Section 1.5 The Factor Game

This material is useful for the study and practice of factoring trinomials, found later in Section 4.3 of this text.

Contents:

Here's a "game" that will prove to be quite useful in Section 4.3, Factoring Trinomials. At this point, that probably has no meaning to you, but it will help you to think mathematically, to help you to develop "number sense."

This game starts by identifying a special number called the **key number** (key #). You are asked to find different factor pairs of this key number. The numbers in the factor pair are then added to get a **sum number** (sum #).

Your job is to find the right factor pair (called the **solution**) that will combine to get the sum #.

The Factor Game Rules:

1. You are given two numbers: the **key number** and the **sum number**;
2. you are to find two **factors** (actually, a factor pair) of the key number that will **add** up to the sum number.

Special Notes:

i) It's actually possible that there is no answer.

ii) If there is an answer, there will be only *one* answer (two numbers—a factor pair) for each set of key and sum numbers.

It's easiest to understand this game through example. Here are two. Look them over carefully, and follow them through to get a full understanding.

Example 1: Key # = 12 and Sum # = 7

We need to find two factors (a factor pair) of the key #, 12, that will add to get 7. You probably already know what the numbers are, but let's look at the full way to find it:

a) look at all of the factors pairs (*not* a factor tree) of the key # 12; we'll start with the smallest factor, 1, and work up one factor at a time:

b) **Factor pairs**:

12		**Sum**		
1	12	13	.....too large	
2	6	8	.....closer	So, for the key # 12 and the sum # 7,
3	4	7	.....**this is it!**	the solution is: **3 and 4**.

Notice that the answer is a pair of numbers; they are a *factor pair* of 12 and have a *sum* of 7.

Example 2: Key # = 36 and Sum # = 15

Procedure: We need to find a factor pair of the key # 36 that will add to get 15. You probably already know what the numbers are, but let's look at a complete way to find it:

a) Look at all of the factors pairs (*not* a factor tree) of the key # 12; we'll start with the smallest factor, 1, and work up one factor at a time:

b) **Factor pairs:**

36

/ \

		Sum	
1	36	37	… too large
2	18	20	… closer
3	12	**15**	… **THIS IS IT!**
4	9	13	… we didn't need to go this far since the
6	6	12	solution has already been found.

So, for the key # 36 and the sum # 15, the solution is: **3 and 12** (or **12 and 3**).

Let's check: $3 \cdot 12 = 36$ (the key #) and $3 + 12 = 15$ (the sum #).

Exercise 1: Find the factor pair that satisfies each Key # and Sum #.

a) Key # = 24, Sum # = 10

b) Key # = 18, Sum # = 11

c) Key # = 24, Sum # = 14

d) Key # = 18, Sum # = 19

e) Key # = 30, Sum # = 11

f) Key # = 14, Sum # = 15

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THE FACTOR GAME AND A NEGATIVE SUM NUMBER

Here is a reminder of a rule presented in Section 1.4:

The Product Rule of Signs	
a)	if the *product* of two factors is **positive**, the *factors* must have the **same sign**
b)	if the *product* of two factors is **negative**, the *factors* must have **different signs**

Let's do a couple of examples of the Factor Game with a very slight change. This time either the key number or the sum number, or both, will be negative.

Example 3: Given the key number and the sum number, find the factor pair of the key number that adds to the sum number. (Notice that each number is designated as positive or negative; this is very important.)

a) Key # = + 12
Sum # = - 7

b) Key # = + 60
Sum # = - 23

Answer: Refer above, as necessary, to the Product Rule of Signs.

a) Key # = + 12 and Sum # = - 7

First, we need to find a factor pair of + 12.
Since this is a positive key number, the two factors will either be both positive or both negative. Think about why this is so. (Ask yourself, "What if the signs of the factors were different: one positive and one negative?")

Second, since the sum number is negative, - 7, the factors must both be negative. Looking at the factor pairs of + 12, knowing that the factors will both be negative, we can write it as

+ 12 **Sum = - 7**
/ \
- 1 - 12
- 2 - 6
- 3 - 4 .....**this is it!**

So, the solution is **- 3 and - 4**.

b) Key # = 60 and Sum # = - 23

Again, the key number is positive (the signs of the factors must be the same) and the sum number is negative (they'll both be negative).

