2.1 Solving Linear Equations, Part 1

INTRODUCTION, ESTABLISHING SOME GROUND RULES

In order to solve equations (explained below) we need to first be reminded of a couple of properties of mathematics. The first is the notion of the identity properties. Here are both identities, one for addition and one for multiplication. The *identity* is a number that, when applied, won't change the value of another number or quantity.

For **addition**, the **identity is 0** (zero), because

a) $6 + 0 = 6$ b) $0 + 15 = 15$

Notice that adding 0 (zero) to a number doesn't change the value of the number.

For **multiplication**, the **identity is 1** (one), because

a) $6 \cdot 1 = 6$ b) $1 \cdot 15 = 15$

Notice that multiplying a number by 1 (one) doesn't change the value of the number.

For algebra, this means that, as you already know, $1 \cdot x = x$; it also means that $x + 0 = x$. In fact, we can get a little extravagant and say that $1 \cdot x + 0 = x$.

To achieve the identity for addition from any number, A, we simply add the opposite of A, - A:

Example 1: The Sum of a number and its opposite is always 0.

a) $A + (-A) = 0$ b) $-B + B = 0$ c) $6 + (-6) = 0$ d) $-3 + 3 = 0$

Exercise 1: Fill in the blank to make each statement true.

a) $7 + (-7) =$ ___   b) $4 +$ ___ $= 0$ c) $-2 + 2 =$ ___

d) $-5 +$ ___ $= 0$ e) ___ $+ (-10) = 0$ f) ___ $+ 9 = 0$

Likewise, to achieve the identity for multiplication from any number, A, we simply multiply by the reciprocal of A, $\frac{1}{A}$. Here is the rule about the product of a number and its reciprocal.

The product of a fraction and its reciprocal is always 1:	$\frac{A}{B} \cdot \frac{B}{A} = 1$

Example 2:

a) $A \cdot \frac{1}{A} = 1$ b) $6 \cdot \frac{1}{6} = 1$ c) $-6 \cdot \frac{-1}{6} = 1$

d) $\frac{1}{3} \cdot 3 = 1$ e) $\frac{5}{4} \cdot \frac{4}{5} = 1$ d) $\frac{-5}{8} \cdot \frac{-8}{5} = 1$

Exercise 2: Fill in the blank to make each statement true.

a) $8 \cdot \frac{1}{8} =$ _____ b) $\frac{-1}{7} \cdot (-7) =$ _____ c) $\frac{3}{4} \cdot \frac{4}{3} =$ ___

d) $-5 \cdot$ ____ $= 1$ e) $\frac{1}{10} \cdot$ _____ $= 1$ f) $\frac{5}{2} \cdot$ ____ $= 1$

g) ____ $\cdot \frac{1}{9} = 1$ h) ___ $\cdot 12 = 1$ i) ___ $\cdot \frac{-5}{3} = 1$

Lastly, in Section 1.3 you were introduced to the notion of "the sign in front of a number" and in Section 1.9 you were introduced to the notion of "the sign in front of a term," as shown here:

The sign in front of a number belongs to that number.

For algebraic expressions, we extend that notion to include terms; it now reads

The sign in front of a *term* belongs to that *term*.

The purpose of this introduction will be clear a little later when we discuss "isolating the variable."

WHAT IS AN EQUATION?

Equations and expressions are *not the same*. You *must* learn the difference. One major distinction is

We ***simplify* expressions** :

- by distributing
- by applying the order of operations
- by combining like terms and like fractions
- by reducing fractions

We ***solve* equations**:

- by distributing
- by clearing fractions
- by combining like terms
- by clearing operations
- by **isolating the variable**

Expressions and equations are related, though, and this might add to the confusion. In fact, an **equation** includes two expressions, one equal to the other. It could look like this:

Equation: left side expression = right side expression

Expressions can be quite complex, such as $3(2x^2 + 4) - 5(6 - x) + 8$, but if there is no equal sign between any two expressions, it isn't an equation.

If this same expression is "teamed up" with another expression, *however simple*, with an equal sign between them, it is then an **equation**:

$$3(2x^2 + 4) - 5(6 - x) + 8 = 1$$ is an equation.

