Section 2.7 Solving Linear Inequalities

Objectives
In this section, you will learn to:

- Add and multiply an inequality.
- Multiply an inequality by -1.
- Switch the sides of an inequality.
- Solve inequalities

To successfully complete this section, you need to understand:

- Solving equations (2.1, 2.2, and 2.3)
- Graphing the solution set of an inequality on the number line (2.6)

INTRODUCTION

In this section, we will learn how to solve an inequality statement, such as $5 - 3x > x - 19$. Just as with equations, the goal is to isolate the variable.

When completely isolated, the inequality will have a single variable on the left side and a constant on the right.

We conclude this by graphing the solution set on the number line, just as we learned in Section 2.6.

In working toward isolating the variable, the intermediate steps are other inequality statements that have the same solution as the others. Together, these reduced inequalities are called **equivalent inequalities**.

$$5 - 3x > x - 19$$

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Some steps occur here, reducing the inequality until the variable is isolated on the left side.

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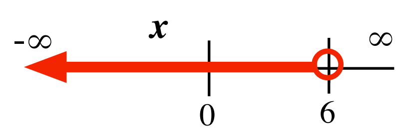
$$x < 6$$



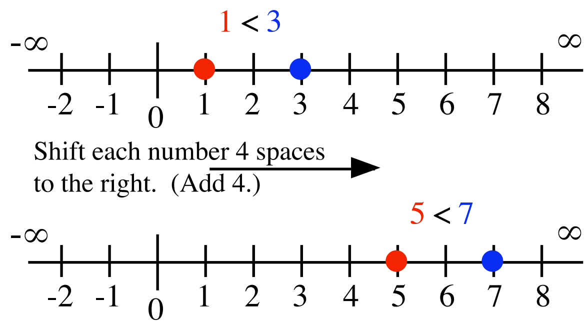
Solving an inequality is much like solving an equation. However, there are some special circumstances that you must be aware of.

In solving an inequality it is possible to *change the direction of the inequality sign*. This means that it is possible to change, for example, a *greater than* sign to a *less than* sign. It doesn't happen arbitrarily, though, and you must be careful whenever doing so.

Before solving inequalities directly, we must first prepare to solve them by learning more about inequality statements and how we can alter them step by step.

MAINTAINING AN INEQUALITY'S STATUS

If one number is less than another, such as $1 < 3$, then we can maintain the inequality's "less than" status by shifting each number by, say, 4 spaces to the right.

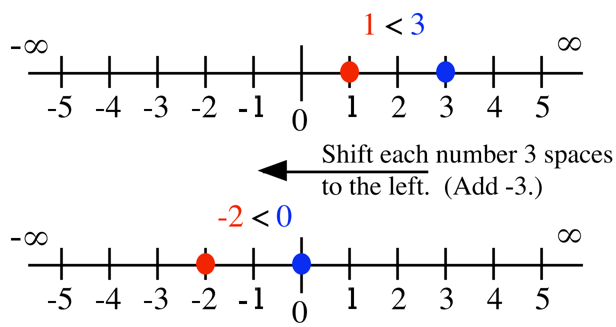


This is the same as adding 4 to each side of the inequality to get

$$1 + 4 \ < \ 3 + 4$$

$$5 \ < \ 7 \ \text{(still less than)} \ \nearrow$$

Starting, again, with $1 < 3$, we can also maintain less than status by shifting the same two numbers three spaces to the left:



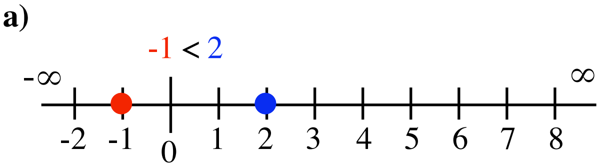
This is the same as adding -3 to each side of the inequality to get

$$1 + (-3) \ < \ 3 + (-3)$$

$$-2 \ < \ 0 \ \text{(still less than)} \ \nearrow$$

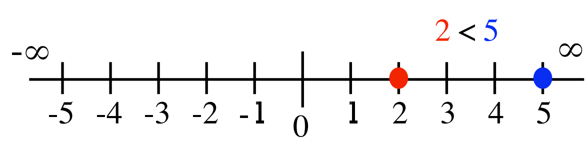
Example 1: Given two points on a number line, shift each value the same number of spaces, according to the directions. State the resulting inequality.

a)



Shift each value 5 spaces to the right. (Add 5.)

