1.9 Algebraic Expressions

TERMS

In algebra, when a constant is multiplied by a variable, like $5 \cdot y$, the constant, *5*, is called a **coefficient**, and the product of a coefficient and a variable create a **term**. So in the *term* 5y, the *coefficient* is 5.

We don't need to restrict *terms* to just the product of a coefficient and a variable, a term can be:

1. the product of two or more variables: $a \cdot b$, $w \cdot z$, or $x \cdot y$ (or just ab, wz, xy);
2. the product of the same variable with itself—any number of times: $x \cdot x \cdot x \cdot x \cdot x \cdot x = x^6$;
3. the product of a constant (coefficient) with a variety of variables: $8 \cdot a^5 \cdot b^3$, or $8a^5b^3$;
4. just a constant with no variable: 6, -5, or $\frac{3}{8}$.

This leads to two new definitions:

1. A **variable cluster** can be just one variable *or* can be the product of more than one variable, *each with its own exponent*. A **variable cluster** does *not* include the coefficient.

2. A **term** can be just a constant *or* can be the product of a constant—called a coefficient—and a variable cluster.

Example 1: Given the term identify the coefficient and the variable cluster.

Term	Coefficient	Variable Cluster	Term	Coefficient	Variable Cluster
$8x^2y^3$	8	x^2y^3	$5x^4$	5	x^4
3a	3	a	$-6x^2$	-6	x^2
$\frac{2}{3}cb^5$	$\frac{2}{3}$	cb^5	7	7 *	(none)

* 7 is a term that is really a *constant* term; it is not, itself, a coefficient as defined above.

Here's a special term: **x**. "What's so special?" you ask? Well, it *appears* that this term has no coefficient (nor any exponent). Actually, it has both. In fact,

- x is the same as 1x or $1 \cdot x$;
- it's also the same as x^1 (read "x to the first power");
- it could even be thought of as $1x^1$.

This may look as if we're getting carried away, but the 1's that you see here are actually very important numbers and deserve more than a passive recognition. In both cases, they are sometimes referred to as, "the invisible 1." Though it may sound a little like the twilight zone, the "invisible 1" will rush to our aid—and make itself visible—in many situations.

Example 2: Given the term identify the coefficient and the variable cluster.

Term	Coefficient	Variable Cluster	Term	Coefficient	Variable Cluster
x^2y^3	1	x^2y^3	$- x^4$	- 1	x^4
- a	- 1	a	x^2	1	x^2

Exercise 1: Given the term identify the coefficient and the variable cluster.

	Term	Coefficient	Variable Cluster		Term	Coefficient	Variable Cluster
a)	$3x^5y$			b)	- 4		
c)	- 2c			d)	$- x^2$		
e)	$\frac{7}{8} c^2 d^5$			f)	w^8		
g)	$6 x^0$			h)	y		
i)	$- n^5$			j)	- y		

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ALGEBRAIC EXPRESSIONS

You've already been introduced to the notion of algebraic expressions. Here it is again with a little explanation.

In algebra we tend to put things, such as terms, together. We connect terms with parentheses and operations and call them **algebraic expressions**. Here are some examples of algebraic expressions:

$9y + 5x$	the sum of two terms
$3a(4b – 7)$	the product of a term and a quantity
$(8x^2y^3)(2xy)$	the product of two terms
$(5x + 6)(2x – 1)$	the product of two quantities
$7x^2 + 6x + 1$	the sum of three terms
$\frac{4b}{9a}$	the quotient of two terms
$6y^3$	A single term is also an algebraic expression
12	Even a constant could be thought of as an algebraic expression.

At this point, for Example 3, we're going to restrict our discussion of algebraic expressions to individual terms and to the *sum* of two or more terms.

Example 3: Identify the terms in each (algebraic) expression.