LINEAR EQUATIONS

There are a variety of equations in algebra, though in this course we concentrate only on **linear equations**, those in which the highest power of the variable is 1, as in $3x^1$, or just $3x$.

A linear equation: $3x + 8 = 2x - 1$

left side expression = right side expression

As shown above, the expression to the left of the equal sign is called, simply, ***the left side*** of the equation; likewise, the expression to the right of the equal sign is called ***the right side*** of the equation.

Example 3: Identify each as either an expression or an equation. State why.

a) $5x + 6 = 2x - 9$ b) $3(4x - 1) + 6(5 - 2x)$

c) $4x + 2 + 3x$ d) $3(2x + 4) + 5(6 - x) = 0$

Answer: Equations have their own equal sign between two expressions. Expressions have no equal sign of their own.

a) $5x + 6 = 2x - 9$ is an equation; it has an equal sign between two expressions.

b) $3(4x - 1) + 6(5 - 2x)$ is an expression; it has no equal sign.
It can be *simplified* by distributing and combining like terms, but there is only one (large) expression here, not two.

c) $4x + 2 + 3x$ is an expression; it has no equal sign.
It can be *simplified by* combining like terms.

d) $3(2x + 4) + 5(6 - x) = 0$ is an equation; it has an equal sign between two expressions, even though the right side expression is a single number. It's true that the left side expression can be simplified (after all, it *is* an expression), but that doesn't take away from the fact that there is an equal sign between two expressions.

Exercise 3: Identify each as either an expression or an equation. State why.

a) $5x - 8 - 6x$ ____

b) $2(3x + 4) + 6(2 - 5x)$ ____

c) $2(3x + 4) = 6(2 - 5x)$ ____

d) $2(3x + 4) - 2 - 5x = -8$ ____

SOLUTIONS

What is the solution of an equation? It is not necessarily "the answer." It is better said as,

A **solution** of an equation is a number that makes the equation **true**.

When solving an equation we will find a numerical value for the variable. This value, called a **replacement value**, can then replace the variable in the equation to see if it leads to a true statement.

For example, 5 is a solution for the **linear** equation $2x = 10$, because when we replace x with 5, we get a **true** statement:

$$2 \cdot (5) = 10$$

$$10 = 10$$

This is a true statement, so the **replacement value**, 5, is a solution.

As stated before, a linear equation's variables have a highest power of 1. Here are some examples of linear equations:

One operation:	**Two operations:**	**Multiple operations:**
$x + 6 = 10$	$3 \cdot c + 7 = -5$	$9y - 14 = 6y + 4$
$w - 9 = -4$	$9 - 5p = 24$	$3(6 - x) = -2(5x - 4)$
$2 \cdot y = -8$	$-4(w + 5) = 32$	$3(2x + 4) + 5(6 - x) = 0$
$\frac{x}{6} = \frac{2}{3}$	$\frac{x}{7} - 8 = -1$	
$m \div 8 = 6$		

As with formulas, when the replacement value for the variable x is 2, then it is 2 for all x's found in the equation.

Example 4: Determine if the **replacement value** shown (after each equation) is a solution of that equation.

a) $-4v + 9 = 1$; **v = 2** b) $2x + 6 = 3x - 5$; **x = 10** c) $4(3 - y) = 2(y + 9)$; **y = -1**

Procedure: Each of these is a linear equation and will have only one solution. Replace each value of x with the replacement value given, and see if it leads to a true statement.

a)

$-4v + 9 = 1$	Replace the v with the value of 2.
$-4(2) + 9 = 1$	Simplify each side using the order of operations.
$-8 + 9 = 1$	
$1 = 1$	This is *true*! **v = 2** is the *solution.*

b)

$2x + 6 = 3x - 5$	Replace *each* x with the value of 10.
$2(10) + 6 = 3(10) - 5$	Simplify each side using the order of operations.
$20 + 6 = 30 - 5$	
$26 = 25$	This is not true! x = 10 is ***not*** the solution.

c)

$4(3 - y) = 2(y + 9)$	Replace *each* y with the value of -1.