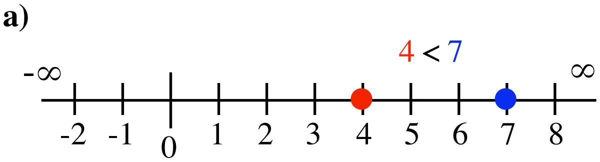
b)



Shift each value 6 spaces to the left. (Add -6.)

Answer:

a)



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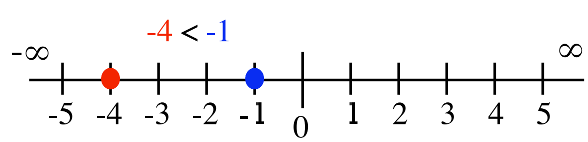
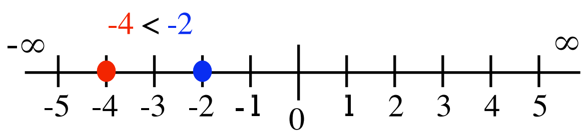


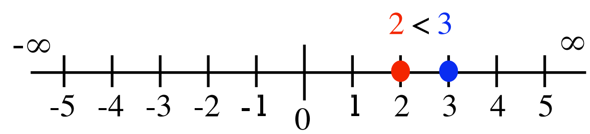
You Try It 1 Given two points on a number line, shift each value the same number of spaces, according to the directions. State the resulting inequality. Use Example 1 as a guide. (Use the same given number line for your answer.)

a)



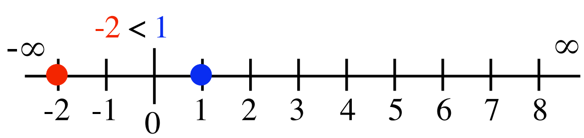
Shift each value 6 spaces to the right. (Add 6.)

b)



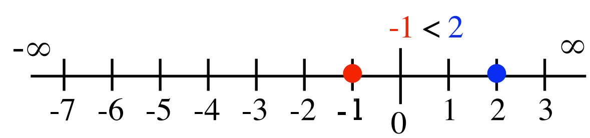
Shift each value 4 spaces to the left. (Add -4.)

c)



Shift each value 5 spaces to the right. (Add 5.)

d)



Shift each value 5 spaces to the left. (Add -5.)

Example 1 and You Try It 1 illustrate a property of inequalities that we will use when solving inequality statements later in this section:

The Addition Property of Inequalities

For all values of a, b, and c,

If $\quad a < b$,

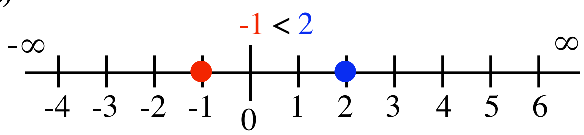
then $\quad a + c < b + c$

This property is true for all inequality forms ($<$, $>$, $\leq$, $\geq$, and $\neq$).

We can also maintain an inequality's status by doubling or tripling each value. In other words, we can find positive multiples of each side without changing the inequality's status. Likewise we can find positive fractional values of each number without changing the inequality's status.

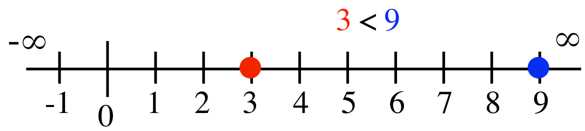
Example 2: Given two points on a number line, find the multiple or fraction of each value, according to the directions. State the resulting inequality.

a)



Triple each number. (Multiply by 3.)

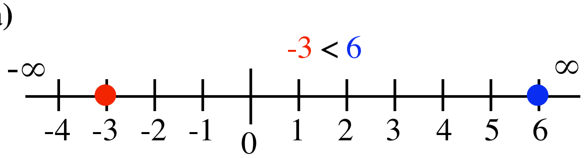
b)



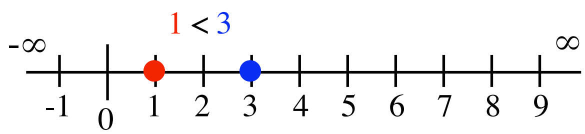
Find one-third of each number.
(Multiply by $\frac{1}{3}$, or divide by 3.)

