1.6 The Order of Operations

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OPERATIONS

Let's be reminded of those operations seen thus far in the course:

Operation	Example	Special name
Multiplication	$3 \cdot 5 = 15$	this is a **product**
Division	$14 \div 2 = 7$	this is a **quotient**
The Division Bar	$\frac{14}{2} = 7$	this is a **fraction**
Subtraction	$12 - 9 = 3$	this is a **difference**
Addition	$2 + 8 = 10$	this is a **sum**
Exponent	$2^3 = 8$	this is a **power**
Radical	$\sqrt{25} = 5$	this is a **square root**
Absolute Value	$\lvert -4 \rvert = 4$	this is the **numerical value**

GROUPING SYMBOLS

As you may recall, there are a variety of symbols that we can use to group two or more values. Grouping symbols create a **quantity**, suggesting that what's inside is *one* value. The parentheses also act as a barrier, a protector of sorts, to outside influence until the single value within is known.

Some grouping symbols are also operators, as shown below.

Grouping Symbols that create quantities:		Grouping symbols that are also operations:	
( )	parentheses (used frequently)	$\lvert\ \ \rvert$	absolute value bars
[ ]	brackets (used at times)	$\sqrt{\ }$	radical
{ }	braces (rarely used)	$\frac{\text{numerator}}{\text{denominator}}$	the division bar

The division bar is quite an accomplished grouping symbol in that it automatically groups three things: the entire numerator, the entire denominator, and even itself.

$$\left(\frac{\text{(numerator)}}{\text{(denominator)}}\right)$$

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THE ORDER OF OPERATIONS

You are already quite familiar with the order of operations, so why are we looking at it again? We now have negative numbers and need to see how they work within this system. In this section you will be introduced to a variety of mathematical formulas that use both positive and negative numbers. Here is the order of operations.

The Order Of Operations

1. Evaluate within all grouping symbols (one at a time), if there are any.
2. Apply any exponents.
3. Apply multiplication and division *reading from left to right.*.
4. Apply addition and subtraction *reading from left to right.*.

When speaking of which operation should come first, we refer to an operation's **rank** For example, because multiplication is applied *before* addition, multiplication has a *higher rank* than addition. Similarly, the exponent has a "higher rank" than multiplication.

Let's practice using the order of operations with a few exercises.

Example 1: Evaluate each expression according to the Order of Operations.

a) $6 + 12 \div 3$ b) $(6 + 12) \div 3^2$ c) $\sqrt{24 + 6 \cdot 2}$

Answer:

a) $6 + 12 \div 3$ Apply division first, then addition.

$= 6 + 4$

$= 10$

b) $(6 + 12) \div 3^2$ Addition has the highest rank because it is within the parentheses.

$= 18 \div 3^2$ The exponent has the higher rank, so apply it before applying the division.

$= 18 \div 9$

$= 2$

c) $\sqrt{24 + 6 \cdot 2}$ The radical is both a grouping symbol and an operation; as a grouping symbol, we must evaluate *within* before applying the square root.

$= \sqrt{24 + 12}$

$= \sqrt{36}$

$= 6$

Exercise 1: Evaluate each expression.

a) $24 \div 6 + 2$

b) $24 \div (6 + 2)$

c) $10 - 3 \cdot 2$

d) $(10 - 3) \cdot 2$

e) $(12 \div 3)^2$

f) $35 \div 5 + 2 \cdot 3$

g) $9 \cdot \sqrt{25}$

h) $\sqrt{100 - 36}$

i) $(6 + 12) \div (2 \cdot 3)$

j) $6 + [12 \div (2 \cdot 3)]$

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Exponents and Negative Numbers

Every pair of negative factors has a product that is positive; for example, $(-2)(-3) = +6$. Multiply that result by another negative factor and the new product is negative; for example, $(+6)(-5) = -30$, though the original product may look like this: $(-2)(-3)(-5) = -30$.

Multiply that product by another negative factor and the new product will return to being positive; for example, since $(-30)(-2) = +60$, we could say that $(-2)(-3)(-5)(-2) = +60$. Hence, an even number of negative factors has a product that is positive, and an *odd* number of negative factors has a product that is *negative*.

