Section 1.7 Formulas

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Introduction

Many formulas—in a variety of fields—require the order of operations for their proper evaluation. For example, if you need to convert a temperature reading from Celsius degrees to Fahrenheit degrees, you need to use the formula

$$F = \frac{9}{5} C + 32,$$

where: F = Fahrenheit degrees
C = Celsius degrees

You' may have seen Celsius temperatures, for example, at bank buildings. The sign out front might say that the temperature is 15° C, but is that t-shirt or jacket weather?

To answer that question, we need to *substitute* the known value of C (15°) into the formula; we then evaluate the expression on the right side using—that's right—the order of operations!

To **substitute** a *value* into a formula means to *replace* the variable with a number; this number is called the **replacement value**. Whatever operation was applied to the variable is now applied to the replacement value.

When we see $F = \frac{9}{5} C + 32$, $\mathbf{C = 15}$ ← this means we replace C with 15.

$F = \frac{9}{5}(15) + 32$ Substitute the value of C = 15.

$F = \frac{9}{5} \cdot \frac{15}{1} + 32$ Now multiply the fractions; cross divide to simplify.

$F = \frac{9}{1} \cdot \frac{3}{1} + 32$ Use the order of operations to complete the evaluation.

$F = 27 + 32$

$F = 59$

We can interpret this as saying 15° C is equivalent to 59° F. By the way, you might want to put a jacket on, it's starting to get a little chilly.

You may have noticed the use of parentheses around the 15. Here, the parentheses are used more as a separator than as grouping symbols. When substituting a value into a formula, it is usually proper to put parentheses around that number, especially when

1) the variable is being multiplied or divided;

2) when the replacement value is a negative number—we need to make sure that the negative sign is grouped with the number;

Many formulas have more than one variable that needs to be replaced. In this case, substitute for all such variables—using their corresponding replacement values—at the same time. Be especially careful in your use of parentheses.

Example 1: Evaluate the numerical value of each formula with the given replacement values.

a) $d = r \cdot t$ $\quad r = 50$, $t = 2$

b) $E = \dfrac{9R}{I}$ $\quad R = 10$, $I = 45$

c) $m = \dfrac{x - p}{y - q}$ $\quad x = -2$, $p = 4$, $y = 6$, $q = -3$

d) $c = \sqrt{a^2 + b^2}$ $\quad a = 3$, $b = 4$

Answer:

a)
$$d = r \cdot t$$
$$d = (50)\cdot(2)$$
$$d = \boxed{\mathbf{100}}$$

b)
$$E = \frac{9R}{I}$$
$$E = \frac{9(10)}{45}$$
$$E = \frac{90}{45} = \boxed{\mathbf{2}}$$

c)
$$m = \frac{(-2) - 4}{6 - (-3)}$$
$$m = \frac{-6}{6 + 3}$$
$$m = \frac{-6}{9}$$
$$m = \boxed{\mathbf{-\frac{2}{3}}}$$

d)
$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{3^2 + 4^2}$$
$$c = \sqrt{9 + 16}$$
$$c = \sqrt{25} = \boxed{\mathbf{5}}$$

Exercise 1: Evaluate the numerical value of each formula with the given replacement values.

a) $d = r \cdot t$ $\quad r = 15$, $t = 3$

b) $A = \dfrac{a + b + c}{3}$ $\quad a = 15$, $b = 23$, $c = 34$

c) $m = \dfrac{x - p}{y - q}$ $\quad x = 10$, $p = -6$, $y = -5$, $q = 7$

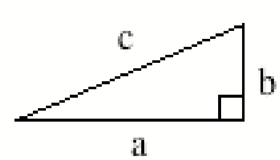
d) $z = \dfrac{x - m}{s}$ $\quad x = 24$, $m = 16$, $s = 4$

From where do we get all of these formulas, anyway? In Example 1 and Exercise 1, we saw six formulas from a variety of disciplines or interests:

