2.2 Solving Linear Equations, Part 2

INTRODUCTION

Here are a few examples of the types of equations you'll be seeing in this section:

(a) $2x + 9 = 3$ (b) $5a + 7 = 13 - a$ (c) $3(a - 1) = 2(6 - a)$ (d) $2v - 1 = \frac{3}{4}v + 9$

Notice that these equations are more involved than the ones seen in Section 2.1. Some of these equations have more than one variable term, and they all involve more than one operation: multiplication along with addition and/or subtraction. Yet, they are still **linear** equations: each variable is to a power of only 1.

Because they are all linear equations, we still seek to **isolate the variable**. Getting there will be a little more challenging, though, so you'll need to be careful and follow the required steps carefully. You'll also need to keep in mind the notion that

the sign in front of the term belongs to the term.

Another idea that you haven't seen for a long time is that of the "main operation." You were first introduced to the main operation in Section 1.6; here is some text taken directly from Section 1.6:

> Every algebraic or numeric expression has one operation that, in a sense, holds the whole expression together; this is called the "main operation." It is the main operation that lets us know if the expression is a sum, difference, product or quotient. The **main operation** is the last operation that is to be applied, according to the order of operations.
>
> For example, in the expression $5 + 6 \cdot 3$, of the two operations present (addition and multiplication), the order of operations has us apply the multiplication first and the *addition last*. Hence, the main operation is *addition* (it is applied last), and we can think of
> $5 + 6 \cdot 3$ as a *sum*; it is a sum of two parts: **5** and $\mathbf{6 \cdot 3}$.

In our linear equations, we'll be looking for the main operation in expressions like $2x + 9$. This expression has two operations: multiplication ($2x$ is $2 \cdot x$) and addition: $2x \mathbf{+} 9$. The main operation is **addition** because it has the lowest rank (according to the order of operations).

Isolating the variable still means getting the variable all by itself. The challenge here is to work with equations that have more than one variable term and more than one operation. Following the guidelines presented in this section should make the solving process simpler than you might imagine.

SOLVING LINEAR EQUATIONS WITH MORE THAN ONE OPERATION

Some equations have more than one operation to clear. When this occurs we must have a consistent method of isolating the variable.

For example, in an equation such as **3x + 5 = 17**, one might ask:

> "To isolate x, which should we clear first, the *constant* term, 5, or the *coefficient*, 3?"

The answer to the question is this: isolate the variable by *first clearing the* ***main operation***. Remember, the *main* operation is the one that has the *lowest rank*, according to the order of operations. In the left side expression, 3x + 5, the main operation is *addition.*

Example 1: Solve each linear equation by clearing both operations, one at a time. Use the solving techniques presented earlier in this section.

a) $3x + 5 = 17$ b) $9 = 7b - 6$

Procedure: First, recognize the operations, multiplication and addition, then decide which is the main operation. Check your answer for accuracy.

a)

$3x + 5 = 17$ Clear the main operation, addition, by *adding the opposite* of 5 to each side.

$\underline{\quad - 5 = -5}$

$3x + 0 = 12$ Adding 5 + (- 5) gives the identity for addition, 0.

$3x = 12$ Now clear the coefficient by dividing each side by 3.

$\dfrac{3x}{3} = \dfrac{12}{3}$ $\dfrac{3x}{3}$ reduces to 1x, or just x.

$x = 4$ **Check** the answer, x = 4: $3(4) - 5 = 7$

$12 - 5 = 7$ is true.

b)

$9 = 7b - 6$ Clear the main operation, subtraction, by *adding* 6 to each side.

$\underline{\quad + 6 = \quad + 6}$

$15 = 7b + 0$ Adding - 6 + 6 gives the identity for addition, 0.

$15 = 7b$ Now clear the coefficient by dividing each side by 3.

$\dfrac{15}{7} = \dfrac{7b}{7}$ **Check** the answer: $9 = 7 \cdot \dfrac{15}{7} - 6$

$\dfrac{15}{7} = b$ $\left(7 \text{ is the same as } \dfrac{7}{1}\right)$ $9 = \dfrac{7}{1} \cdot \dfrac{15}{7} - 6$

$b = \dfrac{15}{7}$ □ $9 = 15 - 6$ This is true.