Let's extend this idea to powers (exponents) of negative numbers. For example, we know that $3^2 = 9$. It's also true that $(-3)^2 = 9$. It's easy to demonstrate this using the definition of exponents: $(-3)^2 = (-3)(-3) = +9$.

What about $(-3)^3$ and $(-3)^4$, and so on? To see their values we need to refer to the definition of exponents, an abbreviation for repeated multiplication:

a) $(-3)^3 = (-3)(-3)\cdot(-3)$ Multiply the first two factors together: $(-3)(-3) = +9$

$= (+9)\cdot(-3)$

$= -27$

b) $(-3)^4 = (-3)(-3)\cdot(-3)(-3)$ Multiply the first two factors *and* the last two factors.

$= (+9) \cdot (+9)$

$= +81$

c) $(-3)^5 = (-3)(-3)(-3)(-3)\cdot(-3)$ We already know that the product of the first four factors is $+81$

$= (+81) \cdot (-3)$

$= -243$

Here is a list of the different powers of -3 (follow the arrows):

$(-3)^1 = -3$ ➔ ➔ $(-3)^2 = +9$

$(-3)^3 = -27$ ➔ ➔ $(-3)^4 = +81$

$(-3)^5 = -243$ ➔ ➔ $(-3)^6 = +729$

Think About It: What pattern do you notice about the power of a negative number?

Answer: For the power of a negative number: if the power (exponent) is *odd* (such as 1, 3, 5 and 7), the result will be *negative*. However, if the power is even (such as 0, 2, 4, 6 and 8), the result will be positive.

Example 2: Evaluate each expression.

a) $(-2)^2$ b) $(-2)^3$ c) $(-2)^4$ d) $(-2)^5$

e) $(-2)^1$ f) $(-7)^2$ g) $(-5)^4$ h) $(-9)^3$

Answer: First decide if the result is going to be positive or negative, based on the exponent. Here are the final results for each one. You might want to multiply them out to verify the actual number.

a) $(-2)^2 = +4$ b) $(-2)^3 = -8$

c) $(-2)^4 = +16$ d) $(-2)^5 = -32$

e) $(-2)^1 = -2$ f) $(-7)^2 = +49$

g) $(-5)^4 = +625$ h) $(-9)^3 = -729$

Exercise 2: Evaluate each expression. (There's a little room near each one to work it out.)

a) $(-5)^2 =$ ______ b) $(-4)^3 =$ ______

c) $(-1)^2 =$ ______ d) $(-1)^3 =$ ______

e) $(-1)^4 =$ ______ f) $(-1)^1 =$ ______

g) $(-8)^2 =$ _______ h) $(-7)^3 =$ _______

i) $(-10)^4 =$ _______ j) $(-10)^5 =$ _______

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NEGATIVE SQUARE ROOTS

We now know that both $3^2 = 9$ and $(-3)^2 = 9$. It's appropriate, therefore, to say that 9 has *two* square roots, a positive square root (+ 3) and a negative square root (- 3).

The Rule of the Square Root:
If $r^2 = P$ then r is *a* square root of P.

To write "The square root of 9," though, is rather cumbersome, so we use a symbol $\sqrt{}$ called the **radical**. The number that fits inside the radical is called the **radicand**. So, "The square root of 9" is written as $\sqrt{9}$. As you probably know, $\sqrt{9} = 3$. But we have new information about the square roots of 9: there are two of them, + 3 and - 3.

Does this change what we've learned about the square root radical? No, $\sqrt{9}$ still means the square root of 9, but now we need to say that it refers only to the **principal square root**—the *positive* square root—and $\sqrt{9} = 3$. We can emphasize, when using the radical, that the square root of a number is the *principal*, or *positive*, square root using the plus sign: $\sqrt{9} = +3$.

So, how can we represent - 3 as a square root of 9? We simply negate the *outside* of the radical:

$$-\sqrt{9} = -3.$$

Notice that the negative is outside of the radical; also, we can rewrite the expression $-\sqrt{9}$, using the "4th meaning of the dash," to be $-1 \cdot \sqrt{9}$. Before we can apply the negative, though, we must first apply the radical—the square root—to 9 because the radical is a grouping symbol.