$4[3 - (-1)] = 2(-1 + 9)$		Simplify each side using the order of operations.
$4(3 + 1) = 2(8)$		$[3 - (-1)]$ could first be rewritten as $[3 + (+1)]$.
$4(4) = 16$		
$16 = 16$		This <u>is</u> true! **y = - 1** is the solution.

Exercise 4: Determine if the **replacement value** shown (after each equation) is a solution of that equation.

a) $3x - 7 = 8$; $\mathbf{x = 5}$

b) $6 - 5w = 2w - 8$; $\mathbf{w = 2}$

c) $2y + (y - 3) = 2 + 5(y - 1)$; $\mathbf{y = 1}$

d) $\frac{3}{4}c + \frac{1}{2}c = \frac{c + 6}{2}$; $\mathbf{c = 4}$

ISOLATING THE VARIABLE

Linear equations involve expressions that have (possibly) both variable terms and constant terms. The variable is treated as an <u>unknown value</u>, and its value must be found. To solve each equation, we must focus on one goal: to isolate the variable.

> To solve a linear equation, you must **ISOLATE THE VARIABLE**.

This principle is so important that you should imagine an **echo** effect each time you think it or read it.

Here are a few examples of the types of equations you'll be seeing in *this* section:

(a) $x + 9 = 17$ (b) $-6 = 2y$ (c) $a - 7 = -3$ (d) $\frac{w}{4} = -9$

The whole idea about **isolating the variable** is to get the variable all by itself on one side of the equation and only a number on the other side. For example, if the variable is x and the solution is 5, then at the end of the solving process we should see either

$$x = 5 \qquad \text{or} \qquad 5 = x$$

In both cases, the variable x is isolated (by itself) and the solution is on the other side. But what is x when it's all by itself?

x can be written as $1 \cdot x$, or just $1x$ because 1 is the **identity for multiplication**

x can be written as $x + 0$ because 0 is the **identity for addition**

and, as mentioned earlier, x can also be written as $1 \cdot x + 0$.

This makes the solution of 5, when isolating the variable

$$1x + 0 = 5 \qquad \text{or} \qquad 5 = 1x + 0$$

So, even though the goal is to get x all by itself, we'll do so by "creating" $\mathbf{1x + 0}$. Let's see how this is done.

To isolate the variable we must learn a few basic properties of equations. The properties listed below say that we can "operate" on each side of an equation, as long as we operate exactly the same way.

1 we can **add** any term to *each side* of an equation;
2. we can **subtract** any term from *each side* of an equation;
3. we can **multiply** the same term (as long as it isn't 0) to *each side* of an equation; and
4. we can **divide** *each side* of an equation by the same term (as long as it isn't 0).

Each time we correctly apply something to each side of the equation, we get a new equation. If we use the rules (presented below) properly, then the new equation is *equivalent* to the previous equation.

In this way, we have, what are called, two **equivalent equations**; this means that—though the equations may look different—the solution hasn't changed by applying the rule properly.

Addition and Subtraction

We may add or subtract any term, C, to *each side* of an equation to get a *new,* **equivalent** equation.

if	$a = b$	if	$a = b$
then	$a + C = b + C$	then	$a - C = b - C$

Actually, you may recall that subtraction is really *adding the opposite*. So, to subtract C from each side could also mean that we add the opposite of C to each side:

$$\text{if} \qquad a = b$$

$$\text{then} \qquad a + (-C) = b + (-C)$$

Multiplication and Division

We may multiply *each side* of an equation by any term, C, (as long as it isn't 0), or we may divide each side of an equation by any term, C, (as long as it isn't 0). In doing so, we get a *new,* **equivalent** equation.

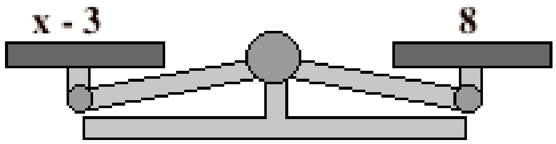
if	$a = b$	if	$a = b$
then	$C \cdot a = C \cdot b$	then	$\frac{a}{C} = \frac{b}{C}$

The purposes of these properties is so that we can manipulate equations but still keep the equation "balanced."

