + 1
	i) - 1	j) + 5	k) - 5	l) + 14
	m) - 14	n) + 44	o) + 24	p) + 16
	q) - 27	r) - 77		

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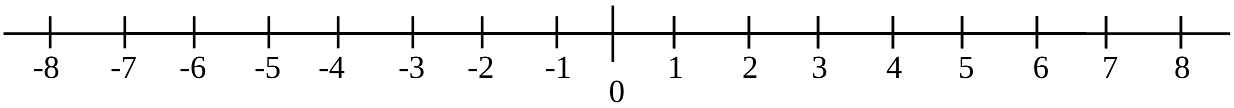
Section 1.2 Focus Exercises

1. Use this number line, as necessary, to help you evaluate the following.



a) -3 + 4 =

b) - 5 + (- 1) =

c) - 4 + (- 3) =

d) - 2 + (- 5) =

e) - 1 + (- 6) =

f) - 8 + 8 =

g) - 4 + 8 =

h) 5 + (- 5) =

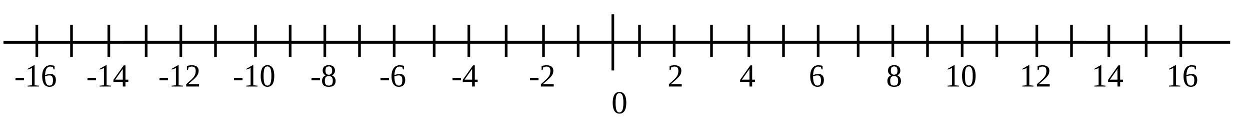
i) - 8 + 10 =

j) 0 + (- 3) =

k) - 6 + 0 =

l) - 6 + 12 =

2. Use the variety of techniques presented to add these signed numbers. A number line is provided here for your convenience.



a) - 10 + 3 =

b) - 1 + (- 11) =

c) - 11 + 5 =

d) - 3 + (- 15) =

e) - 13 + 14 =

f) - 3 + (- 14) =

g) 19 + 1 =

h) - 9 + (- 11) =

i) - 8 + 12 =

j) 17 + (- 4) =

k) - 3 + 16 =

l) 2 + (- 15) =

3. Rewrite each sum using the Commutative Property. Then, evaluate the original sum and the new sum.

a) $-7 + 6 =$ __________

______ = ______

b) $-8 + (-2) =$ __________

______ = ______

c) $3 + (-10) =$ ______

______ = ______

d) $9 + 6 =$ __________

______ = ______

4. Find each sum. You may use any technique presented in this section.

a) $-21 + 8 =$

b) $14 + (-29) =$

c) $-18 + 36 =$

d) $20 + (-19) =$

e) $-15 + (-40) =$

f) $-68 + (-23) =$

g) $-42 + 91 =$

h) $-53 + 37 =$

i) $-57 + (-63) =$

5. Find each sum. These require a little more thinking.

a) $-5.2 + 8.3 =$

b) $-.58 + (-.82) =$

c) $1.6 + (-7.9) =$

d) $-.38 + .14 =$

e) $4.6 + (-3.8) =$

f) $-2.8 + (-0.59) =$

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