Example 3: Evaluate each radical expression.

a) $\sqrt{36}$ b) $-\sqrt{36}$ c) $\sqrt{25}$ d) $-\sqrt{25}$

Answer: Apply the square roots before negating.

a) $\sqrt{36} = +6$ b) $-\sqrt{36} = -6$

c) $\sqrt{25} = +5$ d) $-\sqrt{25} = -5$

Exercise 3: Evaluate each radical expression.

a) $\sqrt{49}$ = ______ b) $-\sqrt{16}$ = ______

c) $-\sqrt{81}$ = ______ d) $\sqrt{64}$ = ______

e) $-\sqrt{100}$ = ______ f) $-\sqrt{1}$ = ______

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Square Root of a Negative Number

We know that $(+3)(+3) = +9$ and $(-3)(-3) = +9$

This suggests that 9 has two square roots, $+3$ and -3. What about -9? Does it have any square roots? Is there any number that when multiplied by itself gives a product of -9?

We have seen that the product of two positive numbers is another positive number, and we have seen that the product of two negative numbers is also a positive number. It would make sense, then, that -9 can't be a perfect square. Therefore, it can't have a square root.

This means that $\sqrt{-9}$ has no real value. For our purposes, we'll say that $\sqrt{-9}$ is undefined. Why isn't this like $-\sqrt{9}$? The difference between the two is when we apply the negative:

(a) in $-\sqrt{9}$, we apply the negative after applying the square root; in other words, we get the square root of 9, which is +3, *before* applying the negative, making it -3.

(b) in $\sqrt{-9}$, we apply the negative before applying the square root; in other words, we have a -9 (a negative radicand) before we get a chance to apply the square root.

It all goes back to the order of operations. We must work within grouping symbols first. The radical is both a grouping symbol and an operation, but we can't apply the operation of "square root" before we evaluate what is "inside" of it. There is no way to first "take" the negative out of the radical.

Example 4: Evaluate each radical expression, *if possible*. If the radical is undefined, say so.

a) $\sqrt{36}$ b) $-\sqrt{36}$ c) $\sqrt{-36}$ d) $-\sqrt{-36}$

Answer: Apply the square roots before negating.

a) $\sqrt{36} = +6$

b) $-\sqrt{36} = -6$

c) $\sqrt{-36}$ is undefined

d) $-\sqrt{-36}$ is also undefined

(a negative undefined value is still undefined.)

Exercise 4: Evaluate each radical expression, if possible. If the radical is undefined, say so.

a) $\sqrt{64}$ = ______ b) $-\sqrt{64}$ = ______

c) $\sqrt{-81}$ = ______ d) $-\sqrt{81}$ = ______

e) $\sqrt{-1}$ = ______ f) $-\sqrt{-1}$ = ______

g) $-\sqrt{100}$ = ______ h) $\sqrt{100}$ = ______

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THE ORDER OF OPERATIONS AND NEGATIVE NUMBERS

When you were first introduced to the order of operations, in Section 0.3, you were asked to evaluate expressions that held only positive numbers. To that point, we had not discussed negative numbers.

Of course, we are now familiar with negative numbers. The order of operations doesn't change because of negative numbers, but we may need to be a little more careful when evaluating such expressions.

Example 5: Evaluate each according to the *Order of Operations*.

a) $3 - 4 \cdot 5$ b) $3 - 4 \cdot (-5)$ c) $(4 - 10)^2$

Procedure: Each of these has two operations; some of them have grouping symbols that will affect the order that the operations are applied.

a)	$3 - 4 \cdot 5$	The minus sign can be thought of as a negative sign; it belongs to the 4. Multiply first: $-4 \cdot 5 = -20$.
	$= 3 - 20$	-20 shows up as "minus 20."
	$= \boxed{-17}$	
b)	$3 - 4 \cdot (-5)$	This time the product, which is applied first, is of -4 and -5. Multiply first: $-4 \cdot (-5) = +20$ (positive 20).
	$= 3 + 20$	Positive 20 shows up as "plus 20."
	$= \boxed{23}$	
c)	$(4 - 10)^2$	Subtract first, but keep the parentheses.
	$= (-6)^2$	The negative is being squared along with the 6. The square of a negative number is a positive number: $(-6)^2 = (-6)(-6) = +36$.
	$= \boxed{36}$	

A challenging expression in algebra is, surprisingly, something as simple looking as -3^2. How is this to be interpreted? What is its value?

