2.3 Solving Linear Equations, Part 3

SOLVING EQUATIONS: THE ULTIMATE GUIDELINES

Here is a summary of the steps involved in solving a variety of linear equations. Not all steps will be necessary for each equation; you should apply the guidelines in the order presented here but may skip any guideline that does not apply. For example, if an equation has no fractions, you may skip guideline (2) and proceed to guideline (3).

Solving Linear Equations: The Ultimate Guidelines

The Preparation:

1. Eliminate any parentheses by distributing. (Be careful to look for negative multipliers.)
2. Clear any fractions or decimals by multiplying each side by the equation's LCD.
3. Combine like terms on each individual side.

Isolating the Variable:

4. If there is more than one variable term, get them together on the same side by adding the opposite of one of the variable terms.
5. Clear the operations; start with the main operation.

THE PREPARATION: EQUATIONS WITH PARENTHESES

If an equation has any parentheses, then we need to "clear" them by distributing before we do anything else.

Example 1: Solve this equation by first clearing the parentheses: $2(x - 1) + 8 = 9 - (x - 3)$

Answer: First recognize that we'll distribute 2 on the left side and - 1 on the right side.

$2(x - 1) + 8 = 9 - 1(x - 3)$ Place a 1 on the right side as a multiplier; distribute.

$2x - 2 + 8 = 9 - x + 3$ Combine like terms on each side.

$2x + 6 = -x + 12$

$+x \quad = +x$ Add x to each side to get one variable term.

$3x + 6 = 12$ Add - 6 to each side to start to isolate the variable.

$-6 = -6$

$3x = 6$ Divide each side by 3.

$\frac{3x}{3} = \frac{6}{3}$

$x = 2$ ✓

Place x = 2 into the original equation:

$2(2 - 1) + 8 = 9 - (2 - 3)$

$2(1) + 8 = 9 - (-1)$

$2 + 8 = 9 + 1$ **true!**

Exercise 1: Solve each equation by first clearing the parentheses. Check your answer to show that it is a solution.

a) $2(x + 9) + 2x = 38$

b) $3(x + 20) + (x + 20) + x = 180$

c) $5(x + 1) - 3x = -19 + 2(4 - 3x)$

d) $3(x - 4) + 2x = 7x - 2(2x + 1)$

THE PREPARATION: EQUATIONS WITH FRACTIONS

We have already worked with clearing the fractions in the previous section. However, in each of those exercises the denominators were all the same. If the denominators are different, though, we need to clear the fractions first by multiplying each side by the **least common denominator** (LCD).

Example 2: Solve each equation by first identifying the LCD and then clearing the fractions.

a) $\frac{x}{3} + 1 = \frac{5x}{6} - 3$ b) $\frac{x}{4} + \frac{1}{12} = \frac{x}{3} - \frac{1}{6}$

Answer:

a) $\frac{x}{3} + 1 = \frac{5x}{6} - 3$ The LCD is 6, so multiply each side by 6 or $\frac{6}{1}$.

$\frac{6}{1} \cdot \left(\frac{x}{3} + 1\right) = \frac{6}{1} \cdot \left(\frac{5x}{6} - 3\right)$ Now distribute....

$\frac{6}{1} \cdot \frac{x}{3} + 6 \cdot 1 = \frac{6}{1} \cdot \frac{5x}{6} - 6 \cdot 3$ .....and simplify.

$2x + 6 = 5x - 18$ Now we can solve, as before, without the fractions getting in the way.

$-2x \quad = -2x$

$6 = 3x - 18$

$+18 = \quad +18$

$24 = 3x$

$\frac{24}{3} = \frac{3x}{3}$

$8 = x$ □

Check the answer; replace each variable in the *original* equation:

$x = 8$: $\frac{8}{3} + 1 = \frac{5(8)}{6} - 3$

$\frac{8}{3} + \frac{3}{3} = \frac{40}{6} - \frac{18}{6}$

$\frac{11}{3} = \frac{22}{6}$ This is true.

b) $\frac{x}{4} + \frac{1}{12} = \frac{x}{3} - \frac{1}{6}$ The LCD is 12, so multiply each side by 12 or $\frac{12}{1}$.

$\frac{12}{1} \cdot \left(\frac{x}{4} + \frac{1}{12}\right) = \frac{12}{1} \cdot \left(\frac{x}{3} - \frac{1}{6}\right)$ Now distribute....

$\frac{12}{1} \cdot \frac{x}{4} + \frac{12}{1} \cdot \frac{1}{12} = \frac{12}{1} \cdot \frac{x}{3} - \frac{12}{1} \cdot \frac{1}{6}$ .....and simplify.

$3x + 1 = 4x - 2$ Now we can solve, as before, without the fractions getting in the way.