	Expression	terms (separated by commas)	
a)	$7x^2 + 6x + 1$	three terms:	$7x^2$, $6x$, 1
b)	$5x + 2$	two terms:	$5x$, 2
c)	$-3a^5 + 2a^3 + 4a + 9$	four terms:	$-3a^5$, $2a^3$, $4a$, 9
d)	$-15y$	one term:	$-15y$

Of course, there are many algebraic expression that have subtraction as the operation within, such as $3x^2 - 8x - 2$. This expression has three terms, but we need to remember a rule from Section 1.2 to know what the terms are:

The sign in front of a number belongs to that number.

For algebraic expressions, we extend that notion to include terms; it now reads

The sign in front of a *term* belongs to that *term*.

So, the terms of $3x^2 - 8x - 2$ are **$3x^2$**, **$-8x$** and **-2**.

Example 4: Identify the terms in each (algebraic) expression.

	Expression	terms (separated by commas)	
a)	$7x^2 - 6x - 1$	three terms:	$+7x^2$, $-6x$ *and* -1
b)	$5x^2 - 2x$	two terms:	$+5x^2$ *and* $-2x$
c)	$3a^5 - 2a^3 - 4a + 9$	four terms:	$+3a^5$, $-2a^3$, $-4a$ *and* $+9$

Exercise 2: Identify the terms in each (algebraic) expression.

	Expression	Terms (separate them by a comma)
a)	$5x^2 - x + 3$	________________
b)	$x^3 - 6x^2 - 8x$	________________
c)	$-2x - 6x^5$	________________
d)	$-3x^2y^2 + 6x^2 + 1x^2 - 4$	________________
e)	$4 - \frac{2}{3}x - y$	________________

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LIKE TERMS

Two things are "like" each other if they have the same characteristics. For example *like* fractions have the same denominator. In algebra, two or more terms are considered to be **like terms** if they have exactly the same variable cluster, even if the coefficients are different. For example, $3x^2y^5$ and $9x^2y^5$ are like terms because they have *exactly* the same variable cluster, x^2y^5.

> Two or more terms are considered to be **like terms** if they have exactly the same *variable cluster*.

Furthermore, constants, which have no variable cluster, are "like" all other constants. Use the example below to carefully examine which terms are "like" and which are *not* like. See if you can decide why some terms are not like others within the same expression.

Example 5: In each expression, identify the like terms.

	Expression		**Like terms**
a)	$7x - 6x + 1$	one pair of terms is **like**:	$7x$ and $-6x$
b)	$5x - 2 + 9$	one pair of terms is **like**:	-2 and 9
c)	$3a^2 + 2b^3 + 4a^2 + 9b^3$	two pairs of terms are **like**:	$3a^2$ and $4a^2$, $2b^3$ and $9b^3$
d)	$-15y + 6y - 3y$	all three terms are **like**:	$-15y$ and $6y$ and $-3y$
e)	$4x + 6y - 7$	no terms are **like**:	(none)
f)	$5ab - 8ab + 3b$	one pair of terms is **like**:	$5ab$ and $-8ab$

(Note: the variable cluster for **3b** is just **b** which is not the same as **ab**.)

g)	$-2x^3 + 6x^2 + 9x$	no terms are **like**:	(none)

(Note: a different exponent on the same variable indicates a different variable cluster.)

Exercise 3: In each expression, identify the like terms.

	Expression	Like Terms
a)	$5x - x + 3$	______________
b)	$x^2 - 6x - 8x^2$	______________
c)	$2x^5 - 6x^5$	______________
d)	$-3x^2y + 6x^2 + 1x^2y - 4$	______________
e)	$4 - \frac{2}{3}y - y$	______________

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COMBINING LIKE TERMS

In algebra you will be asked to "simplify" a great deal. Unfortunately, this may cause some confusion at times because "simplify" will have different meanings based on the expression given and under what circumstances the expression is being used.

Basically, an expression is **simplified** when it is written in the briefest, least complex form. When working with like terms, an expression is said to be simplified when the like terms are "combined." To **combine** like terms means to find their sum or difference, depending on the operation between the terms.

