2.3 Solving Equations Containing Fractions and Decimals

Objectives

In this section, you will learn to:

- Solve equations containing fractions
- Solve equations containing decimals

To successfully complete this section, you need to understand:

- Operations with real numbers (Chapter 1)
- Combining like terms (1.8)
- The Distributive Property (1.9)
- Solving linear equations (2.1 and 2.2)
- Finding the least common denominator (R.4)

INTRODUCTION

In Section 2.1 we solved equations that contained fractions. For example,

To solve $w - \frac{2}{5} = \frac{8}{5}$, we add $\frac{2}{5}$ to each side:	To solve $15 = \frac{-5}{8} y$, we multiply each side by $\frac{-8}{5}$:
$w - \frac{2}{5} = \frac{8}{5}$	$15 = \frac{-5}{8} y$
$w - \frac{2}{5} + \frac{2}{5} = \frac{8}{5} + \frac{2}{5}$	$\frac{-8}{5} \cdot \frac{15}{1} = \frac{-8}{5} \cdot \frac{-5}{8} y$
$w + 0 = \frac{10}{5}$	$\frac{-8}{1} \cdot \frac{3}{1} = 1y$
w = 2	-24 = y
	y = -24

In some equations, though, it is easier—and more efficient—to clear any and all fractions, thereby making all of the constants and coefficients into integers.

For example, the equation $\frac{x}{9} + \frac{4}{3} = \frac{1}{2}x - 1$ is easier to solve after its fractions are cleared and it has only integer constants and coefficients: 2x + 24 = 9x - 18.

What is it that allows us to transform $\frac{x}{9} + \frac{4}{3} = \frac{1}{2}x - 1$ into 2x + 24 = 9x - 18? It is

- 1. our ability to find the least common denominator (LCD) for the three fractions (the LCD is 18);
- 2. our ability to multiply integers and fractions;
- 3. our ability to use the Distributive Property; and
- 4. our ability to apply the Multiplication Property of Equality (Refer to Section 2.1.)

Applying the Multiplication Property of Equality to an equation such as $\frac{x}{9} + \frac{4}{3} = \frac{1}{2}x - 1$ requires that we first prepare the equation by grouping each side as one quantity, using parentheses:

$$\left(\frac{x}{9} + \frac{4}{3}\right) = \left(\frac{1}{2}x - 1\right)$$

It is then that we can apply the Multiplication Property of Equality and multiply each side by 18:

$$18 \cdot \left(\frac{x}{9} + \frac{4}{3}\right) = 18 \cdot \left(\frac{1}{2}x - 1\right)$$

The solving of this equation will be completed later in this section. To learn the process, let's start with some simpler equations.

EQUATIONS CONTAINING FRACTIONS

Let us start with an equation that contains just one fraction, $2x - 1 = \frac{3}{4}x + 9$. It is possible to solve this equation by first adding $-\frac{3}{4}x$ to each side, but to avoid the time-consuming work involved with adding fractions, it is often helpful to first *clear the fraction*—or *clear the denominator*—and work only with integers.

Caution: "Clearing the fractions" requires us to multiply each full side of the equation—each and every term— by the same value, the common denominator. We do not multiply only the terms containing fractions.

As stated above, and as shown in Example 1, we must prepare the equation for multiplication by grouping each side using parentheses:

$$(2x-1) = \left(\frac{3}{4}x+9\right)$$

Example 1: Solve this equation by f	Example 1: Solve this equation by first clearing the fraction(s).			
$2x - 1 = \frac{3}{4}x + 9$				
Procedure: There is only one fraction fraction.	on. The LCD is 4, so multiply each	h side by 4 to clear the		
Answer: $2x - 1 = \frac{3}{4}x + 9$	Prepare the equation by placing p	parentheses around each side.		
$(2x-1) = \left(\frac{3}{4}x+9\right)$	Multiply each side by 4.			
$4(2x-1) = 4\left(\frac{3}{4}x+9\right)$	Distribute 4 on each side, to each to the fraction $\frac{3}{4}x$, it is helpful to	1		
$4 \cdot 2x - 4 \cdot 1 = \frac{4}{1} \cdot \frac{3}{4} x + 4 \cdot 9$	$\frac{4}{1} \cdot \frac{3}{4} x$ simplifies to $3x$.			
8x - 4 = 3x + 36	Reduce this to standard form by adding $-3x$ to each side.	Verify the solution, 8:		
-3x = -3x		$2x - 1 = \frac{3}{4}x + 9$		
5x - 4 = 36 <u>+4 = +4</u>	Isolate the variable term by adding +4 to each side.	$2(8) - 1 = \frac{3}{4}(8) + 9$		
5x = 40	Divide each side by 5.	$16 - 1 = \frac{3}{4} \cdot \frac{8}{1} + 9$		
$\frac{5x}{5} = \frac{40}{5}$	Simplify.	$15 = \frac{3}{1} \cdot \frac{2}{1} + 9$		
<i>x</i> = 8	Verify $x = 8$.	$\frac{?}{15} = 6 + 9$		
		15 = 15 ✓		

