

## Section 2.6 Solving Linear Inequalities

### INTRODUCTION

Solving an inequality is much like solving an equation; there are, though, some special circumstances of which you need to be aware.

In solving an inequality it is possible to *change the direction of the inequality sign*. This means that it is possible to change, for example, a *greater than* sign to a *less than* sign. It doesn't happen arbitrarily, though, and you must be careful whenever doing so.

### SWITCHING SIDES

As was demonstrated in Section 2.5, we can change the direction of the inequality sign by “switching sides”; this means that whatever expressions are on the left and right sides can be swapped, exactly as they are—the right side becomes the left side and vice-versa.

Again, if you choose to do this, you *must* change the direction of the inequality sign. This rule looks like this:

If	$A < B$	if	$C \geq D$
then	$B > A$	then	$D \leq C$

**Example 1:** Switch sides and change the direction of the inequality sign; **do not solve**.

a)  $4 < x + 3$                       b)  $y - 5 \geq 2y$                       c)  $6 - x \leq 2x + 3$

**Answer:** Do not change the value of either side, just change on which side each expression is. Also, change the direction of the inequality sign. (The inequality signs are bold and a little bigger to emphasize the change.)

a)  $x + 3 > 4$                       b)  $2y \leq y - 5$                       c)  $2x + 3 \geq 6 - x$

**Exercise 1:** Switch sides and change the direction of the inequality sign; **do not solve**.

a)  $7 \geq x$                               b)  $6 < w$                               c)  $-8 > y$

d)  $9 > x + 5$                           e)  $-4 \leq d - 8$                           f)  $3x \geq 5x - 2$

g)  $1 - 3x < 4x + 8$                       h)  $2x + 3 > 6 - 4x$                       i)  $5 - 3p \leq 2p - 10$

## MULTIPLYING BY - 1

We can also change the direction of the inequality sign by multiplying each side by  $-1$ . This is useful when the next to last step is, for example,  $-x \leq 5$ . In this case, we need only multiply each side by  $-1$  and—at the same time—*change the direction of the inequality sign*. It look like this:

$$-x \leq 5$$

$$x \geq -5$$

Actually, you can multiply each side of the inequality by  $-1$  at any time in the problem solving process; just be sure that, if you do, you also change the direction of the inequality sign.

Multiplying by  $-1$  is the same as taking the opposite of something. In this case, when we multiply each side of the inequality by  $-1$  we get:

- the opposite of the entire left side
- the opposite of the entire right side
- the opposite direction of the inequality sign.

In this example, multiplying each side by  $-1$  shows up as *changing the sign of each and every term*, and changing the direction of the inequality sign from *greater than* to *less than*.

$$8 - 2x > x + 5$$

$$-8 + 2x < -x - 5$$

**Example 2:** Multiply each side by  $-1$  and change the direction of the inequality sign; do not solve.

a)  $-x < -4$                       b)  $6 - w \leq 3$                       c)  $5 - 3y < 2y + 1$

**Answer:** Multiplying by  $-1$  is the same as taking the opposite of *everything*: the opposite of every term on each side and the opposite direction of the inequality sign.

a)  $x > 4$                               b)  $-6 + w \geq -3$                       c)  $-5 + 3y > -2y - 1$