(a)	$d = r \cdot t$	from physics:	distance = rate · time
(b)	$E = \dfrac{9R}{I}$	from baseball:	a pitcher's "earned run average" R = # runs allowed; I = # innings pitched
(c)	$m = \dfrac{x - p}{y - q}$	from algebra:	the slope of a line
(d)	$A = \dfrac{a + b + c}{3}$	from statistics:	The average of three numbers
(e)	$z = \dfrac{x - m}{s}$	from statistics:	an important conversion formula
(f)	$c = \sqrt{a^2 + b^2}$	from geometry:	the Pythagorean Theorem

c = the length of the longest side of a right triangle and

a and b are the lengths of the other two sides



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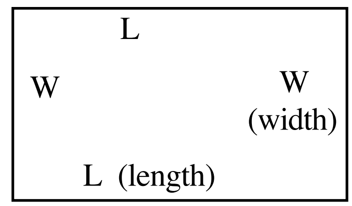
MORE FORMULAS FROM GEOMETRY

Some basic shapes from geometry include the rectangle, the triangle and the circle. For most geometric shapes we are interested in the **perimeter**, the line (or "fencing") around the figure, and the **area**, the inside "fill" of the figure. The perimeter of a circle is called the **circumference**. (**Area** is always represented in **square** units, like feet^2.)

The Rectangle: **Perimeter** **Area**

$P = L + W + L + W$ $\quad$ $\mathbf{A = L \cdot W}$

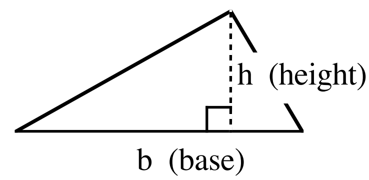
or $\mathbf{P = 2 \cdot L + 2 \cdot W}$



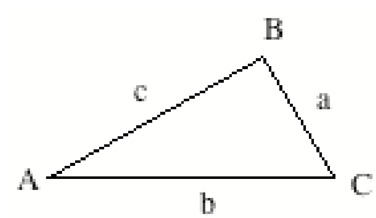
The Triangle: **Area**

$A = \frac{1}{2} \cdot b \cdot h$

or $A = \frac{b \cdot h}{2}$



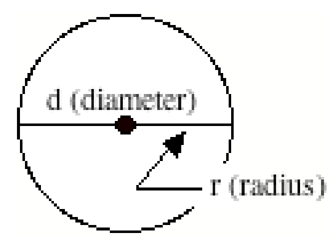
We also have formulas related to the *sides* and *angles* of a **triangle:**



Perimeter	**Sum of angles**
$P = a + b + c$	$A + B + C = 180$ (degrees)
a, b and c are side lengths.	A, B and C are angle measures.

The Circle: **Circumference** **Area**

$C = \pi \cdot d$ $\quad$ $A = \pi \cdot r^2$



In a circle: The *radius* is half of the *diameter*.

Also: π is an irrational number; $\pi \approx 3.14$.

Example 2: Find the following using the appropriate formula and the given values.

a) the perimeter of a rectangle

$L = 8$ ft. and $W = 6$ ft.

b) the area of a triangle

$b = 10$ in. and $h = 7$ in.

Answer: Be sure to write the answer so that the unit of measure is appropriate.

a) $P = 2L + 2W$

$P = 2(8 \text{ ft.}) + 2(6 \text{ ft.})$

$P = 12 \text{ ft.} + 16 \text{ ft.}$

$P = 28$ ft. (feet)

b) $A = \frac{1}{2} \cdot b \cdot h$

$A = \frac{1}{2} \cdot (10 \text{ in.})(7 \text{ in.})$

$A = 5 \text{ in.} \cdot 7 \text{ in.}$

$A = 35 \text{ in.}^2$ (square inches)

Example 3: Find the following using the appropriate formula and the given values.