$-3x \quad = -3x$

$1 = x - 2$

$+2 = \quad +2$

$3 = x$ □

Check the answer; replace each variable in the *original* equation:

$x = 3$: $\frac{3}{4} + \frac{1}{12} = \frac{3}{3} - \frac{1}{6}$

$\frac{9}{12} + \frac{1}{12} = \frac{6}{6} - \frac{1}{6}$

$\frac{10}{12} = \frac{5}{6}$ This is true.

Exercise 2: Solve each equation by first identifying the LCD and clearing the fractions. Check your answer to show that it is a solution.

a) $\frac{3y}{4} - 6 = \frac{y}{8} + 4$

b) $p - \frac{p}{6} = \frac{p}{3} + 2$

c) $\frac{3x}{20} + \frac{1}{10} = \frac{x}{4} - \frac{1}{5}$

d) $\frac{w}{4} + \frac{11}{12} = \frac{1}{2} - \frac{w}{6}$

THE PREPARATION: EQUATIONS WITH DECIMALS

Decimals are special fractions that aren't written with a denominator. For example, $\frac{3}{10}$ can be written as 0.3. This isn't really an abbreviation, it's just a different way of saying "three-tenths."

There are three ways that we can approach an equation involving decimals:

i) we can rewrite each decimal as a fraction and solve by clearing the fractions;
ii) we can get each number to be represented by the same number of decimal places then multiply by a power of 10 to clear the decimals;
iii) we can solve it directly without clearing any decimals, though this can be more challenging.

Though the second option sounds the most challenging, but it is actually the most efficient. This next example will show the first two methods for the same equation.

Example 3: Solve the equation by first writing the decimals as fractions; then, identify the LCD and clear the fraction.

$$0.4x - 1.2 = 0.15x + 0.8$$

Answer: The first two methods will be shown here:

i)

$$0.4x - 1.2 = 0.15x + 0.8$$ Write each decimal as a fraction.

$$\frac{4}{10}x - \frac{12}{10} = \frac{15}{100}x + \frac{8}{10}$$ The LCD is 100; multiply each side by $\frac{100}{1}$.

$$\frac{100}{1}\cdot\left(\frac{4}{10}x - \frac{12}{10}\right) = \frac{100}{1}\cdot\left(\frac{15}{100}x + \frac{8}{10}\right)$$ Distribute.

$$\frac{100}{1}\cdot\frac{4}{10}x - \frac{100}{1}\cdot\frac{12}{10} = \frac{100}{1}\cdot\frac{15}{100}x + \frac{100}{1}\cdot\frac{8}{10}$$ Simplify.

* $$40x - 120 = 15x + 80$$ Whew! Now it's in a more manageable form.

$$-15x \quad = -15x$$

$$25x - 120 = 80$$

$$+120 = +120$$

$$25x = 200$$

$$\frac{25x}{25} = \frac{200}{25}$$

$$x = 8$$

Check x = 8:

$$0.4(8) - 1.2 = 0.15(8) + 0.8$$

$$3.2 - 1.2 = 1.20 + 0.8$$

$$2.0 = 2.0$$

True!

ii)

$$0.4x - 1.2 = 0.15x + 0.8$$ Write each decimal so that they all have two decimal places.

$$0.40x - 1.20 = 0.15x + 0.80$$ (This is the equivalent of identifying the LCD.)

$$100(0.40x - 1.20) = (0.15x + 0.80)100$$ Multiplying each side by 100 (clear the decimals).

* $$40x - 120 = 15x + 80$$ **Whew!**

* This is the same place we got in part (i), so I can trust that you can solve it from here.

Exercise 3: Solve each equation by clearing the decimals. You may use either method illustrated in Example 3. <u>Check your answer to show that it is a solution.</u>

a) $0.2x + 0.5 = 0.7x - 4$

b) $2x - 0.4 = 1 + 1.8x$

c) $0.17x - 0.43 = 0.25x + 0.05$

d) $0.3x + 1.38 = 0.24x + 1.2$

e) $0.27x - 1.6 = 0.32x - 2$

f) $0.1x - 0.006 = 0.08x + 0.134$

FRACTIONS, DECIMALS AND PARENTHESES

The Ultimate Guidelines, presented at the beginning of this section, indicate that parentheses should be cleared first. This is true even if fractions are involved; in other words, if an equation has both fractions (or decimals) and parentheses, then it is best to clear the parentheses before trying to clear the fractions.