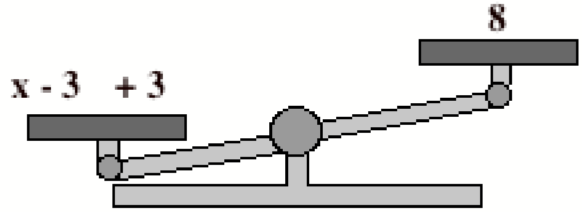
BALANCING AN EQUATION

A way to maintain **equivalent equations**, from step to step, is to keep the equation "balanced." This means that we can manipulate the right side and left side at the same time, but we must "operate" on them in the exact same way.

In other words, if we have a simple equation, like $x - 3 = 8$, we can isolate the variable by somehow eliminating the "- 3" from the left side. It probably makes good sense to eliminate the - 3 by adding 3 to the left side; however, if we don't add 3 to each side, this could leave the equation "out of balance."



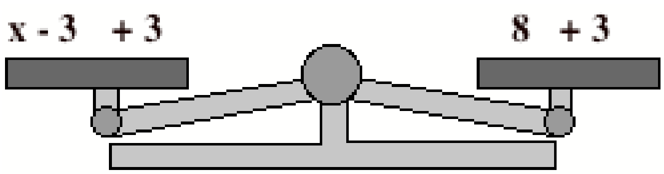
A **balanced** equation.



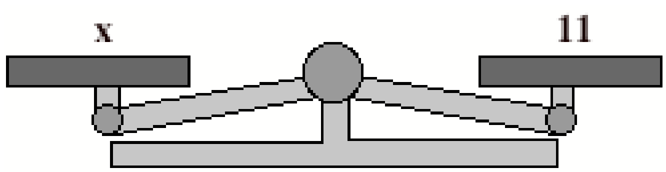
An **out of balance** equation

Adding 3 to only the left side
makes it heavier than the right side.

Instead, we must add 3 to each side:



Adding 3 to each side maintains the balance.



Here are two different ways in which we could choose to show our work as we solve:

1.
$$\begin{aligned} x - 3 &= 8 \\ x - 3 \mathbf{+ 3} &= 8 \mathbf{+ 3} \\ x + 0 &= 11 \\ x &= 11 \end{aligned}$$

2.
$$\begin{aligned} x - 3 &= 8 \\ \underline{\mathbf{+3}} &\underline{= \mathbf{+3}} \\ x + 0 &= 11 \\ x &= 11 \end{aligned}$$

In both cases, we need to add 3 to each side. This isolates the variable and maintains a balance. Both methods of solving are presented in the examples that follow.

SOLVING EQUATIONS

Guidelines to Solving Simple Equations by Isolating the Variable

To get a variable "isolated" in a simple equation we must "clear" any numerical values—either constants or coefficients—by applying the appropriate operation:

1. "Clear" a constant term by *adding its opposite*.
2. "Clear" a coefficient by *dividing by the coefficient*.
3. "Clear" a fraction by *multiplying by the denominator*.

In any case, don't forget to keep the equation *balanced* by operating on each side.

Example 5: Solve each of these "simple" equations by clearing the constant.

a) $x - 6 = 3$ b) $-9 = y + 4$ c) $y - \frac{2}{5} = \frac{8}{5}$

Procedure: Recognize the constant that needs to be cleared, and add its opposite to isolate the variable. Mentally check each answer by using it as a *replacement value* for the variable in the original equation.

a)

$x - 6 = 3$	Clear the constant, - 6, by adding its opposite to each side.
$x - 6 \mathbf{+ 6} = 3 + \mathbf{6}$	(The opposite of - 6 is **+ 6**.)
$x + \mathbf{0} = 9$	- 6 + 6 = **0**, the identity for addition; this is what we want.
$x = 9$	Check the answer: 9 – 6 = 3 is true.

b)

$-9 = y + 4$	Clear the constant, 4, by adding its opposite to each side.
$-9 + \mathbf{(-4)} = y + 4 \mathbf{+ (-4)}$	(The opposite of + 4 is **- 4**.)
$-13 = y + \mathbf{0}$	4 + (- 4) = **0**, the identity for addition; this is what we want.
$-13 = y$	Write the equation with the variable on the left side.
$y = -13$	Check the answer: - 13 + 4 = - 9 is true.