We can use *like measurements*, such as inches and inches or feet and feet, as an illustration:

2 inches + 3 inches = 5 inches.

2 feet + 3 feet = 5 feet.

In that analogy we are able to combine the measurements because they are "like measures," meaning the unit of measurement (inches or feet) is the same. Notice that the result, the sum, is in the same unit of measurement, feet, as the addends. In the very same way, we can combine like terms because the variable cluster is the same:

$$2x + 3x = 5x.$$

Again, the result, **5x**, has a variable cluster that is exactly the same as the terms, **2x** and **3x**. This means that the result is "like" the original two terms. In this way, the only change is the coefficient; to get the coefficient of the result, 5, we had to add the coefficients of the like terms, 3 and 2.

Why is it that combining like terms works this way? Good question. It goes back to the abbreviation called "multiplication." Remember that multiplication is just an abbreviation for repeated addition. So, when we wish to find the sum of $4h$ and $3h$, it's like this:

$4h$ means $4 \cdot h$ which means $h + h + h + h$. Likewise, $3h$ means $3 \cdot h$ which means $h + h + h$. Therefore, when we add,

$$4h \quad + \quad 3h$$

is the same as $(h + h + h + h) + (h + h + h)$

or just $h + h + h + h + h + h + h$ which is abbreviated as $7 \cdot h$, or just $7h$.

This is also why we can't combine $2y + 5x$. It becomes

$$(y + y) + (x + x + x + x + x)$$

In this case, we can't say that we have 7 of anything that is the same.

The combining of like terms is not restricted to addition; we can also *subtract* like terms. In this way it is like eliminating terms, maybe one at a time. In elementary school, subtraction is sometimes referred to as "take aways," such as $5 – 2$ means "5 take away 2". It's like cutting 2 feet off of a 5 foot board.

5 feet – 2 feet = 3 feet.

Or, five x's take away two x's leaves three x's: $5x - 2x = 3x.$

In this case we *combine* like terms by finding the difference between the coefficients.

To **combine** like terms we need only add or subtract their coefficients.

However, if terms are not *like* terms, then they cannot be combined. A somewhat silly example might be to add 6 feet and 5 pounds. They have no relevance to each other and cannot combine. Similarly, the expression $6x + 5y$ cannot be rewritten as one term. $6x + 5y$ is simplified just as it is.

Combining like terms is an art that needs to be mastered in order to be successful in algebra. It is not difficult, but you do need to learn it! Let's practice:

Example 6: Simplify each expression by combining like terms. (Find the sum of the coefficients.)

First, written "vertically":

a) $\begin{array}{r} 10x \\ +\ 4x \\ \hline 14x \end{array}$

b) $\begin{array}{r} 12b^4 \\ +\ 7b^4 \\ \hline 19b^4 \end{array}$

c) $\begin{array}{r} 2h \\ 3h \\ +\ 6h \\ \hline 11h \end{array}$

Second, written "horizontally":

d) $7x + 6x = 13x$

e) $-2 + 9 = 7$

f) $-3a^2 + 4a^2 = 1a^2$ or just a^2

g) $4y + 1y = 5y$

h) $4p + p = 5p$

i) $15y^3 + 6y^3 + 3y^3 = 24y^3$

j) $-2x^2y^3 + 9x^2y^3 = 7x^2y^3$

k) $2x^4y^5 + 9x^5y^4$ <u>cannot combine</u>

Notice that (g) and (h) are virtually the same; what we do see in (g) but don't see in (h) is the "invisible 1," the coefficient of **p**. This is a time when making the "invisible 1" *visible* might be helpful to the process, though it isn't necessary.

Also notice that, in (k), $2x^4y^5$ and $9x^5y^4$ are not like terms, so they can't combine. Look carefully at the variable cluster of each term to see why.