Example 4: Solve each equation by first distributing; then clear the fractions or decimals.

a) $\frac{1}{2}\cdot\left(x+\frac{2}{3}\right) = 3\cdot(x-1)$ b) $0.2\cdot(3x - 5) = 0.15(2x + 3) - 0.85$

Answer:

a)

$$\frac{1}{2}\cdot\left(x+\frac{2}{3}\right) = 3\cdot(x-1)$$ Distribute and simplify; $\frac{1}{2}\cdot\frac{2}{3} = \frac{1}{3}$

$$\frac{1}{2}x + \frac{1}{3} = 3x - 3$$ The LCD is 6; clear the fractions.

$$\frac{6}{1}\cdot\left(\frac{1}{2}x+\frac{1}{3}\right) = 6(3x-3)$$ Simplify: $\frac{6}{1}\cdot\frac{1}{2}x = 3x$ and $\frac{6}{1}\cdot\frac{1}{3} = 2$.

$$3x + 2 = 18x - 18$$
$$-3x = -3x$$
$$2 = 15x - 18$$
$$+18 = +18$$
$$20 = 15x$$
$$\frac{20}{15} = \frac{15x}{15}$$
$$\frac{4}{3} = x \quad \text{Better written as} \quad x = \frac{4}{3}$$

Check $x = \frac{4}{3}$:

$$\frac{1}{2}\cdot\left(\frac{4}{3}+\frac{2}{3}\right) = 3\cdot\left(\frac{4}{3}-1\right)$$
$$\frac{1}{2}\cdot\left(\frac{6}{3}\right) = 3\cdot\left(\frac{4}{3}-\frac{3}{3}\right)$$
$$\frac{6}{6} = \frac{3}{1}\cdot\left(\frac{1}{3}\right)$$
$$1 = 1$$ **True!**

b)

$$0.2\cdot(3x - 5) = 0.15(2x + 3) - 0.85$$ Distribute.

$$0.6x - 1.0 = 0.30x + 0.45 - 0.85$$ Get all the decimals to two decimal places.

$$0.60x - 1.00 = 0.30x + 0.45 - 0.85$$ Multiply each side by 100.

$$100(0.60x - 1.00) = (0.30x + 0.45 - 0.85)100$$

$$60x - 100 = 30x + 45 - 85$$ Combine like terms.

$$60x - 100 = 30x - 40$$
$$-30x = -30x$$
$$30x - 100 = -40$$
$$+100 = +100$$
$$30x = 60$$
$$\frac{30x}{30} = \frac{60}{30}$$

Check $x = 2$:

$$0.2\cdot[3(2) - 5] = 0.15[2(2) + 3] - 0.85$$
$$0.2\cdot[6 - 5] = 0.15[4 + 3] - 0.85$$
$$0.2\cdot(1) = 0.15\cdot(7) - 0.85$$
$$0.2 = 1.05 - 0.85$$
$$0.2 = 0.20$$ **True!**

$x = 2$

Exercise 4: Solve each equation using the methods illustrated in Example 4. Check your answer to show that it is a solution.

a) $\frac{1}{2}(2x - 1) = \frac{1}{3}\left(2x + \frac{1}{2}\right)$

b) $0.5(x + 3) = 3(0.1 + 0.16x)$

c) $\frac{1}{8}(3x + 2) = \frac{1}{4}\left(2x + \frac{1}{2}\right) + \frac{1}{2}$

d) $0.6(10x - 3) = 1.5(x + 2) - 0.3$

Answers to each Exercise

Section 2.3

Exercise 1	a) x = 5	b) x = 20	c) x = - 2	d) x = 5
Exercise 2	a) y = 16	b) p = 4	c) x = 3	d) w = - 1
Exercise 3	a) x = 9	b) x = 7	c) x = - 6	d) x = - 3
	e) x = 8	f) x = 7		
Exercise 4	a) x = 2	b) x = - 60	c) x = - 3	d) x = 1

Section 2.3 Focus Exercises

1. Solve each equation. Check your answer to show that it is a solution.

a) $3x + 4(x + 7) = 56$

b) $5(x - 2) + 2x = 9x - 2(3x - 5)$

c) $p - \frac{p}{8} = \frac{p}{4} - 10$

d) $\frac{5y}{2} - 9 = \frac{2y}{3} + 2$

e) $1 - \frac{5}{8}x = 2 - \frac{2}{3}x$

f) $\frac{3x}{5} + \frac{1}{6} = \frac{x}{2} - \frac{1}{3}$

2. Solve each equation. <u>Check your answer to show that it is a solution.</u>

a) $.6x - 3.2 = .4 - .3x$

b) $.29x - .25 = .43x + .03$

c) $.48x - 1.9 = .54x - 4$

d) $.1x + .008 = .06x - .172$

e) $\frac{1}{6}(1 - 6x) = -\frac{1}{3}\left(6x + \frac{1}{2}\right)$

f) $.3(x + 5) = 5(.1 + .11x)$