c)

$y - \frac{2}{5} = \frac{8}{5}$	Clear the constant, $\frac{2}{5}$, by adding its opposite to each side.
$y - \frac{2}{5} \mathbf{+} \left(\mathbf{\frac{2}{5}}\right) = \frac{8}{5} + \left(\mathbf{\frac{2}{5}}\right)$	(The opposite of $-\frac{2}{5}$ is $\mathbf{+\frac{2}{5}}$.)
$y + \mathbf{0} = \frac{10}{5}$	$\frac{2}{5} + (-\frac{2}{5}) = \mathbf{0}$, the identity for addition; this is what we want.
$y = 2$	Check the answer: $\frac{10}{5} - \frac{2}{5} = \frac{8}{5}$ is true.

Look back at Example 5 and notice these three things:

1. each step in the solving process is directly below the preceding step;
2. the equal signs are lined up, one below the other;
3. we achieved the identity of addition, 0, just as we're supposed to do.

Exercise 5: Solve each of these "simple" equations by clearing the constant. **SHOW ALL STEPS!**

a) $x + 7 = 9$

b) $a - 4 = 11$

c) $-2 = b - 5$

d) $1 = h + 6$

e) $x - 3 = -12$

f) $y + 2 = -8$

g) $-9 = w - 9$

h) $c + 8 = 8$

i) $p - \frac{1}{2} = \frac{5}{2}$

j) $\frac{8}{3} = m + \frac{5}{3}$

Another method—preferred by most students—for isolating the variable by clearing a constant is to add the number to each side *below* the constant term. Here is a repeat of the first two parts of Example 5 (now called Example 6). This time we'll "add the opposite" *below* the number we trying to clear.

Example 6: Solve each of these "simple" equations by clearing the constant.

a) $x - 6 = 3$ b) $-9 = y + 4$

Procedure: We still need to recognize the constant that needs to be cleared and add its opposite to each side.

a)

$$\begin{aligned} x - 6 &= 3 \\ \underline{\mathbf{+6}} &\underline{= \mathbf{+6}} \\ x + \mathbf{0} &= 9 \\ x &= 9 \end{aligned}$$

To clear the - 6 we'll add + 6 to each side.
Notice that we write **+ 6 = + 6**
We still want to get the identity for addition, **0**.
Check the answer: 9 – 6 = 3 is true.

b)

$$\begin{aligned} -9 &= y + 4 \\ \underline{\mathbf{-4}} &\underline{= \quad \mathbf{-4}} \\ -13 &= y + \mathbf{0} \\ -13 &= y \\ y &= -13 \end{aligned}$$

To clear the + 4 we'll add - 4 to each side.
Notice that we write **– 4 = – 4**
We still want to get the identity for addition, **0**.
Check the answer: - 13 + 4 = - 9 is true.

Look back at Example 6 and notice these three things:

1. an equal sign is included between the number that is being added to each side;
2. still, the equal signs are lined up, one below the other;
3. and again, we achieved the identity of addition, 0, just as we're supposed to do.

Exercise 6: Solve each of these "simple" equations by clearing the constant.

a) $x + 5 = 12$

b) $3 = w + 8$

c) $7 = y - 4$

d) $c - 6 = -2$

e) $m + 1 = -9$

f) $-8 = x - 7$

Example 7: Solve each of these "simple" equations by clearing the coefficient.

a) $5x = -20$ b) $12 = -2w$ c) $8x = 5$

Procedure: Divide each side by the coefficient. Remember to keep the equation balanced. **Mentally** check each answer by using it as a replacement value for the variable in the original equation.

a)

$5x = -20$	The coefficient is 5; clear it by dividing each side by 5.
$\frac{5x}{\mathbf{5}} = \frac{-20}{\mathbf{5}}$	Dividing by 5 makes the coefficient = 1.
$1x = -4$	We get the new coefficient 1 because $\frac{5}{5} = 1$.
$x = -4$	Check the answer: $5(-4) = -20$ is true.