a) the area of a rectangle

$L = 9$ yd. and $W = 5$ yd.

b) the circumference of a circle

$d = 5$ m.; use $\pi = 3.14$

Answer: Be sure to write the answer so that the unit of measure is appropriate.

a) $A = L \cdot W$

$A = (9 \text{ yd.}) \cdot (5 \text{ yd.})$

$A = 45 \text{ yd.}^2$ (square yards)

b) $C = \pi \cdot d$

$C \approx (3.14)(5 \text{ m.})$

$C \approx 15.7$ m. (meters)

We use $\approx$ in place of $=$ when the valule is an approximation.

Exercise 2: Find the following using the appropriate formula and the given values. Be sure to write the answer so that the unit of measure is appropriate.

a) the perimeter of a triangle with sides
a = 5.3 inches,
b = 6.4 inches and
c = 3.9 inches

b) the area of a triangle with a **base** of 7 feet and a **height** of 6 feet

c) the area of a circle with a **radius** of 10 inches.

d) the perimeter of a rectangle with a **length** of 3.2 yards and a **width** of 2.8 yards.

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AN IMPORTANT FORMULA FROM STATISTICS

An important concept in statistics is the **mean**, or **average**, of a set of numbers. In statistics, we call a set of numbers "data values." These data values can be collected from a variety of sources.

The formula for the average (mean) of any set of data values is

$$\textbf{Average} = \frac{\text{the sum of all of the data values}}{\text{the number of data values in the set}}$$

Consider the following advertisement for an automobile dealership.

We sell an average of 80 cars a month.
Come see what *Darryl's Autos* has got for you!

How would Darryl know that he sells an average of 80 cars a month? Consider this table, indicating the number of cars sold during the first four months of the year.

Month	January (J)	February (F)	March (M)	April (A)
# Sold	70	78	84	88

The formula for the average is $A = \dfrac{J + F + M + A}{4 \text{ (months)}}$

Replacing J, F, M and A with the values in the table, we get

$$A = \frac{(70 + 78 + 84 + 88) \text{ cars}}{4 \text{ months}} = \frac{320 \text{ cars}}{4 \text{ months}} = 80 \text{ cars per month}$$

He didn't sell exactly 80 cars in any one month, but <u>he sold an average of 80 cars for those four months</u>.

Example 4: If Carla received test scores of 77, 85, and 90, what is her average test score? Write the answer as a complete sentence.

Answer: Find the sum of the scores and <u>divide by **3**</u> (the number of tests taken).

$$A = \frac{77 + 85 + 90}{3} = \frac{252}{3} = 84 \text{ points.}$$

We can interpret this answer as, <u>Carla's average test score is 84 points.</u>

Example 5: Mai is a waitress. In five days, she made daily tips of \$23, \$35, \$27, \$29 and \$31. What was her average daily tip for those five days?

Answer: Find the sum of the scores and divide by **5** (the number of days).

$$A = \frac{(23 + 35 + 27 + 29 + 31) \text{ dollars}}{5 \text{ days}} = \frac{\$145}{5 \text{ days}} = \$29 \text{ per day.}$$

<u>Mai earned an average of \$29 per day.</u>

<u>Exercise 3:</u> Find the average of each situation, as described within.

a) Humberto received the following test scores in his Algebra class: 80, 89, and 95. What is his average score on those three tests?

b) Yat-Sun kept track of how much he was paying for gas each week. For four weeks his gasoline bills came to \$43, \$28, \$35 and \$42. On average, how much did he spend on gas each week?

c) Devon was keeping track of the number of pages he read each day. For seven straight days he read 43, 38, 26, 45, 39, 21 and 33 pages. How many pages, on average, did Devon read each day?

d) Sarah kept track of how many minutes it took her to drive to work each day. For five straight days it took her 17, 19, 16, 23, and 25 minutes to get to work. How many minutes, on average, did it take Sarah to get to work each day?