Note: The two steps of

1. preparing the equation for multiplication by placing parentheses around each side, and

2. showing the multiplication by the LCD

can be combined into one step, just as they are in the next example.

If an equation contains more than one fraction, then to clear the fractions, we can multiply by a common denominator of all of the fractions (usually, the least common denominator—LCD). If the fractions already have a common denominator, then we multiply each side by that common denominator, as shown in Example 2.

Example 2:	Example 2: Solve this equation by first clearing the fractions.			
	$\frac{3w}{2} + 1 = w + \frac{9}{2}$			
Procedure:	There is only one denominato	r, 2. Multiply each side by 2 to	o clear the fractions.	
Answer:	$\frac{3w}{2} + 1 = w + \frac{9}{2}$	Prepare the equation by placing pa around each side. Multiply each s		
2	$\frac{3w}{2}+1\Big) = 2\left(w+\frac{9}{2}\right)$	Distribute 2 to each term on each Write 2 as $\frac{2}{1}$ when multiplying the		
$\frac{2}{1}\cdot\frac{3w}{2}$	$+ 2 \cdot 1 = 2 \cdot \mathbf{w} + \frac{2}{1} \cdot \frac{9}{2}$	$\frac{2}{1} \cdot \frac{3w}{2}$ simplifies to $3w$; $\frac{2}{1} \cdot \frac{9}{2}$ s	implifies to 9.	
	3w + 2 = 2w + 9 -2w = -2w	Reduce this to standard form by adding $-2w$ to each side.	Verify the solution, 7:	
	w + 2 = 9 -2 = -2	Isolate the variable term by adding -2 to each side.	$\frac{3w}{2} + 1 = w + \frac{9}{2}$	
	w = 7	Verify $w = 7$.	$\frac{3(7)}{2} + 1 \stackrel{?}{=} 7 + \frac{9}{2}$	
			$\frac{21}{2} + \frac{2}{2} \stackrel{?}{=} \frac{14}{2} + \frac{9}{2}$	
			$\frac{23}{2} = \frac{23}{2} \checkmark$	

You Try It 1Solve each equation by first clearing the fractions. Verify the solution. Use Examples1 and 2 as guides.

a)
$$\frac{x}{3} = x + 4$$
 b) $m - 3 = \frac{4}{5}m - 2$

c)
$$\frac{x}{5} - 4 = 2 - \frac{2x}{5}$$
 d) $\frac{1}{2} + w = 8 - \frac{3w}{2}$

FRACTIONS WITH DIFFERENT DENOMINATORS

If the denominators are different, we must identify the LCD before we multiply. Then, to clear the fractions, we must multiply each side by the LCD.

Example 3:	Solve the equation by first	Solve the equation by first clearing the fractions.			
	$\frac{x}{3} + 1 = \frac{5x}{6} - 3$				
Procedure:	First identify the LCD, the the fractions.	en multiply each side of	the equation by the LCD to clear		
Answer:					
$\frac{x}{3}$	$+1 = \frac{5x}{6} - 3$	The LCD is 6. Prepare the parentheses around each signature the parentheses around each signature the second each second each signature the second each se	equation by placing de. Multiply each side by 6.		
$6\left(\frac{x}{3}+\right)$	$(-1) = 6\left(\frac{5x}{6} - 3\right)$	Distribute 6, or $\frac{6}{1}$, to each	term on each side.		
$\frac{6}{1}\cdot\frac{x}{3}+6$	$6 \cdot 1 = \frac{6}{1} \cdot \frac{5x}{6} - 6 \cdot 3$	Simplify.			
2x	+6 = 5x - 18	Reduce this to standard for	m by adding $-2x$ to each side.		
-2x	= -2x	Simplify.			
	6 = 3x - 18	Isolate the variable term by	adding 18 to each side.		
±	18 = +18	Simplify.	<u>You finish it:</u>		
	24 = 3x	Divide each side by 3.	Verify that 8 is the solution.		
	$\frac{24}{3} = \frac{3x}{3}$				
	8 = x				
	<i>x</i> = 8	Verify $x = 8$.			