**Exercise 2:** Multiply each side by  $-1$  and change the direction of the inequality sign; **do not solve**.

a)  $-c \geq 6$

b)  $-w < -1$

c)  $10 > -5x$

d)  $-8 \leq -4m$

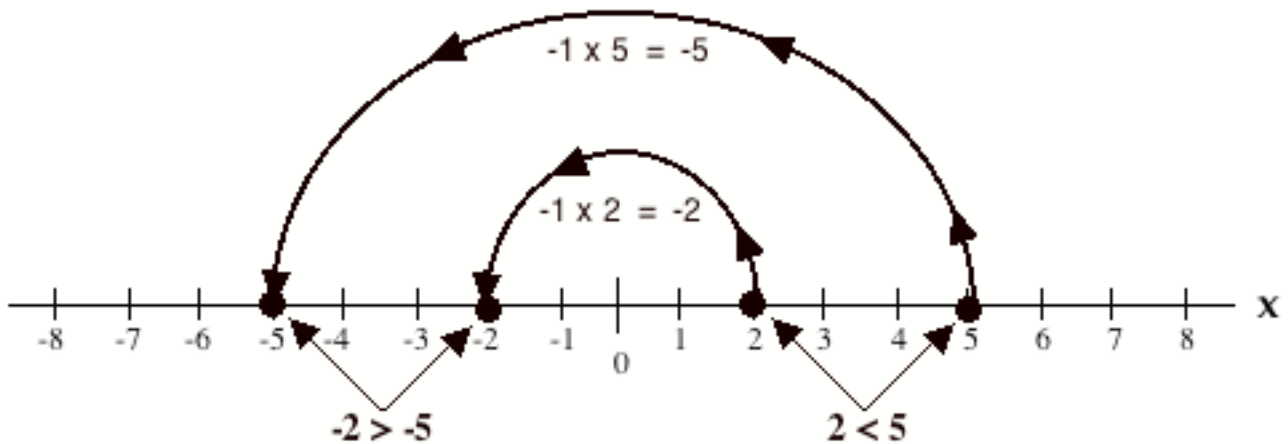
e)  $-x + 3 > 9$

f)  $-7 - y \geq -3y + 4$

To further illustrate the notion of how multiplying by a negative number changes the direction of the inequality sign, here are two diagrams showing what happens when inequalities are multiplied by  $-1$ .

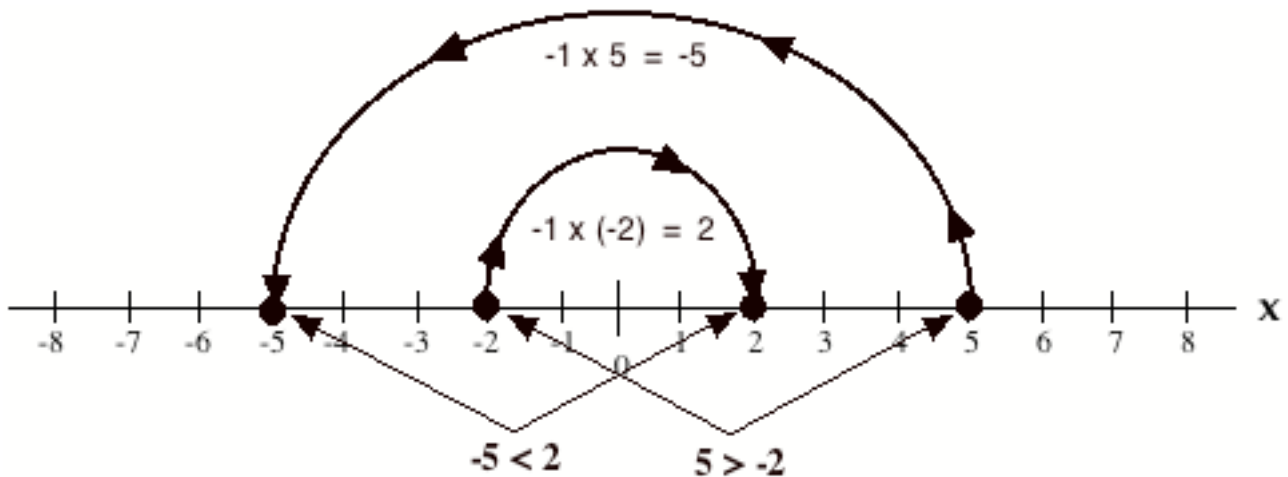
**Example A:** This first diagram shows the effects of multiplying each side of  $2 < 5$  by  $-1$ . Of course, after doing so, the numbers are both negative, but the most important thing to recognize is the change in the inequality sign:  $-2 > -5$ .

Why did this change take place? It all centers around zero (0) and the idea that, “The bigger they are the harder they fall.”



**Example B:** Here we start with  $5 > -2$ . One side is positive and the other is negative. After multiplying each side by  $-1$ , they come out as their opposites, on opposite sides from where they started. And, since no matter the numbers,

every positive  $>$  every negative                      So,  $5 > -2$   
 every negative  $<$  every positive.                      but  $-5 < +2$



## SOLVING AN INEQUALITY: THE GUIDELINES

The general guidelines for solving linear inequalities are shown below. Notice that, with little exception, these guidelines are virtually the same as those used to solve linear equations.

### Solving Linear Inequalities: The Ultimate Guidelines

#### **The Preparation:**

1. Eliminate any parentheses by distributing. (Be careful to look for negative multipliers.)
2. Clear any fractions or decimals by multiplying each side by the equation's LCD.
3. Combine like terms on each individual side.