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THE DISTANCE, RATE AND TIME FORMULA

The formula **d = r·t**, read as "**distance** equals **rate** times **time**" is used for motion problems such as traveling by car, by plane, by train, by bicycle, or by foot. In each case, one travels a certain distance at a certain rate of speed for a certain period of time.

A common use of this formula is related to driving a car where the unit of measure for distance is *miles*, for time is *hours*, and for rate is *miles per hour*. The formula is not restricted to those measures, though. For example, one might want to measure the rate of an ant in terms of *centimeters per second* or in *feet per minute*. What must be present is a measure of length (miles, feet, centimeters, etc.) and of time (seconds, minutes, hours).

Imagine, for example, that you are driving a car at a rate of 60 miles per hour (mph). If you travel for 1 hour, you will have gone 60 miles. Traveling for 2 hours will take you 120 miles.

Using the formula **d = r·t**, we get:

a) r = 60 mph, t = 1 hour

$$d = \frac{60 \text{ miles}}{1 \text{ hour}} \cdot (1 \text{ hour}) = 60 \text{ miles}$$

b) r = 60 mph, t = 2 hours

$$d = \frac{60 \text{ miles}}{1 \text{ hour}} \cdot (2 \text{ hour}) = 120 \text{ miles}$$

We don't need to represent the units of measure (miles) throughout the process, but we should represent it at the end.

Example 6: Use the distance formula to determine the distance traveled using the given rate and time.

a) rate = 15 mph, time = 3 hours

b) rate = 50 mph, time = $\frac{1}{2}$ hour

Answer: Use $d = r \cdot t$

a) $d = 15 \cdot 3 = 45$ miles

b) $d = 50 \cdot \frac{1}{2} = \frac{50}{2} = 25$ miles

The formula $d = r \cdot t$ is useful in finding the distance, d, traveled. If, instead, you wanted to know your average *rate* (speed), r, you can use a variation of that formula ....

$$r = \frac{d}{t}$$

... and if we want to know the *time* it could take us to get from one place to another—depending on how fast we drive—we can use a third formula..............................

$$t = \frac{d}{r}$$

In other words, there are three different forms of the distance formula. Which you use depends on what information you're given:

i) if you're given the rate and time, find the *distance* using $d = r \cdot t$

ii) if you're given the distance and time, find the *rate* using $r = \frac{d}{t}$

iii) if you're given the distance and rate, find the *time* using $t = \frac{d}{r}$

Example 7: Use one of the distances formulas to determine the missing value, which will be either **rate**, **distance** or **time**.

a) rate = 65 mph, time = 4 hr.

b) distance = 12 cm, time = 3 sec.

c) distance = 80 feet, rate = 5 feet per min.

d) distance = 100 mi., time = $\frac{1}{2}$ hr.

Procedure: Use the formula for the measure that is not known: $\mathbf{d = r \cdot t}$, $\mathbf{r = \frac{d}{t}}$ or $\mathbf{t = \frac{d}{r}}$.

Answer: a) We know rate and time; we don't know distance, so we'll use $\mathbf{d = r \cdot t}$

$$\mathbf{d = r \cdot t} = \frac{65 \text{ miles}}{1 \text{ hour}} \cdot 4 \text{ hours} = 260 \text{ miles}$$

b) We know distance and time; we don't know the rate, so we'll use $\mathbf{r = \frac{d}{t}}$

$$\mathbf{r = \frac{d}{t}} = \frac{12 \text{ cm}}{3 \text{ sec}} = 4 \text{ cm per sec } \left(4\ {}^{\text{cm}}/_{\text{sec}}\right)$$

c) We know distance and rate; we don't know the time, so we'll use $\mathbf{t = \frac{d}{r}}$

$$\mathbf{t = \frac{d}{r}} = \frac{80 \text{ feet}}{5 \text{ ft per min.}} = \frac{80}{5} \text{ minutes} = 16 \text{ minutes.}$$

d) We know distance and time; we don't know rate, so we'll use $\mathbf{r = \frac{d}{t}}$

$$\mathbf{r = \frac{d}{t}} = \frac{100 \text{ miles}}{\frac{1}{2} \text{ hour}} = \left(100 \div \frac{1}{2}\right) \text{ mph} = \left(100 \cdot \frac{2}{1}\right) \text{ mph} = 200 \text{ mph}$$