We know, for example, that $(-3)^2 = (-3)(-3) = +9$. What if the parentheses were removed, does -3^2 have the same value of $+9$?

To explore this, let's consider the expression $14 - 3^2$. When we look at this expression we should see two operators, the minus sign and the exponent. Since there are no parentheses, we should apply the exponent first, then the subtraction:

$$14 - 3^2$$
$$= 14 - 9$$
$$= 5$$

Similarly, if the expression were $0 - 3^2$, then we would, again, apply the exponent first and then subtraction:

$$\begin{aligned} & 0 - 3^2 \\ =\ & 0 - 9 \\ =\ & -9 \end{aligned}$$

What you should recognize is that $0 - 3^2$ is the same as just -3^2 (just as $0 - 5 = -5$). The example above says this has the value of -9, so it makes sense that $-3^2 = -9$.

The challenging thing about this is that, at first glance, it looks like the result should be $+9$. We need to recognize, though, that the negative is not being grouped with the 3; that means that the exponent is being applied only to the 3 and not to the negative.

Another way that we can look at this is to use the *4th meaning of the dash*, as "- 1 times." Using this meaning, -3^2 becomes $-1 \cdot 3^2$. Applying the order of operations now we get:

$$\begin{aligned} & -3^2 \\ =\ & -1 \cdot 3^2 \\ =\ & -1 \cdot 9 \\ =\ & -9 \end{aligned}$$

So, after all of the explanation, -3^2 can be thought of as "the opposite of 3^2" which is the opposite of 9, which is -9.

Exercise 5: Evaluate each expression using the techniques learned in this section.

a) $(7 - 9)^2$ b) $7 - 9^2$

c) $0 - 5^2$ d) $(4 - 6)^3$

e) -8^2 f) $(6 - 10)^2$

g) $26 - 4^2$ h) $-30 - 7^2$

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DOUBLE QUANTITIES

Sometimes an expression will have two sets of grouping symbols that are unrelated to each other; in other words, evaluating within one set does not affect the evaluation within the other. This means that some quantities can be evaluated at the same time.

For example, in the expression $(3 - 8) \cdot (12 \div 4)$ we can evaluate within each grouping symbol regardless of what operation each has:

	$(3 - 8) \cdot (12 \div 4)$	Here there are three operations: subtraction, multiplication and division. <u>Subtraction and division have EQUAL rank here because of the parentheses</u>.
=	$(-5) \cdot (3)$	We can evaluate within each grouping separately yet at the same time.
=	-15	

Example 6: Evaluate each according to the *Order of Operations*.

a) $(5 \cdot 6) \div (4 - 7)$ b) $(-8 + 3) \cdot (-14 \div 2)$

c) $(24 \div 6) - \sqrt{5 + 11}$ d) $\sqrt{4^2} - \sqrt{50 \cdot 2}$

Answer: Operations within different grouping symbols have equal rank. They can be applied at the same time.

a)	$(5 \cdot 6) \div (4 - 7)$	We can apply both the multiplication and the subtraction in the same step.
=	$(30) \div (-3)$	A positive divided by a negative results in a negative.
=	$\boxed{-10}$	
b)	$(-8 + 3) \cdot (-14 \div 2)$	We can apply both the addition and the division in the same step.
=	$(-5) \cdot (-7)$	A negative multiplied by a negative results in a positive.
=	$\boxed{+35}$	
c)	$(24 \div 6) - \sqrt{5 + 11}$	We can apply both the division and the addition in the same step.
=	$4 - \sqrt{16}$	Now apply the radical.
=	$4 - 4$	and subtract.
=	$\boxed{0}$	
d)	$\sqrt{4^2} - \sqrt{50 \cdot 2}$	We can apply both the exponent and the multiplication in the same step.
=	$\sqrt{16} - \sqrt{100}$	Now we can apply both radicals at the same time (they have the same rank).
=	$4 - 10$	
=	$\boxed{-6}$	