Example 4:	4: Solve the equation by first clearing the fractions. $\frac{5}{8} + \frac{1}{6}y = \frac{5}{12} + y$			
Procedure:	First identify the LCD, the	n multiply each side of the eq	uation by the LCD to clear	
	the fractions. Note: The c	denominator of the term y is 1	$\frac{y}{1}$.	
Answer:				
$\frac{5}{8}$	$+\frac{1}{6}y = \frac{5}{12} + \frac{y}{1}$	The LCD is 24. Prepare the equat parentheses around each side. Mu		
$24\left(\frac{5}{8}+\right)$	$-\frac{1}{6y} = 24\left(\frac{5}{12} + \frac{y}{1}\right)$	Distribute 24, or $\frac{24}{1}$, to each term	n on each side.	
$\frac{24}{1} \cdot \frac{5}{8} + \frac{24}{1}$	$\cdot \frac{1}{6}y = \frac{24}{1} \cdot \frac{5}{12} + \frac{24}{1} \cdot \frac{y}{1}$	$\frac{y}{1}$ Simplify.		
15	+ 4y = 10 + 24y	Reduce this to standard form by a	dding -4y to each side.	
	-4y = -4y		Verify the solution, $\frac{1}{4}$:	
	15 = 10 + 20y	Isolate the variable term by	$\frac{5}{8} + \frac{1}{6} \cdot \frac{1}{4} = \frac{5}{12} + \frac{1}{4}$	
	-10 = -10	adding -10 to each side.	0 0 7 12 7	
	5 = 20y	Divide each side by 20.	$\frac{5}{8} + \frac{1}{24} \stackrel{?}{=} \frac{5}{12} + \frac{1}{4}$	
	$\frac{5}{20} = \frac{20y}{20}$	Simplify.	$\frac{15}{24} + \frac{1}{24} \stackrel{?}{=} \frac{5}{12} + \frac{3}{12}$	
	$\frac{1}{4} = y$	Write the variable on the left.	$\frac{16}{24} \stackrel{?}{=} \frac{8}{12}$	
	$y = \frac{1}{4}$	Verify $y = \frac{1}{4}$.	$\frac{2}{3} = \frac{2}{3} \checkmark$	

You Try It 2Solve each equation by first identifying the LCD and clearing the fractions. Verify
the solution. Use Examples 3 and 4 as guides.

a)	$\frac{3y}{4} - 6 =$	$\frac{y}{8}$ + 4	b)	$p - \frac{p}{6}$	$=\frac{p}{3}$	+ 2
<i>a)</i>	4 - 0 -	8 + +	0)	P = 6	- 3	Τ 4

c)
$$\frac{3x}{20} + \frac{1}{10} = \frac{x}{4} - \frac{1}{5}$$
 d) $\frac{w}{4} + \frac{11}{12} = \frac{1}{2} - \frac{w}{6}$

EQUATIONS CONTAINING DECIMALS

Recall (1.2) that terminating decimals are rational numbers (fractions) in which the denominators are powers of 10, such as 10, 100, and so on. For example, $0.3 = \frac{3}{10}$ and $0.25 = \frac{25}{100}$.

Consider an equation that contains these two fraction: $\frac{3}{10}x = \frac{25}{100}x + 1$. We can clear the fractions by multiplying each side by the LCD of 100, changing it to an equation of integers: 30x = 25x + 100.

If this same equation is written with decimals instead of fractions, it would be 0.3x = 0.25x + 1. Because this is the same equation, we also can multiply each side by 100, but this time we will *clear the decimals*.