Example 7: Simplify each expression by combining like terms. Find the difference of the coefficients.

a) $7w - 2w = 5w$

b) $3x^2 - 2x^2 = 1x^2$ or just x^2 — The coefficient 1 doesn't need to be written.

c) $18x - 6x = 12x$

d) $9y - y = 9y - 1y = 8y$ — Here, it might be to your advantage to include the coefficient 1 before subtracting.

Of course, since the new rule states that "the sign in front of the term belongs to that term," even subtraction of terms is an addition process. It's just like adding (or subtracting) two signed numbers.

Just as $3 - 8$ can be thought of as $+3$ and -8 (this adds to -5),

so can $3x - 8x$ can be thought of as $+3x$ and $-8x$ (this adds to $-5x$).

Here's the bottom line on combining like terms:

> To **combine like terms**, simply *add (or subtract) the numerical coefficients*; the result you get will be "like" the original terms. In other words, the variable cluster will remain the same. If a variable cluster has no visible coefficient, then the coefficient is 1.

Example 8: Simplify each expression by combining like terms.

a) $-7w - 2w = -9w$

b) $3x^2 - 5x^2 = -2x^2$

c) $-18x + 6x = -12x$

d) $-9y + y = -9y + 1y = -8y$

e) $-2p + 3p = 1p$ or just p

f) $-c + 7c = -1c + 7c = 6c$

g) $-m - (-6m) = -1m + 6m = 5m$

h) $3x - (-4x) = 3x + 4x = 7x$

i) $-4x + 4x = 0x = 0$

h) $5y^2 + 3y$ cannot combine (unlike terms)

Exercise 4: Simplify each expression by combining like terms. If the terms are *not* like terms, then write "cannot combine."

a) $3y + 9y =$ ______

b) $7y - 2y =$ ______

c) $-4w + (-2w) =$ ______

d) $5w^3 - 9w^3 =$ ______

e) $-6c^2 + 7c^2 =$ ______

f) $-c - 8c =$ ______

g) $p + (-4p) =$ ______

h) $5p^2 - 6p =$ ______

i) $-8x^2y + 2x^2y =$ ______

j) $-4x - (-x) =$ ______

k) $6m + (-6m) =$ ______

l) $2s - 4m =$ ______

m) $-3v + 3v =$ ______

n) $-v - v =$ ______

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THE PRODUCT OF TWO TERMS

The product of two terms is another term. The associative property, the commutative property and the definition of exponents combine to make multiplying two terms a relatively easy process.

For example, we can multiply (3x)(2x) using the properties mentioned above. A word of caution, though: this is behind the scenes work to explain what's happening; it is not intended that students will copy this process. Instead, your process will be much simpler than this.

The explanation:

		$(3x)(2x)$	
can be rewritten as	=	$3 \cdot x \cdot 2 \cdot x$	by the associative property
	=	$3 \cdot 2 \cdot x \cdot x$	by the commutative property
	=	$6 \cdot x^2$	$x \cdot x = x^2$
	=	$6x^2$.	

Of course, we would normally do this in just one step: $(3x)(2x) = 6x^2$. The result is the product of the coefficients and the product of any variable factors present.

Example 9: Multiply and write the product as one term.

a) $(4x)(5x) = 20x^2$ b) $(-3y)(8y) = -24y^2$

c) $(w)(-2w) = -2w^2$ d) $3(6x^2) = 18x^2$

e) $(-4y)(-2) = 8y$ f) $(8w)(-4y) = -32wy$

Exercise 5: Multiply and write the product as one term.

a) $(9c)(4c) =$ b) $-6(5y) =$ c) $(3a)(-12a) =$

d) $(-7x)(-x) =$ e) $5y(-4) =$ f) $(-2b)(6c) =$

g) $-9(-5x^2) =$ h) $x(-3) =$ i) $(4d)(-1) =$

j) $w(-w) =$ k) $-y(y) =$ l) $(-x)(-x) =$

m) $3w(-1) =$ n) $-4y(1) =$ o) $(-2x)(-1) =$

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THE DISTRIBUTIVE PROPERTY

There is a property in mathematics that applies to both multiplication and addition at the same time. It is called the **Distributive Property**. It is called this because, as you will see, we actually distribute one term to two (or more) terms, just as a mail carrier might distribute the same advertisement to two neighbors' houses.