The reason we say that **r** represents an *average* rate of speed is because we don't consider the speed every second of every minute of the trip. We don't see the slowing down due to heavy traffic, the stopping at a traffic signal, the acceleration when the traffic is lighter. Instead, the formula $r = \frac{d}{t}$ asks you to consider the average rate of speed over the entire trip (or over a certain portion of the trip).

It's very much like a numerical grade in class; your grade at the end is based on your accumulated grade from (possibly) all of your test, quizzes, homework and projects. The grade you receive at the end is just an average of all of the little grades you received along the way.

Exercise 4: Ben had to drive from San Diego to Los Angeles. The drive is 147 miles long. The traffic was so rough it took him 3 hours to get there. What was his average rate (speed) for the trip?

Exercise 5: Beatriz was flying from San Francisco to New Orleans. No, not as a passenger, she's the pilot. She averaged 240 miles per hour in traveling the 1,920 miles. How many hours did the flight take?

Exercise 6: Banjo the Beagle loves to chase tennis balls. While playing at an empty football field one day, his owner timed him while he was chasing balls. On one chase, Banjo ran 36 yards in $4\frac{1}{2}$ seconds. What was Banjo's rate on that run? (**Hint**: first convert the time into an improper fraction.)

Exercise 7: Kinde is going to run in a 12 kilometer (km) race. She is usually able to average 18 km per hour. How long should it take her to complete the race?

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SIMPLE INTEREST

In the world of banking and finance, money that is put away for awhile—in a special savings account or other financial investment—usually gains **interest**, an amount of money that is automatically added to the account after a period of time. There are two types of interest: **simple interest**, which occurs when the money is left alone for one year or less, and **compound interest** which occurs when money is left alone for longer than one year. In this course we will concentrate on simple interest only.

The formula for **simple interest** is **I = P·r·t**. The money you put in at the start is called the **principal**. The *amount of interest*, **I**, you earn is based on the *principal*, **P**, the **interest rate**, **r** (always a *percent*) and the amount of **time**, **t**, (in a fraction of 1 year) it is left in the bank.

Example 8: Find the amount of interest earned given the principal, the interest rate and the amount of time the money is left in the account.

a) P = \$1,000	**b)** P = \$3,500	**c)** P = \$4,000
r = 8%	r = 10%	r = 6%
t = 1 year	t = $\frac{1}{2}$ year	t = 8 months

Answer: Rewrite each percent as a decimal, then multiply appropriately.

a)

I = P·r·t	Substitute the numbers into the formula; 8% = .08
$I = 1000 \cdot (.08) \cdot 1$	Multiply P · r first: $1000 \cdot (.08) = 80$
$I = 80 \cdot 1$	Now multiply $80 \cdot 1 = 80$
I = 80 dollars	

b)

I = P·r·t	10% = .10
$I = 3500 \cdot (.10) \cdot \frac{1}{2}$	Multiply P · r first: $3500 \cdot (.10) = 350$
$I = 350 \cdot \frac{1}{2}$	Now multiply $350 \cdot \frac{1}{2} = 175$
I = 175 dollars	

c)

I = P·r·t	Rewrite 8 months in terms of years: 8 months = $\frac{8}{12}$ years = $\frac{2}{3}$ years.
$I = 4000 \cdot (.06) \cdot \frac{2}{3}$	Multiply P · r first: $4000 \cdot (.06) = 240$
$I = 240 \cdot \frac{2}{3}$	Multiply $240 \cdot \frac{2}{3} = \frac{240}{1} \cdot \frac{2}{3} = \frac{80}{1} \cdot \frac{2}{1} = 160$.
I = 160 dollars	