One major distinction, when clearing decimals, is to prepare the equation by first *writing each constant* and coefficient with the same number of decimal places.

For example, each number in the equation 0.3x = 0.25x + 1 can be written with two decimal places:

- For 0.3, we can place one zero at the end of the number: 0.3 = 0.30
- For 1, we can place a decimal point and two zeros at the end of the number: 1 = 1.00
- 0.25 already has two decimal places, so no change is necessary.

The equation becomes 0.30x = 0.25x + 1.00. Now having two decimal places, each number is in terms of *hundredths*, and we can clear the decimals by multiplying each side by 100:

100(0.30x) = 100(0.25x + 1.00) Multiplying by 100 has the effect of moving the decimal point two places to the right.

30x = 25x + 100

It is now an equation of integers, and we can solve it using the techniques learned earlier in this chapter.

Preparing an equation by creating an equal number of decimal places is an important first step when clearing decimals in an equation.

Example 5: For each equation,					
Prepare	e the equ which p	ation by build	aces each constant and coefficient should ing up each number, as necessary. ould be multiplied to each side of the equ		
	a)	0.4x - 1.2 =	0.15x + 0.8 b) $0.12y - 1 = 0.12y$	0.095y - 0.9	
Procedure:		· · · ·	he constant or coefficient with the highes many decimal places each number shou		
	a) 0.15 has two decimal places, so we should build up each number to have two decimal places				
b) 0.095 has three decimal places, so we should build up each number to have three decimal places					
Number ofMultiplydecimal placesPrepared equationeach side by					
Answer:	a)	Two	0.40x - 1.20 = 0.15x + 0.80	100	
	b)	Three	0.120y - 1.000 = 0.095y - 0.900	1,000	

You Try It 3 For each equation,

- Decide how many decimal places each constant and coefficient should have;
- Prepare the equation by building up each number, as necessary; and
- Decide which power of 10 should be multiplied to each side of the equation to clear the decimals.

		Number of decimal places	Prepared equation	Multiply each side by
a)	2w - 0.4 = 1 + 1.8w			
b)	0.17k - 0.43 = 0.25k + 0.05			
c)	0.27v - 1.6 = 0.32v - 2			
d)	0.1x - 0.006 = 0.08x + 0.134			

Example 6:	Solve the equation by first cl	learing the decimals.	
	a) $0.4x - 1.2 = 0.15x + 0.3$	8 b) 0.12y –	1 = 0.095y - 0.9
Procedure:	Use the guidelines from the p decimals.	previous example to prepa	are the equation for clearing the
Answer:			
a) 0.4 <i>x</i>	x - 1.2 = 0.15x + 0.8	Write each decimal so that it has two decimal places.	
0.40 <i>x</i> -	-1.20 = 0.15x + 0.80	Prepare the equation by placi around each side. Multiply ea	
100(0.40 <i>x</i> –	1.20) = 100(0.15x + 0.80)	Distribute. Multiplying by 10	0 will clear all decimals.
	-120 = 15x + 80	Reduce this to standard form	by adding $-15x$ to each side.
25 <i>x</i>	= -15x - 120 = 80	Isolate the variable term by a	dding 120 to each side.
	+120 = +120		Verify the solution, 8:
	25x = 200	Divide each side by 25.	$\begin{array}{rcl} ?\\ 0.4(8) - 1.2 &=& 0.15(8) + 0.8 \end{array}$
	$\frac{25x}{25} = \frac{200}{25}$	$200 \div 25 = 8$? 3.2 - 1.2 = 1.20 + 0.8
	x = 8	Verify $x = 8$.	2.0 = 2.0 🗸

b) 0.12	2y - 1 = 0.095y - 0.9	Write each decimal so has three decimal plac	
0.120y - 1	1.000 = 0.095y - 0.900	Prepare the equation b around each side. Mul	y placing parentheses tiply each side by 1,000.
1,000 (0.120y – 1.0	(000) = 1,000(0.095y -	- 0.900) Distribute. Mul 1,000 will clear	
120y – 1	,000 = 95y - 900	Reduce this to standar	d form by adding -95y to each side.
<u>-95y</u>	= -95y		
25y - 1	,000 = -900	Isolate the variable ter	m by adding 1,000 to each side.
<u>+1</u>	<u>,000 = +1,000</u>		<u>You finish it:</u>
	25y = 100	Divide each side by 25.	Verify that 4 is the solution.
	$\frac{25y}{25} = \frac{100}{25}$	$100 \div 25 = 4$	
	y = 4	Verify $y = 4$.	
L			