The Distributive Property		
1.	$\mathbf{b \cdot (c + d)}$	$\mathbf{= b \cdot c + b \cdot d}$
2.	$\mathbf{(c + d) \cdot b}$	$\mathbf{= c \cdot b + d \cdot b}$

Notice that what is actually being distributed is more than just a number, **b**; the multiplication symbol is being distributed along with the b; so really, "**b·**" (*b times*) is being distributed. We also have a name for the number that is distributed; it is called the **multiplier**.

> The number (or term) that is distributed is called the **multiplier**.

Let's see why the distributive property works the way it does. Consider the example **3** · (x + 6).

Since this is multiplication, and multiplication is an abbreviation for repeated addition, this means

"The sum of **three** (x + 6)'s."

This can be written as $= (x + 6) + (x + 6) + (x + 6)$.

The associative property can be used to write that expression without parentheses:

$$= x + 6 + x + 6 + x + 6$$

We can then use the commutative property to reorganize the numbers:

$$= x + x + x + 6 + 6 + 6$$

Let's group the x's separately from the 6's: $= (x + x + x) + (6 + 6 + 6)$

Each grouping has *three* of something: = **three** x's + **three** 6's

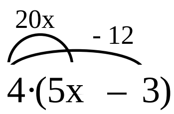
$$= \mathbf{3} \cdot x + \mathbf{3} \cdot 6$$

Hence, from the beginning: $\mathbf{3} \cdot (x + 6) = \mathbf{3} \cdot x + \mathbf{3} \cdot 6$.

Behold, the distributive property.

> **Interesting note:** the distributive property changes the main operation from multiplication to addition.

Technically, the distributive property is the product of a term (the multiplier) and a quantity (a sum of two or more terms). However, now that we are familiar with negative numbers and how to multiply with them, we can slightly modify the distributive property to include subtraction.



For example, if we wish to distribute the multiplier 4 through the quantity $(5x - 3)$, the product, at first, looks like this: $4\cdot(5x - 3)$. We can, however, think of the quantity $5x - 3$ as $5x + (-3)$, effectively changing the product to $4\cdot(5x + (-3))$. Let's look at it one step at a time.

$4\cdot(5x - 3)$	
	Subtraction can be rewritten as the sum: "adding the opposite."
$= 4\cdot(5x + (-3))$	
	We can distribute the multiplier through to the sum
$= 4\cdot5x + 4\cdot(-3)$	
	Of course, $4 \cdot (-3) = -12$
$= 20x + (-12)$	
	Rewrite the terms in a more simplified form.
$= 20x - 12$	

Actually, we don't need to go through all of that work if we remember two things:

(1) <u>the sign in front of a term belongs to that term.</u> So, in the quantity $(5x - 3)$, the 5x is considered to be positive and the 3 is negative.

(2) The four meanings of the dash:
- (a) minus
- (b) negative
- (c) "the opposite of"
- (d) - 1 times.

So, in effect, when distributing the multiplier 4 through to the sum $(5x - 3)$ we are multiplying 4 by both $+5x$ and -3. As we multiply 4 by $(+5x)$ we get, of course, $+20x$; and as we multiply 4 by (-3) we get -12. The $+20x$ shows up as just **20x**, but the **- 12** shows up as *minus* 12.

In other words,

$4\cdot(5x - 3)$	Treat *5x* as <u>+ 5x</u> and *minus 3* as <u>- 3</u>.
$= 20x - 12$	$4 \cdot (+5x) = +20x$; $4 \cdot (-3) = -12$, or *minus 12*.