Exercise 1: Solve each linear equation by clearing both operations, one at a time. <u>Check your answer to show that it is a solution.</u>

a) $3x - 5 = 19$

b) $7w + 3 = 24$

c) $-29 = 4y - 5$

d) $-26 = 7 + 3p$

e) $-2x + 9 = 1$

f) $6 - 5w = -34$

g) $-14 = -3x - 8$

h) $13 = -5 - 9c$

SOLVING LINEAR EQUATIONS WITH MORE THAN ONE VARIABLE TERM

Some equations have more than one variable term, oftentimes one on each side.

For example, **$6x + 5 = 4x + 17$** has a variable term on each side. One might ask:

"To solve this equation, which variable should we isolate?"

To answer this question we must remember that any solution number we find becomes a *replacement value*. This replacement value replaces the variable in *both* variable terms (as was demonstrated in Section 2.1); therefore, they are the *same* variable and must be combined as one term before we can truly isolate the variable.

However, because they are in different expressions (on different sides) the only way we can combine them is to *add the opposite* of one of the terms to each side. It's sometimes best (but not necessary) to add the opposite of the term that is the lesser of the two; this way the resulting term will be positive.

Example 2: Solve each linear equation by first combining the variable terms. Then use the solving techniques presented earlier in this section.

a) $6x + 5 = 4x + 17$ b) $-3y - 8 = 2y + 27$

Procedure: First, combine the variable terms by adding or subtracting the lesser of the two.

a)

$6x + 5 = 4x + 17$	4x is the lesser of the two variable terms; add - 4x to each side.
$-4x \quad = -4x$	This time, we line up the - 4x underneath the variable terms so that we may combine them easily; $4x + (-4x) = 0x$, or just 0.
$2x + 5 = 0 + 17$	
$2x + 5 = 17$	Now isolate the variable by clearing the main operation.
$-5 = -5$	Add *the opposite of* + 5 to each side; $+5 + (-5) = 0$
$2x + 0 = 12$	Now clear the coefficient by dividing each side by 2.
$\frac{2x}{2} = \frac{12}{2}$	**Check** the answer, x = 6: $6(6) + 5 = 4(6) + 17$
$x = 6$ ▯	$36 + 5 = 24 + 17$
	$41 = 41$ is true.

b)

$-3y - 8 = 2y + 27$	- 3y is the lesser of the two variable terms; add 3y to each side.
$+3y \quad = +3y$	
$0 - 8 = 5y + 27$	Now isolate the variable by clearing the main operation.
$-8 = 5y + 27$	
$-27 = \quad -27$	
$-35 = 5y + 0$	Now clear the coefficient by dividing each side by 5.

$$\frac{-35}{5} = \frac{5y}{5}$$

$$-7 = y$$

$$y = -7$$ ▯

Check the answer, y = - 7:

$$-3(-7) - 8 = 2(-7) + 27$$

$$21 - 8 = -14 + 27$$

$$13 = 13$$ is true.

Exercise 2: Solve each linear equation by first combining the variable terms. Check you answer to show that it is a solution.

a) $9x - 5 = 10 + 6x$

b) $2p + 5 = -4p + 23$

c) $4w - 5 = 4 + 7w$

d) $3y + 9 = -5y + 41$

e) $x + 10 = 2 - 3x$

f) $2y - 4 = -y - 25$

Sometimes, before we can start the process of "isolating the variable," we need to simplify one (or both) of the sides; after all, each side is an expression, and it's common for us to simplify expressions. For example, we may need to distribute a number through to a quantity, as in $4(2x + 3)$; or, we may need to combine like terms, as in $2x + 6 - 8x$.

In either situation, we need to simplify each side completely before we start to isolate the variable.