+ 60 **Sum = - 23**
/ \
- 1 - 60
- 2 - 30
- 3 - 20 .....**this is it!**

We don't need to look at any other factors because we've already found the solution: **- 3 and - 20**.

Exercise 2: Find the factor pair that satisfies each Key # and Sum #.

a) Key # = 25, Sum # = - 10

b) Key # = 28, Sum # = - 11

c) Key # = 40, Sum # = - 14

d) Key # = 60, Sum # = - 19

e) Key # = 30, Sum # = - 17

f) Key # = 10, Sum # = - 11

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THE FACTOR GAME AND A NEGATIVE KEY NUMBER

What happens if the key # is negative? This means that the *product* is negative; this can happen only if the factors have *different* signs—one positive and one negative.

Think about this, though: the *sum* of a positive and negative number (as learned in Section 1.2) could be *either* positive or negative; it's the number with the largest numerical value that gets to determine the sign of the sum. For example,

the sum of **+ 9** and **- 5** is . . . + 4, The larger *value* is positive, so the result is positive.

but the sum of **- 10** and **+ 3** is . . . - 7. The larger *value* is negative, so the result is negative.

Remember that the sum (addition) of two numbers of different signs is the same as subtracting the smaller numerical value from the larger numerical value and giving the result the sign of the larger.

So, in a sense, when trying to find two factors of, say, - 12 that *add* to get + 4, we need to recognize that the signs of the factors will be *different* and that we really need to look for factors of 12 that have a *difference* of 4. Those factors are 6 and 2.

However, the sum of + 6 and + 2 is actually + 8, so we really need to consider the sum of + 6 and - 2 in order to get a sum of + 4. In this way, the factors + 6 and - 2 have different signs and their product is negative: - 12.

Example 4: Given the key number and the sum number, find the factor pair of the key number that adds to the sum number.

a) Key # = - 12
Sum # = + 4

b) Key # = - 30
Sum # = - 13

Answer: a) Key # = - 12 and Sum # = + 4

First, since the key number is negative, the signs of the factors will be *different*. (This means we're really looking for a *difference* between the factors, even though we call it the *sum number*.)

Second, since the sum is positive, the larger factor will be positive and the smaller factor will be negative; so, let's start our factor pair tree with that in mind.

Factor pairs: We've already established that the larger will be positive and the smaller negative. We'll identify factor pairs until we get to the desired sum number:

- 12	**Sum**
/ \	
- 1 + 12	+ 11
- 2 + 6	+ 4

So, the answer is: **- 2 and + 6**.

b) Key # = - 30 and Sum # = - 13

Again, the key number is negative, the signs of the factors will be *different*..

And, since the sum is negative, the larger factor will be negative and the smaller factor will be positive ; so, let's start our factor pair tree with that in mind.

Factor pairs: Again, one will be negative and the other positive. Remember, though we're looking for a *sum* of - 13, we really want to find two factors of 30 whose difference is 13.

- 30	**"Sum"**	
/ \		
1 - 30	- 29	
2 - 15	- 13	.....**this is it!** (no need to continue, really)
3 - 10	- 7	
5 - 6	- 1	

So, the answer is: + **2 and - 15**.

Example 5: Given the key number and the sum number, find the factor pair of the key number that adds to the sum number.

a) Key # = + 20
Sum # = - 12

b) Key # = - 24
Sum # = - 12

Answer: a) Key # = + 20 and Sum # = - 12

First, since the key number is positive, the signs of the factors will be *the same*.

Second, since the sum is negative, the factors will both be negative; so, let's start our factor pair tree with that in mind.

Factor pairs: Do not write out all possible factor pairs unless you aren't certain of the correct pair; we're looking for two factors that have the same sign (positive key number), and they'll both be negative (negative sum number).

+ 20	**Sum = - 12**
/ \	
- 2 - 10	- 12

The answer is: **- 2 and - 10**.

b) Key # = - 24 and Sum # = - 12

Factor pairs: Signs are different (key number is negative); the larger is negative (the sum number is negative).

- 24	**Sum = - 12**	
/ \		
+ 1 - 24	- 23	Nope
+ 2 - 12	- 10	Nein
+ 3 - 8	- 5	Nyet
+ 4 - 6	- 2	No way!

None of these factor pairs give the correct sum, so there is **no answer**.