#### **Isolating the Variable:**

4. If there is more than one variable term, get them together on the same side.
5. If the coefficient is negative, multiply each side by  $-1$  and change the direction of the inequality sign.
6. Clear the operations; start with the main operation.
7. If necessary, "switch sides" so that the variable term is on the left. (Don't forget to change the direction of the inequality sign.)
8. Graph the solution on the number line.

Here's a partial example using **Guidelines 4 and 5**:

If you are asked to solve the inequality	$2 + 3x < 5x - 8$
and if you add $-5x$ to each side ( <b>Guideline 4</b> ):	$\underline{-5x \leftrightarrow -5x}$
then, since the variable term, $-2x$ , is now negative, we	$2 - 2x < -8$
can multiply each side by $-1$ and change the	
direction of the inequality sign ( <b>Guideline 5</b> ):	$-2 + 2x > 8$
and so on....	(now isolate the variable)

Here's a partial example using **Guidelines 4 and 7**:

If you are asked to solve the inequality	$2 + 3x < 5x - 8$
and if you add $-3x$ to each side ( <b>Guideline 4</b> ):	$\underline{-3x \leftrightarrow -3x}$
then, since the variable term is on the right side,	$2 < 2x - 8$
we can switch sides and change the direction	
of the inequality sign ( <b>Guideline 7</b> ):	$2x - 8 > 2$
and so on....	(now isolate the variable)

By the way, in using either of these methods, the answer is the same:  $x > 5$ .

Let's put the guidelines into practice.

**Example 3:** Solve each of these inequalities. Be sure to write the solution with the *variable on the left* and draw its graph on a number line.

a)  $3x - 5 < x + 3$                       b)  $17 \leq 5w + 2$

**Answer:** Use only those guidelines, above, that are necessary. Be careful at the end; you may or may not need to change the direction of the inequality.

a)	$3x - 5 < x + 3$ $\begin{array}{r} -x \quad \leftrightarrow \quad -x \\ \hline 2x - 5 < 3 \end{array}$ $\begin{array}{r} +5 \leftrightarrow +5 \\ \hline 2x < 8 \end{array}$ $\frac{2x}{2} < \frac{8}{2}$ $x < 4$	<p>Get the variable terms together adding <math>-x</math> to each side.</p> <p>Get the constant terms together by adding <math>+5</math> to each side.</p> <p>The coefficient is positive. Great!</p> <p>Dividing each side by <math>2</math> and <math>x</math> is already on the left side.</p>
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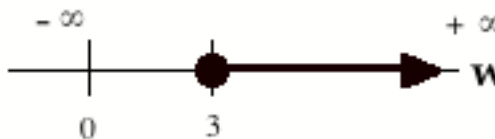
(The little arrows ( $\leftrightarrow$ ) help to separate the two sides.)

Now graph the solution. The inequality is *less than* but is *not* equal to, so the circle is an open circle:



b)	$17 \leq 5w + 2$ $5w + 2 \geq 17$ $\begin{array}{r} -2 \leftrightarrow -2 \\ \hline 5w \geq 15 \end{array}$ $\frac{5w}{5} \geq \frac{15}{5}$ $w \geq 3$	<p>Only one variable term, but it's on the right side; we can switch now (or later). Now add <math>-2</math> to each side.</p> <p>Dividing each side by <math>5</math>.</p> <p>This leaves us with <math>w</math> on the left side, just like we want it</p>
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Now graph the solution. The inequality is *greater than or equal to*, so the circle is a closed (solid) circle:



**Example 4:** Solve each of these inequalities. Be sure to write the solution with the *variable on the left* and draw its graph on a number line.

a)  $2(y - 6) - 6y > 16$                       b)  $-3b - 9 \geq 2b - \frac{3}{2}$

**Answer:** Use only those guidelines, above, that are necessary. Be careful at the end; you may or may not need to change the direction of the inequality.

a) 
$$\begin{aligned} 2(y - 6) - 6y &> 16 \\ 2y - 12 - 6y &> 16 \\ -4y - 12 &> 16 \\ \underline{+ 12 \leftrightarrow + 12} & \\ -4y &> 28 \\ 4y &< -28 \\ \frac{4y}{4} &< \frac{-28}{4} \\ y &< -7 \end{aligned}$$

First, clear the parentheses.

Next, combine like terms.

Now add +12 to each side.

The coefficient is negative, so let's multiply each side by -1 and change the inequality.

Divide each side by 4.

The variable is already on the left side.