Answer:

a)

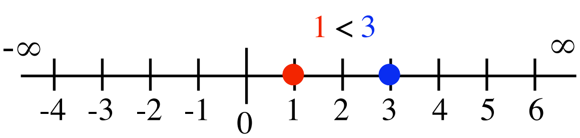


b)



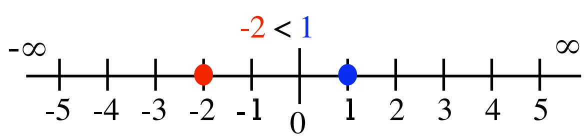
You Try It 2 Given two points on a number line, find the multiple or fraction of each value, according to the directions. State the resulting inequality. Use Example 2 as a guide. (Use the same given number line for your answer.)

a)



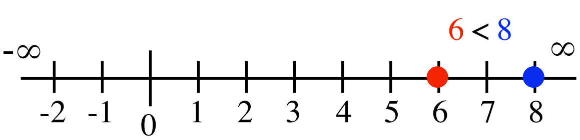
Double each number. (Multiply by 2.)

b)



Double each number. (Multiply by 2.)

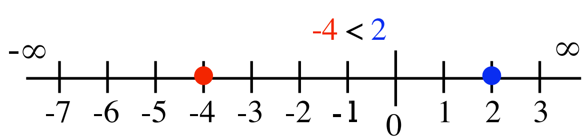
c)



Find one-half of each number.

(Multiply by $\frac{1}{2}$, or divide by 2.)

d)



Find three-halves of each number. (Multiply by $\frac{3}{2}$.)

Example 2 and You Try It 2 illustrate a property of inequalities that we will be using when solving inequality statements later in this section:

The Multiplication Property of Inequalities for *Positive* Multiples

For all values of a and b, and for all positive values of c,

If $a < b$,	If $a < b$,
then $c \cdot a < c \cdot b$	then $\frac{a}{c} < \frac{b}{c}$

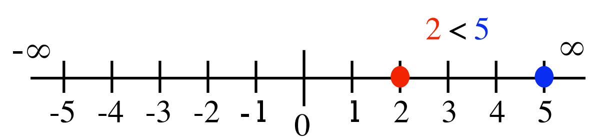
This property is true for all inequality forms ($<$, $>$, $\leq$, $\geq$, and $\neq$) as long as c is positive.

Caution: It's important to note that the direction of the inequality sign does not change when multiplying or dividing by a *positive* number.

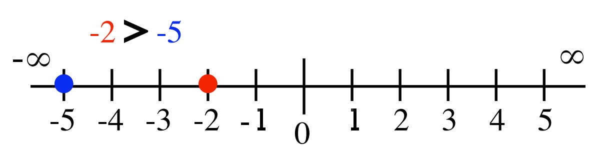
MULTIPLYING BY -1

We can explore what effect a negative multiplier has on the status of an inequality by multiplying each side by -1. What we will discover is that, as we multiply each side by -1, the direction of the inequality sign switches; it switches from *less than* to greater than, or from greater than to *less than*.

For example, 2 < 5 because 2 is to the left of 5 on the number line:



When we multiply each side by -1 the numbers are -2 and -5. However, -2 is to the right of -5, so -2 > -5:

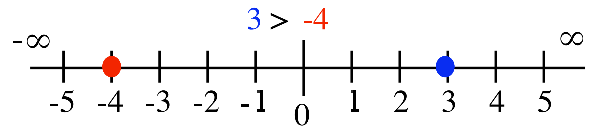


Algebraically, it looks like this: $2 < 5$

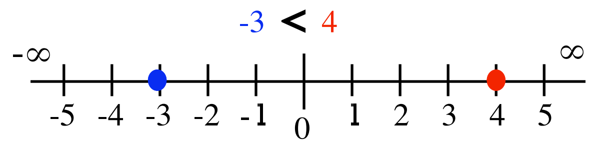
Multiply each side by -1: $-2 > -5$

Multiplying each side by -1 requires that we change the direction of the inequality sign from *less than* to greater than.