Example 6: Given the key number and the sum number, find the factor pair of the key number that adds to the sum number.

a) Key # = - 36
Sum # = 0

b) Key # = + 25
Sum # = 0

Answer: a) Key # = - 36 and Sum # = 0

Factor pairs: Signs are different (key number is negative), but what does that sum number mean? Answer to follow.

- 36	**Sum = 0**
/ \	
+ 1 - 36	- 35
+ 2 - 18	- 16
+ 3 - 12	- 9
+ 4 - 9	- 5
+ 6 - 6	0

The answer is **+ 6 and - 6**. Yes, the only way to get a sum of zero (0) is for the factors to be *opposites*. Notice, though, that the product of any two opposites will be a negative number.

b) Key # = + 25 and Sum # = 0

Factor pairs: Signs are the *same* (key number is positive), sum number is 0, so the factors must be *opposite,* they must be *different*.. Is this possible? Answer to follow.

+ 25	**Sum**
/ \	
+ 1 + 25	+ 26
+ 5 + 5	+ 10
- 1 - 25	- 26
- 5 - 5	- 10

Again, there is **no answer**. That's right, it *isn't* possible for two factors to have the same sign and add to 0.

Exercise 3: Find the solution for the given Key number and Sum number.

a) Key # = + 21, Sum # = + 10:

b) Key # = + 18, Sum # = + 9:

c) Key # = + 45, Sum # = + 14:

d) Key # = + 3, Sum # = + 4:

e) Key # = + 30, Sum # = - 11:

f) Key # = + 14, Sum # = - 15:

g) Key # = - 36, Sum # = + 5:

h) Key # = - 45, Sum # = + 4:

i) Key # = - 24, Sum # = - 10:

j) Key # = - 30, Sum # = - 1:

k) Key # = - 18, Sum # = + 9:

l) Key # = - 20, Sum # = + 1:

m) Key # = - 16, Sum # = 0:

n) Key # = + 60, Sum # = + 23:

o) Key # = - 45, Sum # = - 12:

p) Key # = + 18, Sum # = - 9:

q) Key # = + 40, Sum # = - 22:

r) Key # = + 9, Sum # = 0:

s) Key # = - 36, Sum # = - 9:

t) Key # = + 30, Sum # = - 13:

Answers to each Exercise

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Exercise 1:	a)	6 and 4	b)	9 and 2	c)	12 and 2
	d)	18 and 1	e)	5 and 6	f)	14 and 1
Exercise 2:	a)	- 5 and - 5	b)	- 7 and - 4	c)	- 4 and - 10
	d)	- 15 and - 4	e)	- 15 and - 2	f)	- 10 and - 1
Exercise 3:	a)	+ 3 and + 7	b)	+ 6 and + 3	c)	+ 9 and + 5
	d)	+ 3 and + 1	e)	- 5 and - 6	f)	- 14 and - 1
	g)	+ 9 and - 4	h)	+ 9 and - 5	i)	- 12 and + 2
	j)	- 6 and + 5	k)	no solution	l)	+ 5 and - 4
	m)	- 4 and + 4	n)	+ 20 and + 3	o)	- 15 and + 3
	p)	- 6 and - 3	q)	- 20 and - 2	r)	no solution
	s)	- 12 and + 3	t)	- 10 and - 3		

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Section 1.5 Focus Exercises

Find the factor pair that satisfies each Key # and Sum #. If it is not possible, write "no solution."

1. Key # = 30, Sum # = 13

2. Key # = 36, Sum # = 13

3. Key # = 16, Sum # = - 10

4. Key # = 48, Sum # = - 14

5. Key # = 15, Sum # = 16

6. Key # = 40, Sum # = 14

7. Key # = - 20, Sum # = 9

8. Key # = - 60, Sum # = 4

9. Key # = - 90, Sum # = - 1

10. Key # = - 42, Sum # = - 13

11. Key # = - 49, Sum # = 0

12. Key # = 4, Sum # = 0

13. Key # = 20, Sum # = - 21

14. Key # = 54, Sum # = - 3

15. Key # = - 28, Sum # = 12

16. Key # = - 30, Sum # = 13

17. Key # = - 24, Sum # = - 5

18. Key # = - 60, Sum # = - 11

19. Key # = 36, Sum # = 12

20. Key # = 25, Sum # = - 10

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