b)

$12 = -2w$	The coefficient is -2; clear it by dividing each side by -2.
$\frac{12}{\mathbf{-2}} = \frac{-2w}{\mathbf{-2}}$	Dividing by -2 makes the coefficient = 1.
$-6 = 1w$	We get the new coefficient 1 because $\frac{-2}{-2} = 1$.
$-6 = w$	Write the equation with the variable on the left side.
$w = -6$	Check the answer: $12 = -2\cdot(-6)$ is true.

c)

$8x = 5$	The coefficient is 8; clear it by dividing each side by 8.
$\frac{8x}{\mathbf{8}} = \frac{5}{\mathbf{8}}$	Dividing by 8 makes the coefficient = 1.
$1x = \frac{5}{8}$	The right side can't simplify.
$x = \frac{5}{8}$	Check the answer: $8 \cdot \frac{5}{8} = 5$ is true.

Look back at Example 7 and notice these three things:

1. again, each step in the solving process is directly below the preceding step;
2. and again, the equal signs are lined up, one below the other;
3. this time we achieved the identity of multiplication, 1, just as we're supposed to do.

Exercise 7: Solve each of these "simple" equations by clearing the coefficient.

a) $3y = 24$

b) $-20 = 5x$

c) $-2y = 18$

d) $-28 = -4w$

e) $15 = 6y$

f) $10x = 5$

g) $9x = -14$

h) $-7w = -6$

i) $-12 = 18c$

j) $-21p = -7$

FRACTIONAL COEFFICIENTS

If an equation contains a fraction as the coefficient of the variable, then we modify our approach just a little bit. We still need to isolate the variable by getting a new coefficient of **1**, but we get it by multiplying by the reciprocal of the coefficient.

Example 8: Solve each of these "simple" equations by multiplying the coefficient by its reciprocal.

a) $21 = \frac{7}{2}x$ b) $\frac{x}{4} = 6$ c) $5x = -20$

Procedure: Recognize the constant that needs to be cleared, and add its opposite to isolate the variable. Mentally check each answer by using it as a replacement value for the variable in the original equation.

a)

$21 = \frac{7}{2}x$	The coefficient is $\frac{7}{2}$; clear it by multiplying each side by $\frac{2}{7}$, the reciprocal of $\frac{7}{2}$.
$\frac{21}{1} \cdot \frac{2}{7} = \frac{2}{7} \cdot \frac{7}{2}x$	Make 21 into $\frac{21}{1}$ so that we can cross divide by a factor of 7.
$\frac{3}{1} \cdot \frac{2}{1} = 1 \cdot x$	We get the new coefficient 1 because $\frac{2}{7} \cdot \frac{7}{2} = 1$.
$6 = x$	
$x = 6$	Check the answer: $\frac{7}{2} \cdot 6 = 21$ is true.

b)

$\frac{x}{4} = 6$	$\frac{x}{4}$ can be rewritten as $\frac{1}{4}$ x, so the coefficient of x is $\frac{1}{4}$.
$\frac{1}{4}x = 6$	Multiply each side by 4, the reciprocal of $\frac{1}{4}$.
$\frac{4}{1} \cdot \frac{1}{4}x = 6 \cdot 4$	4 can be written as $\frac{4}{1}$ on the left side.
$1x = 24$	We get the new coefficient 1 because $\frac{4}{1} \cdot \frac{1}{4} = 1$.
$x = 24$	Check x = 24 into the original equation: $\frac{24}{4} = 6$. **True!**

c)

$5x = -20$	The coefficient is 5; clear it by multiplying each side by $\frac{1}{5}$, the reciprocal of $\frac{1}{4}$.
$\frac{1}{5} \cdot \frac{5}{1}x = \frac{1}{5} \cdot \frac{-20}{1}$	Make 5 into $\frac{5}{1}$ and make - 20 into $\frac{-20}{1}$ so that we can cross divide by a factor of 5.
$1x = \frac{1}{1} \cdot \frac{-4}{1}$	We get the new coefficient 1 because $\frac{1}{5} \cdot \frac{5}{1} = 1$.
$x = -4$	Check the answer: $5(-4) = -20$ is true.