Exercise 8: For each, find the simple interest based on the given information.

a) Karin put \$2,000 (this is the *principal*) in a savings account that gained interest at a rate of 6% (this is the *rate*). How much interest did the account gain after 1 year (this is the *time*)?

b) Padeep put \$3,000 in a special account that gained interest at a rate of 7%. How much interest did the account gain after $\frac{1}{2}$ year?

c) Sondra put \$5,000 in a special account that gained interest at a rate of 8%. How much interest did the account gain after 9 *months*?

d) Lupe put \$8,000 in a Certificate of Deposit that gained interest at a rate of 12%. How much interest did the account gain after 4 *months*?

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Answers to each Exercise

Section 1.7

Exercise 1: a) $d = 45$ b) $A = 24$ c) $m = -\frac{16}{12} = -\frac{4}{3}$ d) $z = 2$

Exercise 2:
a) $P = 15.6$ inches
b) $A = 21$ square feet
c) $A = 100\pi$ square feet or $A \approx 314$ square feet
d) $P = 12$ yards

Exercise 3:
a) Humberto's average score is 88 points
b) Yat-Sun spent an average of $37 each week for gas.
c) Devon read, on average, 35 pages each day.
d) It took Sarah an average of 20 minutes each day to drive to work.

Exercise 4: Ben averaged 49 miles per hour for the trip.

Exercise 5: It took Beatriz 8 hours to fly from San Francisco to New Orleans.

Exercise 6: Banjo ran 8 yards per second on that run.

Exercise 7: It should take Kinde $\frac{2}{3}$ of an hour (40 minutes) to complete the race.

Exercise 8:
a) Karen's account gained $120 after 1 year.
b) Padeep's account gained $105 after $\frac{1}{2}$ year.
c) Jaime's account gained $300 after 9 months.
d) Lupe's account gained $320 after 4 months.

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Section 1.7 Focus Exercises

1. Evaluate the numerical value of each formula with the given replacement values.

a) $z = \dfrac{x - m}{s}$ $x = 16$, $m = 25$, $s = 3$

b) $A = \dfrac{a + b + c}{3}$ $a = 13$, $b = 41$, $c = 33$

c) $a = \sqrt{c^2 - b^2}$ $c = 13$, $b = 12$

d) $A = \frac{1}{2} \cdot b \cdot h$ $b = 5$, $h = 8$

e) $P = 2 \cdot L + 2 \cdot W$ $L = 13$, $W = 8$

f) $r = \dfrac{d}{t}$ $d = 24$, $t = \frac{3}{4}$

g) $A = \frac{1}{2} \cdot h \cdot (b + B)$ $h = 3$, $b = 5$, $B = 7$

h) $I = P \cdot r \cdot t$ $P = 500$, $r = .08$, $t = \frac{1}{2}$

i) $m = \dfrac{y - w}{x - v}$ $y = 1$, $w = -8$, $x = -6$, $v = (-3)$

j) $c = \sqrt{a^2 + b^2}$ $a = -4$, $b = 3$

3. Find the simple interest based on the given information. $I = P \cdot r \cdot t$

a) Sally put $800 in a special account that gained 9% interest. How much interest did the account gain after 1 year?

b) Mark put $5,000 in a special account that gained 6% interest. How much interest did the account gain after 8 months?

3. April needed to travel 335 miles by car. She was able to make the trip in 5 hours. What was her average rate of speed? Use $\text{rate} = \dfrac{\text{distance}}{\text{time}}$

4. Reggie needed to go 9 miles on his bike. He was able to make the trip in $\dfrac{3}{4}$ hours. What was his average rate of speed? Use $\text{rate} = \dfrac{\text{distance}}{\text{time}}$

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