Now graph the solution:



b) 
$$\begin{aligned} -3b - 9 &\geq 2b - \frac{3}{2} \\ 2(-3b - 9) &\geq 2\left(2b - \frac{3}{2}\right) \\ -6b - 18 &\geq 4b - 3 \\ \underline{+ 6b \leftrightarrow + 6b} & \\ -18 &\geq 10b - 3 \\ \underline{+ 3 \leftrightarrow + 3} & \\ -15 &\geq 10b \\ \frac{-15}{10} &\geq \frac{10b}{10} \\ \frac{-3}{2} &\geq b \\ b &\leq \frac{-3}{2} \end{aligned}$$

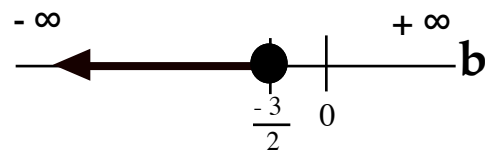
Clear the fraction; multiply each side by 2.

Let's add +6b to each side.

Now add +3 to each side.

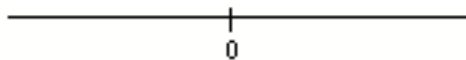
Now divide by 10.

Now switch sides and graph the solution.

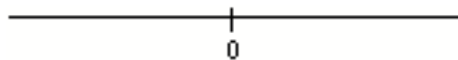


**Exercise 3:** Solve each of these inequalities. Be sure to write the solution with the *variable on the left* and draw its graph on the number line provided.

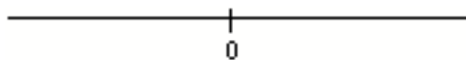
a)  $x + 5 > 9$



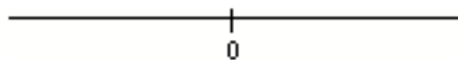
b)  $-y + 3 > 10$



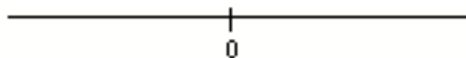
c)  $-5p < 10$



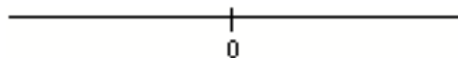
d)  $-4 \leq v - 8$



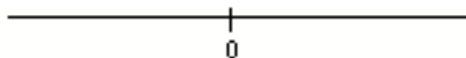
e)  $-8 \leq -4m$



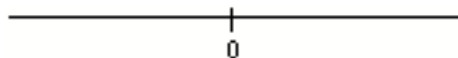
f)  $3 \geq 4 - w$



g)  $3x \geq 5x - 2$

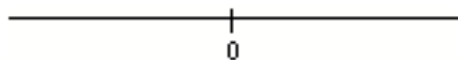
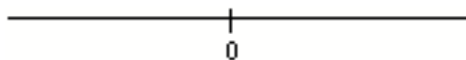


h)  $5v - 7 > 3v - 3$



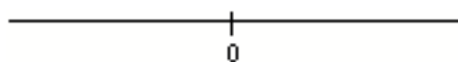
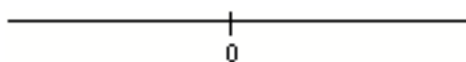
i)  $3(2n - 3) < 1 + 4n$

j)  $1 - 3p \leq 4(p + 2)$



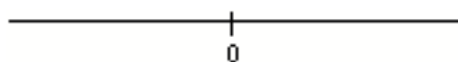
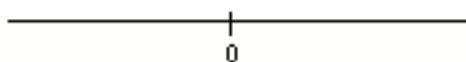
k)  $2(4 - u) + 1 > u + 6$

l)  $\frac{1}{3}m + \frac{1}{2} > 1 - \frac{2}{3}m$



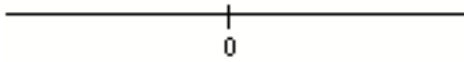
m)  $\frac{1}{2} - \frac{3}{5}c \leq \frac{2}{5}c - 1$

n)  $-7 - 3y \geq -2(y - 4)$

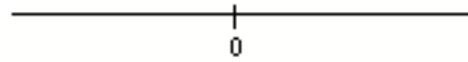


o)  $-(2x - 3) \leq 4(x - 3)$

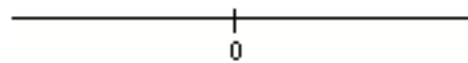
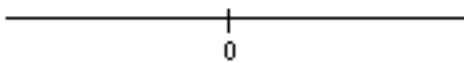
p)  $6\left(\frac{1}{2}x - \frac{2}{3}\right) < 8\left(\frac{5}{4} - \frac{1}{2}x\right)$