Example 3: Solve each linear equation by first simplifying each side. Then isolate the variable using techniques learned earlier in this section.

a) $3(a - 1) = 2(6 - a)$ b) $2x + 6 - 8x = 10 - 3x + 2$

Answer:

a)

$3(a - 1) = 2(6 - a)$	First distribute on each side. Do not try to clear any terms just yet.
$3a - 3 = 12 - 2a$	Now we can start the process of isolating the variable.
$\underline{+ 2a \quad = \quad + 2a}$	Get the variable terms together by adding the opposite of - 2a (the lesser).
$5a - 3 = 12 + 0$	- 2a + 2a = 0a, or just 0.
$5a - 3 = 12$	Now isolate the variable by clearing the main operation (subtraction).
$\underline{+3 = +3}$	Add *the opposite of* - 3 to each side; - 3 + 3 = 0
$5a + 0 = 15$	
	Now clear the coefficient by dividing each side by 5.
$\frac{5a}{5} = \frac{15}{5}$	**Check** the answer, a = 3: $3(3 - 1) = 2(6 - 3)$
$x = 3$ ▯	$3(2) = 2(3)$
	$6 = 6$ is true.

b)

$2x + 6 - 8x = 10 - 3x + 2$	First combine like terms, separately, on each side
$-6x + 6 = 12 - 3x$	- 6x is the lesser of the two variable terms; add 6x to each side.
$\underline{+6x \quad = \quad +6x}$	
$0 + 6 = 12 + 3x$	
$6 = 12 + 3x$	Now isolate the variable by clearing the main operation; add the opposite of + 12.
$\underline{-12 = -12}$	
$-6 = 0 + 3x$	Now clear the coefficient by dividing each side by 3.
$\frac{-6}{3} = \frac{3x}{3}$	**Check** the answer, x = - 2, into <u>the very original equation</u>:
$-2 = x$	$2(-2) + 6 - 8(-2) = 10 - 3(-2) + 2$
$x = -2$ ▯	$-4 + 6 + 16 = 10 + 6 + 2$
	$+18 = 18$ is true.

Exercise 3: Solve each linear equation by first simplifying each side. Then isolate the variable using techniques learned earlier in this section.

a) $3x + 6 + 2x = 8 + x - 10$

b) $x + 9 - 5x = 1 - 2x + 10$

c) $4(y + 1) = 3(2y - 2)$

d) $-2(3w + 5) = 4(-1 - 2w)$

EQUATIONS WITH FRACTIONS; CLEARING THE DENOMINATOR

There are a number of approaches that one could take to solving linear equations that have fractions.

For example, to solve $2x - 1 = \frac{3}{4}x + 9$ we could use the methods learned earlier in this section.

This might require, though, that we add, subtract or multiply *fractions* to finally isolate the variable.

To avoid having to apply all of the operations with fractions it is usually better to **clear the fractions** first by multiplying each side by the least common denominator (LCD).

Example 4: Solve each equation by first clearing the fractions.

a) $2x - 1 = \frac{3}{4}x + 9$ b) $\frac{3w}{2} + 1 = w + \frac{9}{2}$

Procedure: In each case, there is only one value in the denominator, so that is the LCD; In multiplying each side, be sure to use parentheses and distribute.

a)

$$2x - 1 = \frac{3}{4}x + 9$$

The LCD is 4; multiply each side by 4.

$$4\cdot(2x - 1) = 4\cdot\left(\frac{3}{4}x + 9\right)$$

Distribute the 4, on each side, to each term. The toughest one is

$$8x - 4 = \frac{4}{1}\cdot\frac{3}{4}x + 36$$

$4\cdot\frac{3}{4}x$; we'll make this $\frac{4}{1}\cdot\frac{3}{4}x$ which reduces to just $3x$.

$$8x - 4 = 3x + 36$$

Now we can solve, as before, without the fraction getting in the way.

$$\underline{-3x \quad = -3x}$$

Get the variable terms together by adding the opposite of $3x$ to each side.