You Try It 4Solve the equation by first clearing the decimals. Verify the solution. Use Example 6
as a guide.

b) 2w - 0.4 = 1 + 1.8w b) 0.17k - 0.43 = 0.25k + 0.05

c) 0.27v - 1.6 = 0.32v - 2

GUIDELINES FOR SOLVING EQUATIONS

Here is a summary of the steps involved in solving a variety of linear equations. Not all steps will be necessary for each equation; you should apply the guidelines in the order presented here but may skip any guideline that does not apply. For example, if an equation has no fractions or decimals, you may skip guideline (2) and proceed to guideline (3).

Guidelines for Solving Linear Equations

The Preparation:

- 1. Eliminate any parentheses by distributing.
- 2. Clear any fractions or decimals by multiplying each side by the equation's LCD.
- 3. Combine like terms on each individual side.

Isolating the Variable:

- 4. If necessary, reduce the equation to standard form.
- 5. If necessary, isolate the variable term and then finish solving.

The guidelines say that parentheses should be cleared first. This is true even if fractions are involved; in other words, if an equation has both fractions (or decimals) and parentheses, then it is best to clear the parentheses before trying to clear any fractions (or decimals).

Example 7:	Solve each equation and	verify the solution.	
	$\frac{1}{2}\left(x+\frac{2}{3}\right) = 3(x-1)$		
Procedure:	First distribute, then clea	ar the fractions or decimals.	
Answer: $\frac{1}{2}(,$	$\left(x+\frac{2}{3}\right) = 3(x-1)$	Distribute and simplify; $\frac{1}{2} \cdot \frac{2}{3}$	$=\frac{1}{3}$
$\frac{1}{2}x$	$x + \frac{1}{3} = 3x - 3$	The LCD is 6. Prepare the equat parentheses around each side. M	tion by placing Iultiply each side by 6.
$\frac{6}{1}(\frac{1}{2})$	$\frac{6}{1}\left(\frac{1}{2}x + \frac{1}{3}\right) = 6(3x - 3)$ Simplify: $\frac{6}{1} \cdot \frac{1}{2}x = 3x$ and $\frac{6}{1} \cdot \frac{1}{3} = 2$.		
	4x + 2 = 18x - 18	Reduce this to standard form by	adding $-3x$ to each side.
	$\frac{3x}{2} = \frac{-3x}{15x - 18}$	Isolate the variable term by addi	-
	+18 = +18 20 = 15x	Divide each side by 15.	Verify the solution $\frac{4}{3}$: $\frac{1}{2}\left(x+\frac{2}{3}\right) = 3(x-1)$
	$\frac{20}{15} = \frac{15x}{15}$	Simplify.	$\frac{1}{2}\left(\frac{4}{3} + \frac{2}{3}\right) = 3\left(\frac{4}{3} - 1\right)$
	$\frac{4}{3} = x$		$\frac{1}{2}\left(\frac{6}{3}\right) = 3\left(\frac{4}{3} - \frac{3}{3}\right)$
	$x = \frac{4}{3}$	Verify $x = \frac{4}{3}$	$\frac{6}{6} = \frac{3}{1} \left(\frac{1}{3} \right)$ $1 = 1 \checkmark$

Example 8:	Solve each equation	on and verify	the solution.	
	0.2(3y-5) = 0.13	5(2y+3) - 0.	85	
Procedure:	First distribute, the	en clear the fr	actions or decimals	S.
Answer: 0.2((3y-5) = 0.15(2y +	- 3) - 0.85	Distribute.	
0.6	6y - 1.0 = 0.30y + 0	0.45 – 0.85	Write each decimal it has two decimal p	
0.60y	y - 1.00 = 0.30y + 0	0.45 – 0.85		n by placing parentheses Iultiply each side by 100.
100(0.60 <i>y</i>	-1.00 = (0.30y +	0.45 – 0.85) 1	.00	
60	y - 100 = 30y + 45	5 – 85	Combine like terms	on the right side.
60	y - 100 = 30y - 40)	Reduce this to stand	lard form by adding -30y to each side.
<u>-30</u>	y = -30y			You finish it:
30	y - 100 = -40	Add 10	00 to each side.	Verify that 2 is the solution.
	+100 = +100			
	30y = 60	Divide	e each side by 30.	
	$\frac{30y}{30} = \frac{60}{30}$			
	<i>y</i> = 2	Verify	<i>y</i> = 2. <i>▶</i>	