Notice that this requires fewer steps than before. This leads to two new forms of the distributive property:

The Distributive Property with Subtraction

1.	$\mathbf{b\cdot(c - d)}$	$\mathbf{= b\cdot c - b\cdot d}$
2.	$\mathbf{(c - d)\cdot b}$	$\mathbf{= c\cdot b - d\cdot b}$

Example 10: Apply the distributive property to each expression.

a) $5(7y - 3)$ b) $2(4 - 6c)$ c) $(5w - 2)\cdot 8$

Answer:

a) $5(7y - 3)$ — 5 is the *multiplier*

$= 5\cdot 7y - 5\cdot 3$ — This step isn't necessary, but it shows the multiplication.

$= 35y - 15$ — Distribute the "5 times" through to both terms.

b) $2(4 - 6c)$ — The *multiplier* is 2.

$= 2\cdot 4 - 2\cdot 6c$ — Actually, even this step isn't necessary; it's just showing what needs to be done, but this can be done in your head.

$= 8 - 12c$

c) $(5w - 2)\cdot 8$ — This time, only the necessary steps are included.

$= 40w - 16$ — You can distribute and multiply directly without writing the multiplication step down.

Exercise 6: Apply the distributive property to each expression.

a) $6(4x - 3)$ b) $7(9y - 4)$

c) $(2m - 6)5$ d) $4(6 - 5p)$

e) $9(3 - 3w)$ f) $3(8 - 9v)$

g) $2(2d - 5k)$ h) $1(3x - 4y)$

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THE DISTRIBUTIVE PROPERTY WITH A NEGATIVE MULTIPLIER

We can use the distributive property even when the multiplier is negative. We need to be more careful when multiplying each term by a negative number, and we need to continually remember the two ideas mentioned earlier:

(1) the four meanings of the dash and

(2) the sign in front of a number belongs to that number.

- 8x   - 6
$-2\cdot(4x + 3)$

Consider, for example, the product of the multiplier -2 and the sum $(4 + 3)$. As a product, this looks like $-2\cdot(4 + 3)$. According to the distributive property, the multiplier -2 is to be multiplied by both the $+4$ and the $+3$. As you know, $-2\cdot(+4) = -8$ and $-2\cdot(+3) = -6$. Written out, the -8 will show up as itself, -8; however, the -6 will show up as *minus* 6. Here it is step by step:

In other words, $-2(4x + 3)$ — Treat *4x* as <u>+ 4x</u> and *3* as <u>+ 3</u>.

$= -8x - 6$ — $-2\cdot(+4) = -8$; $-2\cdot(+3) = -6$, or *minus 6*.

Let's practice using the distributive property:

Example 11: Apply the distributive property to each expression.

a) $-5(7x + 3)$ b) $-2(4c - 6)$

c) $-8(-5 + 2y)$ d) $-3(-6 - y)$

Answer:

a) $-5(7x + 3)$ $-5(+7x) = -35x$; $-5(+3) = -15$, which shows up as *minus* 15.

$= -35x - 15$

b) $-2(4c - 6)$ $-2(+4c) = -8c$; $-2(-6) = +12$, which shows up as *plus* 12.

$= -8c + 12$

c) $-8(-5 + 2y)$ $-8(-5) = +40$; $-8(+2y) = -16y$, . which shows up as *minus* 16y

$= 40 - 16y$

d) $-3(-6 - y)$ $-3(-6) = +18$; $-3(-y) = +3y$, which shows up as *plus* 3y.

$= 18 + 3y$

Exercise 7: Apply the distributive property to each expression.

a) $-5(6x + 3)$

b) $-2(8k + 10)$

c) $-2(8m - 1)$

d) $-4(y - 7)$

e) $-6(-10 + 4w)$

f) $-3(-9 + 5c)$

g) $-4(-11 - 5p)$

h) $-7(-4 - 2d)$

It's also possible that the multiplier is a variable. When that is the case, we need to rely not only on our understanding of the distributive property, but also on our skills of multiplying terms. You may wish to review Example 8 before continuing with Example 11.