Another example, 3 > -4 because 3 is to the right of -4 on the number line:



When we multiply each side by -1 the numbers are -3 and +4. However, -3 is to the left of 4, so -3 < 4:

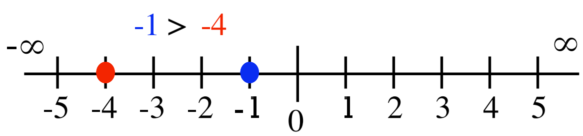


Algebraically, it looks like this: $3 > -4$

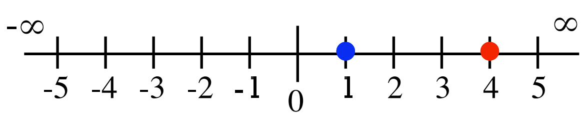
Multiply each side by -1: $-3 < 4$

Multiplying each side by -1 requires that we change the direction of the inequality sign from <u>greater than</u> to *less than*.

Example 3: Given two points on a number line, -1 and -4, and the inequality between them, -1 > -4, multiply each side of the inequality by -1 and state the resulting inequality. Use the number line as a guide.



Procedure: Multiply each side by -1 and place the results on the number line. Notice how they compare to each other before placing the inequality sign. Multiplying by -1 also changes the direction of the inequality sign.

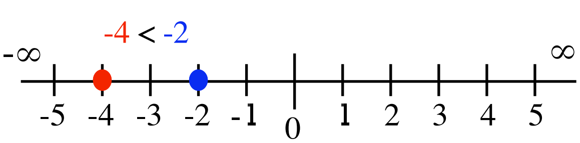


Original inequality: **-1 > -4**

Answer: The new inequality: **1 < 4**

<u>**You Try It 3**</u> Given two points on a number line, multiply each by -1 and state the resulting inequality. Use Example 3 as a guide. (You may use the same given number line for your answer.)

a)



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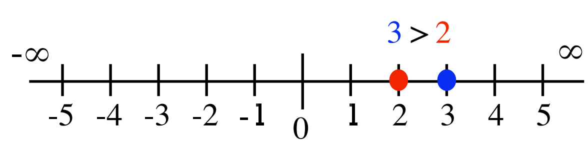
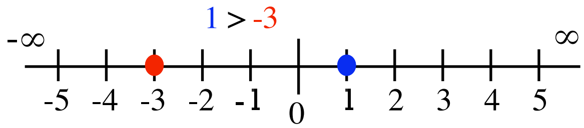
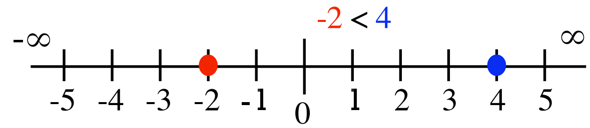


c)



d)



Example 3 and You Try It 3 illustrate the idea that, if needed, we can multiply each side by -1. However, in doing so, we must change the direction of the inequality sign at the same time.

For example, to isolate the variable in this inequality, $-x \leq 5$, we must multiply each side by -1 and—at the same time—*change the direction of the inequality sign*. It looks like this:

$$-x \leq 5$$

Multiply each side by -1. Change the direction of the inequality sign from ≤ to ≥.

$$x \geq -5$$

Multiplying by -1 is the same as <u>taking the opposite of</u> something. In the case of inequalities, when we multiply each side by -1 we get:

- the opposite of the entire left side
- the opposite of the entire right side
- the opposite direction of the inequality sign.

Note:	Multiplying each side of the inequality by -1 can occur at any time in the problem solving process. Whenever we do so, we must also change the direction of the inequality sign.

Consider, for example, $8 - 2x > x + 5$. Multiplying each side by -1 shows up as changing the sign of each and every term, *and* changing the direction of the inequality sign (in this case from <u>greater than</u> to *less than*).

$$8 - 2x > x + 5$$

Multiply each side by -1. Change the direction of the inequality sign from > to <.

$$-8 + 2x < -x - 5$$

Example 4:	Multiply each side by -1 and change the direction of the inequality sign; do not solve.		
	a) $-2x < -8$	b) $6 - w \geq 3$	c) $5 - 3y > 2y + 1$
Procedure:	Multiplying by -1 is the same as taking the opposite of everything: the opposite of every term on each side and the opposite direction of the inequality sign.		
Answer:	a) $2x > 8$	b) $-6 + w \leq -3$	c) $-5 + 3y < -2y - 1$

You Try It 4 Multiply each side by -1 and change the direction of the inequality sign; **do not solve**. Use Example 4 as a guide.

a) $-p \geq 6$

b) $-w < -1$

c) $10 > -5x$

d) $-8 \leq -4m$

e) $-x + 3 > 9$

f) $-7 - y \geq -3y + 4$

If the variable term is isolated, but the coefficient is negative, such as $-5x > 30$, then we must clear both the negative sign and the coefficient 5. This can be a three-step process or a two-step process.