Exercise 6: Evaluate each expression according to the *Order of Operations*.

a) $(7 + 3) \cdot (9 - 4)$

b) $(2 \cdot 3) + (-42 \div 6)$

c) $(6 - 13) \cdot (-5 + 2)$

d) $(-2 \cdot 12) \div (-4 - 2)$

e) $(6 + 3)^2 - \sqrt{-4 + 5}$

f) $\sqrt{12 \cdot 3} - \sqrt{8 + 41}$

g) $(7 + 3)^2 \div (9 - 4)^2$

h) $(2 - 3)^3 - (-12 \div 6)^3$

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THE FRACTION BAR

As was mentioned at the beginning of this section, the fraction bar is a grouping symbol. It groups the numerator separately from the denominator. So, when evaluating an expression that involves a fraction bar, you may treat the numerator and denominator as if they were grouped separately and evaluate within them separately.

For example, in the fraction $\frac{3 \cdot 8}{5 + 1}$ the operations multiplication (in the numerator) and addition (in the denominator) have equal rank because of the groupings: $\frac{(3 \cdot 8)}{(5 + 1)}$. So, we can apply both multiplication and addition at the same time to get $\frac{24}{6}$. This fraction reduces to 4. The parentheses shown in $\frac{(3 \cdot 8)}{(5 + 1)}$ are not necessary; they are there to emphasize the grouping nature of the fraction bar.

Example 7: Evaluate each completely.

a) $\dfrac{56 \div 7}{3 - 1}$ b) $\dfrac{12 \cdot 5}{8 - 12}$

c) $\dfrac{2 - 3^2}{4 - 5^2}$ d) $\dfrac{\sqrt{16} - 5}{7 - 2}$

Procedure: Remember, the radical is both a grouping symbol and an operation—part (d).

a) $\dfrac{56 \div 7}{3 - 1}$ First, apply the division and subtraction individually, but at the same time.

$= \dfrac{8}{2}$ Either think of it as a fraction that needs to be reduced (by a factor of 2) or think of it as division: $8 \div 2 = 4$

$= \boxed{4}$

b) $\dfrac{12 \cdot 5}{8 - 12}$ Apply both multiplication and subtraction at the same time.

$= \dfrac{60}{-4}$ Either think of it as a fraction that needs to be reduced (by a factor of - 4) or think of it as division: $(+60) \div (-4) = -15$

$= \boxed{-15}$

c) $\dfrac{2 - 3^2}{4 - 5^2}$ First apply the exponents individually, but at the same time.

$= \dfrac{2 - 9}{4 - 25}$ Next apply both subtractions individually, but at the same time.

$= \dfrac{-7}{-21}$ We may think of this only as a fraction that needs to be reduced (by a factor of - 7).

$= \boxed{+\dfrac{1}{3}}$

d) $\dfrac{\sqrt{16} - 5}{7 - 2}$ First apply the radical in the numerator and subtraction in the denominator.

$= \dfrac{4 - 5}{5}$ Next apply subtraction in the numerator.

$= \boxed{\dfrac{-1}{5}}$ This fraction cannot simplify, so we're done.

Exercise 7: Evaluate each expression according to the *Order of Operations*.

a) $\dfrac{7 + 3}{9 - 4}$ b) $\dfrac{2 \cdot 3}{-42 \div 7}$

c) $\dfrac{8 \cdot 6}{7 + 3^2}$ d) $\dfrac{3^2 - 7^2}{-20 \div 4}$

e) $\dfrac{5 \cdot 2}{\sqrt{4} + 8}$ f) $\dfrac{4^2 - 6}{2^2 + 1}$

Now let's look at some examples that contain other grouping symbols.

Example 8: Evaluate each completely. Remember, the radical is both a grouping symbol and an operation; the same is true for the absolute value bars.

a) $\sqrt{-5 - 11}$ b) $-\sqrt{9} + 8$ c) $|4 - 6| - |-3|$

Procedure: The radical and the absolute value are both grouping symbols and operations.

(a) $\sqrt{-5 - 11}$ First, evaluate within the radical.