Example 12: Simplify each expression by using the distribute property.

a)	$x(x + 4)$	b)	$-3w(w - 2)$	c)	$2y(8y - 7)$
d)	$x(y + 2)$	e)	$w(9w - 1)$	f)	$4\cdot(x^2 + x)$

Answer:

a) $x(x + 4)$
$= x^2 + 4x$

b) $-3w(w - 2)$
$= -3w^2 + 6w$

c) $2y(8y - 7)$
$= 16y^2 - 14y$

d) $x(y + 2)$
$= xy + 2x$

e) $w\cdot(9w - 1)$
$= 9w^2 - w$

f) $4\cdot(x^2 + x)$
$= 4x^2 + 4x$

Exercise 8: Simplify each expression by using the distribute property

a) $x(x + 3)$

b) $y(x - 10)$

c) $-2a(a + 1)$

d) $-3(y^2 - 7y)$

e) $w(3w + 4)$

f) $6(5w^2 - 1)$

g) $-4m(6m + 1)$

h) $-1(8m - 5x)$

i) $3c(3c + 6)$

j) $-k(4k - 5)$

k) $-5x(-2x + 3)$

l) $-4y(3k - 2)$

m) $-6p(-p - 5)$

n) $-4w(w - 1)$

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Answers to each Exercise

Exercise 1:

	Term	Coefficient	Variable Cluster
a)	$3x^5y$	3	x^5y
c)	$-2c$	-2	c
e)	$\frac{7}{8}c^2d^5$	$\frac{7}{8}$	c^2d^5
g)	$6x^0$	6	x^0
i)	$-n^5$	-1	n^5

	Term	Coefficient	Variable Cluster
b)	-4	-4	(none)
d)	$-x^2$	-1	x^2
f)	w^8	$+1$	w^8
h)	y	$+1$	y
j)	$-y$	-1	y

Exercise 2: a) $5x^2, -x, 3$ b) $x^3, -6x^2, -8x$ c) $-2x, -6x^5$

d) $-3x^2y^2, 6x^2, 1x^2, -4$ e) $4, -\frac{2}{3}x, -y$

Exercise 3: a) $5x$ and $-x$ are like terms b) x^2 and $-8x^2$ are like terms

c) $2x^5$ and $-6x^5$ are like terms d) $-3x^2y$ and $1x^2y$ are like terms

e) $-\frac{2}{3}y$ and $-y$ are like terms

Exercise 4: a) $12y$ b) $5y$ c) $-6w$ d) $-4w^3$

e) $1c^2$ or c^2 f) $-9c$ g) $-3p$ h) cannot combine

i) $-6x^2y$ j) $-3x$ k) 0 l) cannot combine

m) 0 n) $-2v$

Exercise 5:

a)	$36c^2$	b)	$-30y$	c)	$-36a^2$	d)	$7x^2$
e)	$-20y$	f)	$-12bc$	g)	$45x^2$	h)	$-3x$
i)	$-4d$	j)	$-w^2$	k)	$-y^2$	l)	x^2
m)	$-3w$	n)	$-4y$	o)	$2x$		