As a three-step process, we: $-5x > 30$

1. Multiply by -1 and change the direction of the inequality sign. $5x < -30$
2. Divide each side by 5. $\frac{5x}{\mathbf{5}} < \frac{-30}{\mathbf{5}}$
3. Simplify the fractions. $x < -6$

As a two-step process, we: $-5x > 30$

1. Divide each side by -5 and change the direction of the inequality sign. $\frac{-5x}{\mathbf{-5}} < \frac{30}{\mathbf{-5}}$
2. Simplify the fractions. $x < -6$

In the two-step process, we are combining the multiplication by -1 with the division by 5 into one step. Because -5 contains a factor of -1, the step of taking the opposite of everything (including the inequality sign) is included with this division process.

Note:	When dividing by a negative coefficient, we must immediately change the direction of the inequality sign at the same time we write the division.

Think About It 1: Consider the steps shown at right. Fill in the boxes with the correct inequality signs.

In the boxes, did you fill in the same sign or different signs? Discuss your answer with a classmate.

$$-3x < -12$$

$$\frac{-3x}{\mathbf{-3}} \;\square\; \frac{-12}{\mathbf{-3}}$$

$$x \;\square\; 4$$

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The Multiplication Property of Inequality for *Negative* Multiples

For all values of a and b, if $-c$ is a negative value, then

If $a < b$,	If $a < b$,
then $-c \cdot a > -c \cdot b$	then $\frac{a}{-c} > \frac{b}{-c}$

This property is also true for the inequality forms of $\leq$ and $\geq$.

So, if a negative coefficient has not been cleared in an earlier step, then it can be divided out at the end, but the direction of the inequality sign must change.

Think About It 2: To isolate the variable in $4x \geq -20$, if we divide each side by 4, do we need to change the direction of the inequality sign (from $\geq$ to $\leq$)? Why or why not?

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Example 5: Isolate the variable by dividing each side by the coefficient; do not graph the solution set.

a) $-10x \geq -20$ b) $-4w < 4$ c) $2y > -14$

Procedure: If the coefficient is positive, then the direction of the inequality sign does not change. If the coefficient is negative, then the direction of the inequality sign does change.

Answer:

a) $-10x \geq -20$ Divide each side by -10; the direction of the inequality sign changes immediately.

$$\frac{-10x}{\mathbf{-10}} \leq \frac{-20}{\mathbf{-10}}$$ Simplify each side; the inequality sign will remain the same while we simplify.

$x \leq 2$

b) $-4w < 4$ Divide each side by -4; the direction of the inequality sign changes immediately.

$$\frac{-4x}{\mathbf{-4}} > \frac{4}{\mathbf{-4}}$$ Simplify each side; the inequality sign will remain the same while we simplify.

$x > -1$

c) $2y > -14$ Divide each side by 2; the direction of the inequality sign *does not* change.

$$\frac{2x}{\mathbf{2}} > \frac{-14}{\mathbf{2}}$$ Simplify each side.

$x > -7$

You Try It 5 Isolate the variable by dividing each side by the coefficient; do not graph the solution set. Use Example 5 as a guide.

a) $-3p \geq 6$ b) $7w < 21$ c) $9y > -45$ d) $-8x \leq -32$

SWITCHING SIDES

As was demonstrated in Section 2.6, we also can change the direction of the inequality sign by *switching sides*; this means that whichever expressions are on the left and right sides can be swapped, exactly as they are—the right side becomes the left side and vice-versa. However, in doing so, the direction of the inequality sign must also change.

Switching Sides of an Inequality

For all values of a and b,

If $a < b$,

then $b > a$

This property is true for all inequality forms ($<$, $>$, $\leq$, and $\geq$).