$= \sqrt{-16}$ We cannot apply the square root to a negative number.

is $\boxed{\text{undefined}}$

(b) $-\sqrt{9} + 8$ Here, the negative is outside of the radical; So, we first apply the radical to the 9.

$= -3 + 8$ Next apply the addition.

$= \boxed{5}$

(c) $|4 - 6| - |-3|$ In (c), The absolute value bars are the only grouping symbol, but for the second set, $|-3|$, there is nothing to evaluate *inside*; instead, evaluate within the first set only: $|4 - 6| = |-2|$.

$= |-2| - |-3|$ Now we can *apply* the absolute value, not as a grouping symbol but as an operation. Lastly, apply subtraction to get -1.

$= 2 - 3$

$= \boxed{-1}$

Important note: Again, the key to successfully applying the order of operations is to do so one step at a time. It's very important to show your work every step of the way. This will lead to accurate answers and enable others to read and learn from your work.

Exercise 8: Evaluate each according to the *Order of Operations*.

a) $-6 + \sqrt{16}$

b) $-9 \cdot \sqrt{25}$

c) $3 - \sqrt{81}$

d) $-\sqrt{-4 \cdot (-9)}$

e) $\sqrt{-1 - 12 \cdot 4}$

f) $\sqrt{(6-2) \cdot 5^2}$

g) $|-9 + 3|$

h) $|-12| - |-5|$

i) $(6 - 8) \cdot |5 - 10|$

j) $|7 - 4| - |-9|$

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THE MAIN OPERATION

Every algebraic or numeric expression has one operation that, in a sense, holds the whole expression together; this is called the "main operation." It is the main operation that lets us know if the expression is a sum, difference, product or quotient. The **main operation** is the last operation that is to be applied, according to the order of operations.

For example, in the expression $5 + 6 \cdot 3$, of the two operations present (addition and multiplication), the order of operations has us apply the multiplication first and the *addition last*. Hence, the *main operation* is addition (it is applied last), and we can think of $5 + 6 \cdot 3$ as a *sum*:

it is a sum of two parts: 5 *and* $6 \cdot 3$.

However, if the expression had been written, with parentheses, as $(5 + 6) \cdot 3$, then the addition must be applied first—because of the grouping symbols—and multiplication is applied last, so the main operation is *multiplication*. This means that $(5 + 6) \cdot 3$ is a *product:*

It is a product of two *factors*, $(5 + 6)$ *and* 3.

These two expressions illustrate a rule about the main operation:

> If there are no grouping symbols, then the **main operation** will be the one present that has the lowest rank.
>
> If grouping symbols *are* present, then any operations inside must be applied before any operations outside of them, and then one of the "outside" operations will be the **main operation**.

Example 9: Identify the *main operation* in each expression and state whether it is a sum, difference, product, quotient, power or square root. DO NOT EVALUATE THE EXPRESSIONS.

a) $6 + 12 \div 3$ b) $52 - 8$ c) $24 \div 6 \cdot 2$

d) $(9 - 5)^2$ e) $(6 + 12) \div 3$

Procedure: Identify the last operation to be done IF you were to evaluate:

	Expression	**Main Operation**	**the expression is a:**
a)	$6 + 12 \div 3$	addition	sum
b)	$5^2 - 8$	subtraction	difference
c)	$24 \div 6 \cdot 2$	multiplication	product
d)	$(9 - 5)^2$	the power of 2 (square)	power
e)	$(6 + 12) \div 3$	division	quotient

Exercise 9: Identify the *main operation* in each expression and state whether it is a sum, difference, product, quotient or power. DO NOT EVALUATE THE EXPRESSIONS.