q)  $\frac{1}{4}(8y + 5) > \frac{1}{2}(2y - 1)$



r)  $\frac{1}{2}(4b - 3) - 5b \geq 3\left(b - \frac{3}{2}\right)$



## Answers to each Exercise

### Section 2.6

#### Exercise 1

- |                      |                      |                          |
|----------------------|----------------------|--------------------------|
| a) $x \leq 7$        | b) $w > 6$           | c) $y < -8$              |
| d) $x + 5 < 9$       | e) $d - 8 \geq -4$   | f) $5x - 2 \leq 3x$      |
| g) $4x + 8 > 1 - 3x$ | h) $6 - 4x < 2x + 3$ | i) $2p - 10 \geq 5 - 3p$ |

#### Exercise 2

- |                |                 |                        |
|----------------|-----------------|------------------------|
| a) $c \leq -6$ | b) $w > 1$      | c) $-10 < 5x$          |
| d) $8 \geq 4m$ | e) $x - 3 < -9$ | f) $7 + y \leq 3y - 4$ |

#### Exercise 3

(The graphs are not shown.)

- |                         |                       |                         |
|-------------------------|-----------------------|-------------------------|
| a) $x > 4$              | b) $y < -7$           | c) $p > -2$             |
| d) $v \geq 4$           | e) $m \leq 2$         | f) $w \geq 1$           |
| g) $x \leq 1$           | h) $v > 2$            | i) $n < 5$              |
| j) $p \geq -1$          | k) $u < 1$            | l) $m > \frac{1}{2}$    |
| m) $c \geq \frac{3}{2}$ | n) $y \leq -15$       | o) $x \geq \frac{5}{2}$ |
| p) $x < 2$              | q) $y > -\frac{7}{4}$ | r) $b \leq \frac{1}{2}$ |

## Section 2.6 Focus Exercises

1. Switch sides and change the direction of the inequality sign; **do not solve**.

a)  $-3 < x$

b)  $6 \geq y - 8$

c)  $5 - 4m > 2m - 3$

2. Multiply each side by -1 and change the direction of the inequality sign; **do not solve**.

a)  $-w < 9$

b)  $-x \geq -3$

c)  $12 \leq -4x$

d)  $-15 > -3y$

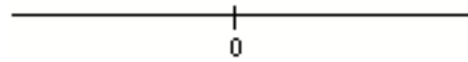
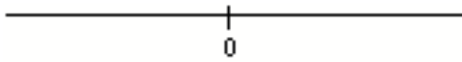
e)  $-2p + 7 \leq -9$

f)  $7 - k < -4k - 8$

3. Solve each of these inequalities. Be sure to write the solution with the *variable on the left* and draw its graph on the number line provided.

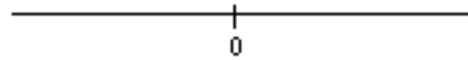
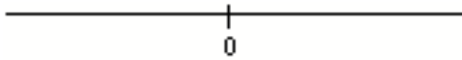
a)  $y - 3 \geq 2$

b)  $-x + 1 \leq -7$



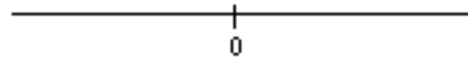
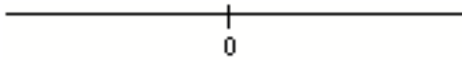
c)  $-3m > -6$

d)  $-10 < c - 6$



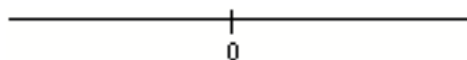
e)  $10 > -5p$

f)  $-6 \leq 2 - x$

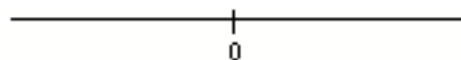


4. Solve each of these inequalities. Be sure to write the solution with the *variable on the left* and draw its graph on the number line provided.

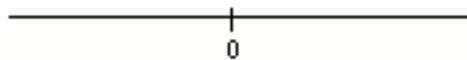
a)  $4x < 3x - 6$



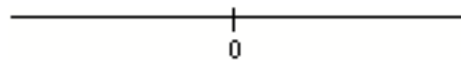
b)  $x - 6 \leq 2x + 5$



c)  $-2(y - 3) \geq 1 + 3y$



d)  $11 - 4x > 3(x - 1)$



e)  $\frac{3}{4}x + \frac{4}{3} \leq 1 + \frac{5}{6}x$