Exercise 8 Solve each of these "simple" equations by multiplying the coefficient by its reciprocal.

a) $\frac{3}{8}y = 15$

b) $\frac{5}{6}w = -10$

c) $-6 = \frac{1}{3}x$

d) $\frac{1}{4}p = \frac{3}{2}$

e) $\frac{w}{5} = 7$

f) $6 = \frac{3x}{4}$

g) $-2 = \frac{4}{5}y$

h) $7c = 5$

i) $\frac{-2}{3}x = 8$

j) $-14 = \frac{-7}{10}p$

NEGATIVE COEFFICIENTS

If an equation contains a negative coefficient, then we can divide it outright, as we have done in some of the examples and exercises. Another option, though, is to first *clear the negative* by multiplying each side by - 1.

For example, in the equation $-3x = 12$, we can recognize that the coefficient is negative and decide to multiply each side by - 1 to help us in isolating the variable:

Given:	$-3x = 12$
Multiply each side by - 1:	$-1(-3x) = 12\cdot(-1)$
This equation has a positive coefficient:	$3x = -12$
Divide each side by 3 to isolate the variable:	$\frac{3x}{3} = \frac{-12}{3}$
Simplify the fractions to find the solution:	$x = -4$

It seems as though we're creating more steps than are necessary; however, there are times when being able to multiply each side of an equation by - 1 will come in especially handy, so this is just a preparation for those times. The benefit of multiplying by - 1 is that we are able to work with a positive coefficient, and let's face it, positive numbers are easier to work with than negative numbers.

Do note that multiplying by - 1 has the same effect as "taking the opposite of." Taking the opposite of each side is actually the desired effect of multiplying each side by - 1. However, there is no special symbol for "taking the opposite of" except for multiplying by - 1.

Let's do the same example without actually multiplying by - 1 but by still taking the opposite of each side:

Given:	$-3x = 12$
Find the opposite of each side:	$3x = -12$
Divide each side by 3 to isolate the variable:	$\frac{3x}{3} = \frac{-12}{3}$
Simplify to find the solution:	$x = -4$

This step in isolating the variable is particularly helpful when the coefficient is - 1 itself:

Given:	$-x = -6$	
Find the opposite of each side:	$x = 6$	and we're done!

Example 9: Solve each equation by first taking the opposite of each side (because the coefficient is negative).

a) $-12 = -2w$ b) $-x = 20$ c) $\frac{-7}{2}x = 21$

Procedure: Recognize the constant that needs to be cleared, and add its opposite to isolate the variable. Mentally check each answer by using it as a replacement value for the variable in the original equation.

a)

$-12 = -2w$ The coefficient is negative, so let's take the opposite of each side.

$12 = 2w$ We may now divide each side by 2

$\frac{12}{2} = \frac{2w}{2}$

$6 = w$ Check the answer: $-12 = -2(6)$ is true.

b)

$-x = 20$ The coefficient is negative, so let's take the opposite of each side.

$x = -20$ And we're done; the solution is $x = -20$

c)

$\frac{-7}{2}x = 21$ The coefficient is negative; take the opposite of each side.

$\frac{7}{2}x = -21$ Now clear the coefficient by multiplying each side by $\frac{2}{7}$.

$\frac{2}{7} \cdot \frac{7}{2}x = \frac{2}{7} \cdot \frac{-21}{1}$ Make 21 into $\frac{21}{1}$ so that we can cross divide by a factor of 7.

$1 \cdot x = \frac{2}{1} \cdot \frac{-3}{1}$ We get the new coefficient 1 because $\frac{2}{7} \cdot \frac{7}{2} = 1$.

$x = -6$ Check the answer: $\frac{-7}{2} \cdot (-6) = 21$ is true.