	Expression	Main Operation	the expression is a:
a)	$24 \div 6 + 2$	______________	______________
b)	$24 \div (6 + 2)$	______________	______________
c)	$10 - 3 \cdot 2$	______________	______________
d)	$(10 - 3) \cdot 2$	______________	______________
e)	$(12 \div 2)^2$	______________	______________
f)	$35 \div 5 + 2 \cdot 3$	______________	______________
g)	$(6 + 12) \div (2 \cdot 3)$	______________	______________
h)	$6 + [12 \div (2 \cdot 3)]$	______________	______________
i)	$9 \cdot \sqrt{25}$	______________	______________

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Answers to each Exercise

Section 1.6

Exercise 1:	a)	6	b)	3	c)	4	d)	14
	e)	16	f)	13	g)	45	h)	8
	i)	3	j)	8				
Exercise 2:	a)	+ 25	b)	- 64	c)	+ 1	d)	- 1
	e)	+ 1	f)	- 1	g)	+ 64	h)	- 343
	i)	+ 10,000	j)	- 100,000				
Exercise 3:	a)	+ 7	b)	- 4	c)	- 9	d)	+ 8
	e)	- 10	f)	- 1				
Exercise 4:	a)	+ 8	b)	- 8	c)	Undefined	d)	- 9
	e)	Undefined	f)	Undefined	g)	- 10	h)	+ 10
Exercise 5:	a)	+ 4	b)	- 74	c)	- 25	d)	- 8
	e)	- 64	f)	+ 16	g)	+ 10	h)	- 79
Exercise 6:	a)	+ 50	b)	- 1	c)	+ 21	d)	+ 4
	e)	+ 80	f)	- 1	g)	+ 4	h)	+ 7
Exercise 7:	a)	+ 2	b)	- 1	c)	+ 3	d)	+ 8
	e)	+ 1	f)	+ 2				
Exercise 8:	a)	- 2	b)	- 45	c)	- 6	d)	- 6
	e)	Undefined	f)	+ 10	g)	+ 6	h)	+ 7
	i)	- 10	j)	- 6				

Exercise 9:	a)	addition; sum	b)	division; quotient
	c)	subtraction; difference	d)	multiplication; product
	e)	exponent; power	f)	addition; sum
	g)	division; quotient	h)	addition; sum
	i)	multiplication; product		

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Section 1.6 Focus Exercises

1. Evaluate each expression.

a) $(-9)^2 =$ ________ b) $(-2)^3 =$ ________ c) $(-1)^4 =$ ________

d) $(5)^3 =$ ________ e) $(10)^1 =$ ________ f) $(-10)^5 =$ ________

g) $(-3)^4 =$ ________ h) $(-6)^1 =$ ________ i) $(-1)^7 =$ ________

j) $-5^2 =$ ________ k) $-8^1 =$ ________ l) $-4^3 =$ ________

2. Evaluate each radical expression, if possible. If the radical is undefined, say so.

a) $\sqrt{-25} =$ ________ b) $-\sqrt{49} =$ ________ c) $\sqrt{100} =$ ________

d) $-\sqrt{16} =$ ________ e) $\sqrt{-4} =$ ________ f) $-\sqrt{-9} =$ ________

3. Evaluate each expression.

a) $(2-7)^2$ b) $2^2 - 7^2$ c) $2 - 7^2$ d) -7^2

4. Evaluate each expression according to the *Order of Operations*.

a) $30 \div 5 \cdot 2$

b) $-4 \cdot \sqrt{9}$

c) $15 - 6 \cdot 3$

d) $30 \div 10 - 4$

e) $(6-4)^2$

f) $36 \div 3 + 6 \div 2$

5. Evaluate each expression according to the *Order of Operations*.

a) $(6 \cdot 5) \div (5 - 2)$

b) $\dfrac{48 - 2 \cdot 12}{2}$

c) $\dfrac{\;\dfrac{29 - 25}{8}\;}{\sqrt{4}}$

d) $9 - \dfrac{2 \cdot 6}{\sqrt{16}}$

e) $\sqrt{6^2 + 8^2}$

f) $|2 - 8| - |-9|$

5. Identify the *main operation* in each expression and state whether it is a *sum*, *difference*, *product*, *quotient* or *power*. DO NOT EVALUATE THE EXPRESSIONS.

	Expression	Main Operation	the expression is a:
a)	$30 \div 5 \cdot 2$	________	________
b)	$15 - 6 \cdot 3$	________	________
c)	$(6 - 4)^2$	________	________
d)	$36 \div 3 + 6 \div 2$	________	________
e)	$-4 \cdot \sqrt{9}$	________	________

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