

Section 5.1 Dividing by a Monomial

Before we can begin dividing polynomials, we need to be sure we have the proper preparation. What follows are some basic ideas with which you are already familiar, but they will help create the foundation used to develop our topic.

SIMPLIFYING FRACTIONS

We can simplify, or reduce, any fraction by recognizing common factors. These factors may be numerical or they may be variable.

Example 1: Simplify each fraction completely.

a) $\frac{18}{24}$

b) $\frac{3x^2}{5x}$

c) $\frac{10x^5}{2x^2}$

d) $\frac{-6p^5}{6p^4}$

Procedure: Divide out the factors of the numerator and denominator.

a) $\frac{18}{24} = \frac{3 \cdot 6}{4 \cdot 6} = \frac{3}{4}$

b) $\frac{3x^2}{5x} = \frac{3 \cdot x \cdot x}{5 \cdot x} = \frac{3x}{5}$

c) $\frac{10x^5}{2x^2} = \frac{2 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x}{2 \cdot x \cdot x} = \frac{5 \cdot x \cdot x \cdot x}{1} = 5x^3$

or, using the quotient rule of exponents: $\frac{10x^5}{2x^2} = \frac{10x^{5-2}}{2} = 5x^3$

d) $\frac{-6p^5}{6p^4} = \frac{-6}{6} \cdot \frac{p^5}{p^4} = -1 \cdot p^{5-4} = -p$

Exercise 1

Simplify each fraction completely.

a) $\frac{35}{14}$

b) $\frac{12y^2}{6y^2}$

c) $\frac{14x^3}{-7x}$

d) $\frac{3c^4}{3c}$

e) $\frac{-15p^2}{3p^2}$

f) $\frac{-8m^6}{-2m}$

DIVIDING BY A MONOMIAL

Just as we can distribute using multiplication, as in

$$\begin{aligned} & 2(x + 5) \\ &= 2 \cdot x + 2 \cdot 5 \\ &= 2x + 10 \end{aligned}$$

we can also distribute using division. For example, consider $(8 + 6) \div 2$ (which is $14 \div 2 = 7$)

$$\begin{aligned} & (8 + 6) \div 2 \\ &= 8 \div 2 + 6 \div 2 \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

This division, and others like it, can also be expressed as a fraction; the distribution would be the same:

$$\begin{aligned} & \frac{8 + 6}{2} \\ &= \frac{8}{2} + \frac{6}{2} \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

As you can see, in this type of distribution, the denominator gets distributed to every term in the numerator.

This type of distribution can also be called **separating the fraction**.

This is more evident when we see a polynomial, such as $15x^3 + 10x^2$ divided by $5x$. Written as a fraction this becomes

$$\begin{aligned} & \frac{15x^3 + 10x^2}{5x} \\ \text{Distribute the denominator to both terms in the numerator} &= \frac{15x^3}{5x} + \frac{10x^2}{5x} \\ \text{Simplify each individual fraction} &= 3x^2 + 2x \end{aligned}$$

Example 2: Divide using distribution.

a) $\frac{12y^5 - 8y^2}{4y^2}$

b) $\frac{21m^3 + 9m^2 - 6m}{-3m}$

Procedure: Distribute the denominator then simplify each fraction.

a) $\frac{12y^5 - 8y^2}{4y^2}$

$$\begin{aligned} &= \frac{12y^5}{4y^2} - \frac{8y^2}{4y^2} \\ &= 3y^3 - 2 \end{aligned}$$

b) $\frac{21m^3 + 9m^2 - 6m}{-3m}$

$$\begin{aligned} &= \frac{21m^3}{-3m} + \frac{9m^2}{-3m} - \frac{6m}{-3m} \\ &= -7m^2 - 3m + 2 \end{aligned}$$

Exercise 2

Divide using distribution.

a) $\frac{6x^4 + 12x^2}{6x}$

b) $\frac{25y^7 - 10y^5}{5y^3}$

c) $\frac{4p^7 - 2p^6}{-2p^4}$

d) $\frac{24m^6 - 4m^4 + 16m^2}{4m^2}$

e) $\frac{-14c^3 - 21c^2 + 7c}{-7c}$

f) $\frac{3x^2y^4 - 15xy^3 + 6xy^2}{3xy^2}$

Answers to each Exercise**Section 5.1**

Exercise 1: a) $\frac{5}{2}$ b) 2 c) $-2x^2$ d) c^3

e) -5 f) $4m^5$

Exercise 2: a) $x^3 + 2x$ b) $5y^4 - 2y^2$ c) $-2p^3 + p^2$
d) $6m^4 - m^2 + 4$ e) $2c^2 + 3c - 1$ f) $xy^2 - 5y + 2$

Section 5.1 Focus Exercises

1. Divide using distribution.

a) $\frac{5x^4 + 15x^2}{5x}$

b) $\frac{27w^7 - 9w^3}{-9w^3}$

c) $\frac{24m^9 - 3m^6 + 36m^3}{3m^3}$

d) $\frac{8v^8 - 16v^4 + 4v^2}{-8v^2}$

e) $\frac{-14cd^4 - 2c^2d^3 - 10c^3d^2}{-2cd}$

f) $\frac{6a^4b^3 - 12a^3b + 3a^2b^2}{6a^2b}$