$$5x - 4 = 0 + 36$$

$$5x - 4 = 36$$

$$\underline{+4 = +4}$$

$$5x + 0 = 40$$

$$\frac{5x}{5} = \frac{40}{5}$$

$$x = 8$$

Check the answer; replace each variable in the *original* equation:

$x = 8$: $2(8) - 1 = \frac{3}{4}(8) + 9$

$16 - 1 = \frac{3}{4}\cdot\frac{8}{1} + 9$

$15 = \frac{3}{1}\cdot\frac{2}{1} + 9$

$15 = 6 + 9$ This is true.

b)

$$\frac{3w}{2} + 1 = w + \frac{9}{2}$$

The LCD is 2; multiply each side by 2 to clear the fractions.

$$2\cdot\left(\frac{3w}{2} + 1\right) = 2\cdot\left(w + \frac{9}{2}\right)$$

Distribute the 2, on each side, to each term. Make 2 into $\frac{2}{1}$ when multiplying by the fractions.

$$\frac{2}{1}\cdot\frac{3w}{2} + 2\cdot1 = 2\cdot w + \frac{2}{1}\cdot\frac{9}{2}$$

$$3w + 2 = 2w + 9$$

We have *cleared* the fractions and can now solve as before.

$$\underline{-2w \quad = -2w}$$

Get the variable terms together by adding the opposite of $2w$ to each side.

$$1w + 2 = 0 + 9$$

$$w + 2 = 9$$

$$\underline{-2 = -2}$$

$$w + 0 = 7$$

$$w = 7$$

Check the answer; replace each variable in the *original* equation:

$w = 7$: $\frac{3(7)}{2} + 1 = 7 + \frac{9}{2}$

write each with a denominator of 2: $\frac{21}{2} + \frac{2}{2} = \frac{14}{2} + \frac{9}{2}$

$\frac{23}{2} = \frac{23}{2}$ This is true.

Exercise 4: Solve each equation by first clearing the fractions. Check you answer to show that it is a solution.

a) $\frac{x}{3} = x + 4$

b) $\frac{4}{9} m = \frac{7}{9} m - 2$

c) $y + \frac{1}{4} = \frac{3y}{4} + 1$

d) $\frac{w}{2} + 3 = \frac{3w}{2} - 1$

e) $\frac{x}{5} + 3 = 3 - \frac{2x}{5}$

f) $\frac{y}{6} + 2 = \frac{5y}{6} + 1$

Answers to each Exercise

Section 2.2

Exercise 1:	a) $x = 8$	b) $w = 3$	c) $y = -6$	d) $p = -11$
	e) $x = 4$	f) $w = 8$	g) $x = 2$	h) $c = -2$
Exercise 2:	a) $x = 5$	b) $p = 3$	c) $w = -3$	d) $y = 4$
	e) $x = -2$	f) $y = -7$		
Exercise 3:	a) $x = -2$	b) $x = -1$	c) $y = 5$	d) $w = 3$
Exercise 4:	a) $x = -6$	b) $m = 6$	c) $y = 3$	d) $w = 4$
	e) $x = 0$	f) $y = \frac{3}{2}$		

Section 2.2 Focus Exercises

1. Solve each linear equation by isolating the variable. Check your answer to show that it is a solution.

a) $6m - 4 = 20$

b) $2 + 6y = -13$

c) $9 - 8x = -23$

d) $y + 19 = 5 - 6y$

e) $4y - 8 = 7 + 9y$

f) $5p + 8 = -7p + 56$

g) $3(2 - w) = 2(10 - 5w)$

h) $y - 4 - 8y = 1 - 5y + 10$

2. Solve each equation by first clearing the fractions. Check you answer to show that it is a solution.

a) $\frac{y}{6} = y + 5$

b) $\frac{7}{4}h = \frac{1}{4}h - 12$

c) $w + \frac{1}{7} = \frac{6w}{7} - 1$

d) $6 + \frac{x}{5} = \frac{4x}{5} - 3$

e) $\frac{y}{8} + 6 = 6 - \frac{5y}{8}$

f) $\frac{w}{3} + 5 = \frac{8w}{3} - 2$