Exercise 9: Solve each equation by first taking the opposite of each side (because the coefficient is negative).

a) $-2y = 18$

b) $-4x = -28$

c) $-7 = -21p$

d) $-6 = -7w$

e) $\frac{-2}{3}x = 10$

f) $\frac{-1}{10}c = -3$

g) $15 = -\frac{5}{4}y$

h) $-6 = -\frac{3}{8}m$

Answers to each Exercise

Section 2.1

Exercise 1

a)	0	b)	- 4	c)	0	d)	5
e)	10	f)	- 9				

Exercise 2

a)	1	b)	1	c)	1	d)	$\frac{-1}{5}$
e)	10	f)	$\frac{2}{5}$	g)	9	h)	$\frac{1}{12}$
i)	$\frac{-3}{5}$						

Exercise 3

a) 5x – 8 – 6x is an expression because it has no equal sign.

b) 2(3x + 4) + 6(2 – 5x) is an expression because it has no equal sign.

c) 2(3x + 4) = 6(2 – 5x) is an equation; it has an equal sign between two expressions.

d) 2(3x + 4) – 2 – 5x = - 8 is an equation; it has an equal sign between two expressions.

Exercise 4

a) The replacement value gives 8 = 8, so x = 5 **IS** the solution.

b) The replacement value gives - 4 = - 4, so w = 2 **IS** the solution.

c) The replacement value gives 0 = 2, so y = 1 is **NOT** the solution.

d) The replacement value gives 5 = 5, so c = 4 **IS** the solution.

Exercise 5

a)	x = 2	b)	a = 15	c)	b = 3	d)	h = - 5
e)	x = - 9	f)	y = - 10	g)	w = 0	h)	c = 0
i)	p = 3	j)	m = 1				

Exercise 6	a) $x = 7$	b) $w = -5$	c) $y = 11$	d) $c = 4$
	e) $m = -10$	f) $x = -1$		
Exercise 7	a) $y = 8$	b) $x = -4$	c) $y = -9$	d) $w = 7$
	e) $y = \frac{5}{2}$	f) $x = \frac{1}{2}$	g) $x = \frac{-14}{9}$	h) $w = \frac{6}{7}$
	i) $c = \frac{-2}{3}$	j) $p = \frac{1}{3}$		
Exercise 8	a) $y = 40$	b) $w = -12$	c) $x = -18$	d) $p = 6$
	e) $w = 35$	f) $x = 8$	g) $y = \frac{-5}{2}$	h) $c = \frac{5}{7}$
	i) $x = -12$	j) $p = 20$		
Exercise 9	a) $y = -9$	b) $x = 7$	c) $p = \frac{1}{3}$	d) $w = \frac{6}{7}$
	e) $x = -15$	f) $c = 30$	g) $y = -12$	h) $m = 16$

Section 2.1 Focus Exercises

1. Determine if the **replacement value** shown (after each equation) is a solution of that equation.

a) $3p - 2 = 6$; $\mathbf{p = 3}$

b) $9 - 3k = 7k - 11$; $\mathbf{k = 2}$

c) $5m + (4 - m) = 3(m - 2) - 2$; $\mathbf{m = -4}$

d) $\frac{1}{6}x - \frac{1}{3}x = \frac{x - 6}{3}$; $\mathbf{x = 12}$

2. Solve each equation by isolating the variable. SHOW ALL STEPS!

a) $p + 2 = 4$

b) $x - 8 = 9$

c) $y - 9 = -6$

d) $b + 1 = -5$

e) $r - 4 = -4$

f) $w + 3 = 3$

g) $m - \frac{5}{6} = \frac{9}{6}$

h) $k + \frac{9}{12} = \frac{3}{12}$

i) $y + 2 = -8$

j) $c - 1 = -6$

3. Solve each equation by isolating the variable. SHOW ALL STEPS!

a) $7x = 56$

b) $9m = -63$

c) $-6x = 24$

d) $-12p = -36$

e) $10x = 15$

f) $5m = -9$

g) $\frac{7}{3}x = 28$

h) $\frac{3}{2}y = -30$

i) $\frac{4}{7}n = -2$

j) $\frac{5}{8}v = \frac{15}{4}$

k) $\frac{y}{9} = 2$

l) $\frac{7n}{8} = -1$

m) $-6m = 4$

n) $-8x = -18$

o) $\frac{-6}{7}x = -12$

p) $\frac{-5}{4}k = \frac{15}{2}$