Exercise 6:

a) $6(4x - 3) = 24x - 18$ b) $7(9y - 4) = 63y - 28$

c) $(2m - 6)\cdot 5 = 10m - 30$ d) $4(6 - 5p) = 24 - 20p$

e) $9(3 - 3w) = 27 - 27w$ f) $3(8 - 9v) = 24 - 27v$

g) $2(2d - 5k) = 4d - 10k$ h) $1(3x - 4y) = 3x - 4y$

Exercise 7:

a) $-5(6x + 3) = -30x - 15$ b) $-2(8k + 10) = -16k - 20$

c) $-2(8m - 1) = -16m + 2$ d) $-4(y - 7) = -4y + 28$

e) $-6(-10 + 4w) = 60 - 24w$ f) $-3(-9 + 5c) = 27 - 15c$

g) $-4(-11 - 5p) = 44 + 20p$ h) $-7(-4 - 2d) = 28 + 14d$

Exercise 8:

a) $x(x + 3) = x^2 + 3x$ b) $y(x - 10) = yx - 10y$ or $xy - 10y$

c) $-2a(a + 1) = -2a^2 - 2a$ d) $-3(y^2 - 7y) = -3y^2 + 21y$

e) $w(3w + 4) = 3w^2 + 4w$ f) $6(5w^2 - 1) = 30w^2 - 6$

g) $-4m(6m + 1) = -24m^2 - 4m$ h) $-1(8m - 5x) = -8m + 5x$

i) $3c(3c + 6) = 9c^2 + 18c$ j) $-k(4k - 5) = -4k^2 + 5k$

k) $-5x(-2x + 3) = 10x^2 - 15x$ l) $-4y(3k - 2) = -12yk + 8y$

m) $-6p(-p - 5) = 6p^2 + 30p$ n) $-4w(w - 1) = -4w^2 + 4w$

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Section 1.9 Focus Exercises

1. Given the term identify the coefficient and the variable cluster.

	Term	Coefficient	Variable Cluster
a)	$-5p^4q^3$		
c)	$\frac{3}{4}xy^2$		
e)	$-x^0$		

	Term	Coefficient	Variable Cluster
b)	$-2w$		
d)	x^2		
f)	-9		

2. Identify the terms in each algebraic expression. Separate them by a comma on the line below the expression.

a) $-6x^3 + 4x - 1$

____

b) $x^2 - 5x + 8$

____

c) $2x^5 - 3x^4 - \frac{5}{4}x + 12$

____

d) $7x^3 - 6x^2y + xy^2 - 4y^3$

____

3. Simplify each expression by combining like terms. If the terms are *not* like terms, then write "cannot combine."

a) $4b^3 + 3b^3 =$ ________________

b) $-b - 6b =$ ________________

c) $-2y + 2y =$ ________________

d) $-y - y =$ ________________

e) $n + (-5n) =$ ________________

f) $7n^3 - 4n =$ ________________

g) $-6h - (-3h) =$ ________________

h) $2h^2 - 5h^2 =$ ________________

i) $-8x^3r + 4x^3r =$ ________________

j) $3v - 6p =$ ________________

k) $4p + (-4p) =$ ________________

l) $-2w + 0w =$ ________________

4. Multiply and write the product as one term.

a)	$(7k)(6k) =$	b)	$-3(8c) =$	c)	$(2p)(-14p) =$
d)	$(-9x)(-x) =$	e)	$8c(-6) =$	f)	$(-4m)(3k) =$
g)	$-7(-8x^4) =$	h)	$x(-2) =$	i)	$(6n)(-5) =$
j)	$y(-y) =$	k)	$-c(c) =$	l)	$(-x)(-x) =$
m)	$2y(-5) =$	n)	$-6c(5) =$	o)	$(-4x)(-5) =$

5. Apply the distributive property to each expression.

a)	$5(7x - 8)$	b)	$8(1 - 2v)$
c)	$(9m - 5)6$	d)	$3(8x - 7y)$
e)	$-6(5x + 8)$	f)	$-7(y - 4)$
g)	$-5(-3 + 7w)$	h)	$-2(-8p - 9d)$
i)	$x(x + 4)$	j)	$2y(3y - 10)$
k)	$-3a(a + 1)$	l)	$-4y(y^3 - 7y)$
m)	$-5m(7m + 1)$	n)	$-1(9m - 6x)$
o)	$-k(-5k - 6)$	p)	$-7p(-p + 3)$

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