You Try It 5 Solve each equation and verify the solution. Use Examples 7 and 8 as guides.

a)
$$\frac{1}{2}(2h-1) = \frac{1}{3}\left(2h+\frac{1}{2}\right)$$
 b) $0.5(p+3) = 3(0.1+0.16p)$

c)
$$\frac{1}{8}(3y+2) = \frac{1}{4}(2y+\frac{1}{2}) + \frac{1}{2}$$
 d) $0.6(10n-3) = 1.5(n+2) - 0.3$

You Try It Answers

You Try It 1:	a)	x = -6	b)	m = 5	c)	x = 10	d)	w = 3
You Try It 2:	a)	<i>y</i> = 16	b)	p = 4	c)	x = 3	d)	w = -1
You Try It 3:	a)	One; $2.0w - 0.4 = 1.0 + 1.8w$; 10						
	b)	Two; $0.17k - 0.43 = 0.25k + 0.05$; 100						
	c)	Two; $0.27v - 1.60 = 0.32v - 2.00; 100$						
	d)	Three; $0.100x - 0.006 = 0.080x + 0.134$; 1,000						
You Try It 4:	a)	w = 7	b)	k = -6	c)	v = 8	d)	<i>x</i> = 7
You Try It 5:	a)	h = 2	b)	p = -60	c)	<i>y</i> = -3	d)	n = 1

Section 2.3 Exercises

Think Again.

- 1. Consider the equation $2x + 1 = \frac{1}{4} \left(\frac{1}{2}x + 4 \right)$. What is the least common denominator on the right side of this equation?
- **2.** If an equation contains decimals, why is it helpful for all of the constants and coefficients to have the same number of decimal places?

Focus Exercises.

Solve each equation. Verify your answer.

 $x + \frac{3x}{4} = 7$ 4. $y - \frac{y}{4} = 12$ 3. $z + \frac{8}{5} = \frac{z}{5}$ 6. $\frac{7}{4}h = \frac{1}{4}h - 12$ 5. $w + \frac{1}{7} = \frac{6w}{7} - 1$ 8. $\frac{y}{8} + 6 = 6 - \frac{5y}{8}$ 7. $2 - \frac{n}{8} = 4n + \frac{5}{8}$ 10. $\frac{5y}{2} - 9 = \frac{2y}{3} + 2$ 9. $1 - \frac{5}{8}x = 2 - \frac{2}{3}x$ 12. $\frac{2r}{3} + \frac{1}{9} = 2r - \frac{2r}{9}$ 11. $\frac{3x}{5} + \frac{1}{6} = \frac{x}{2} - \frac{1}{3}$ $\frac{1}{3}x + \frac{3}{5}x = \frac{9}{10}x - \frac{1}{15}$ 14. 13. 0.6x - 3.2 = 0.4 - 0.3x16. 0.2x + 0.5 = 0.7x - 415. 0.2v - 0.3 = 0.4 - 0.5v17. 18. 1.6 + 0.9w = 7.6 + 2.4w0.4p - 0.9 = -1.26 + 0.28p19. 0.128 - 0.035v = 0.072v + 0.23520. 0.51x - 2.2 = 0.6x + 2.30.48x - 1.4 = 0.54x - 221. 22. $\frac{1}{5}(5x-3) = \frac{2}{3}\left(x+\frac{3}{5}\right)$ 24. $\frac{1}{6}(1-6x) = \frac{1}{3}\left(6x+\frac{1}{2}\right)$ 23.

Think Outside the Box:

Solve each.

25.
$$\frac{2x-18}{4} = \frac{3x+1}{2}$$
 26. $\frac{x+9}{5} = \frac{x-7}{10}$

27.
$$\frac{x+7}{8} - \frac{x}{2} = 5$$
 28. $\frac{x-5}{6} = \frac{x}{4} - 1$