Example 6: Switch sides and change the direction of the inequality sign. **Do not solve at this time**.

a) $4 < x + 3$ b) $y - 5 \geq 2y$ c) $6 - x \leq 2x + 3$

Procedure: Do not change the value of either side, just change the side each expression is on. Also, change the direction of the inequality sign.

Answer: a) $x + 3 > 4$ b) $2y \leq y - 5$ c) $2x + 3 \geq 6 - x$

You Try It 6 Switch sides and change the direction of the inequality sign. **Do not solve at this time.** Use Example 6 as a guide.

a) $7 \geq x$ b) $6 < w$ c) $-8 > y$

d) $9 > x + 5$ e) $-4 \leq d - 8$ f) $3x \geq 5x - 2$

g) $1 - 3x < 4x + 8$ h) $2x + 3 > 6 - 4x$ i) $5 - 3p \leq 2p - 10$

Think About It 3: Under what circumstances do we change the direction of the inequality sign?

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Now that we are finished with the preparation, let's take a look at how to fully solve linear inequalities.

SOLVING AN INEQUALITY: THE GUIDELINES

The general guidelines for solving linear inequalities are shown below. Notice that, with little exception, these guidelines are virtually the same as those used to solve linear equations. We must be careful about two things:

1. There are times when we must change the direction of the inequality sign.

2. The solving is complete when we have isolated the variable on the *left side* of the inequality.

Guidelines for Solving Linear Inequalities

The Preparation:

1. Eliminate any parentheses by distributing.
2. Clear any fractions or decimals by multiplying each side by the equation's LCD.
3. Combine like terms on each individual side.

Isolating the Variable:

4. If necessary, reduce the inequality to standard form.
5. If the coefficient is negative, multiply each side by -1 and change the direction of the inequality sign. (This step can, instead, be combined with clearing the coefficient.)
6. Clear the operations; start with the main operation.
7. If necessary, "switch sides" so that the variable term is on the left. (Don't forget to change the direction of the inequality sign.)
8. Graph the solution set on the number line.

Note: Earlier in this chapter, when adding terms to each side of the equation, it was appropriate to write an equal sign between the added values. For example, adding 5 to each side of this equation looks like this:

$$\begin{array}{rcl} 3x - 5 &=& 19 \\ \underline{\quad +5} &\underline{=}& \underline{+5} \\ 3x &=& 24 \end{array}$$

and we would solve from here.

When solving an inequality, though, it is not recommended to write the equal sign between the added values because the equal sign might cause confusion. So, instead you will see the "double arrow" between the added values, like this:

$$\begin{array}{rcl} 3x - 5 &<& 19 \\ \underline{\quad +5} &\underline{\leftrightarrow}& \underline{+5} \\ 3x &<& 24 \end{array}$$

and we would solve from here.

Notice that the arrow does not affect the inequality symbol in any way.

Here's a partial example using **Guidelines 4 and 5**:

If we are asked to solve the inequality	$2 + 3x < 5x - 8$
and if we add $-5x$ to each side (**Guideline 4**):	$\underline{\quad \mathbf{-5x} \leftrightarrow \mathbf{-5x}\quad}$
then, because the variable term $-2x$ is now negative, we can multiply each side by -1 and change the	$2 - 2x < -8$
direction of the inequality sign (**Guideline 5**):	$-2 + 2x > 8$
and so on....	(now isolate the variable)

Here's a partial example using **Guidelines 4 and 7**:

If we are asked to solve the inequality	$2 + 3x < 5x - 8$
and if we add $-3x$ to each side (**Guideline 4**):	$\underline{\quad \mathbf{-3x} \leftrightarrow \mathbf{-3x}\quad}$
then, because the variable term is on the right side, we can switch sides and change the direction	$2 < 2x - 8$
of the inequality sign (**Guideline 7**):	$2x - 8 > 2$
and so on....	(now isolate the variable)

By the way, in using either of these methods, the answer is the same: $x > 5$.

Let's put the guidelines into practice.

Example 7: Solve each inequality. Draw the graph of the solution set on a number line.

a) $3x - 5 < x + 3$ b) $17 \leq 5w + 2$

Procedure: Use only those guidelines that are necessary.

Answer: a)

$$3x - 5 < x + 3$$
$$\underline{\mathbf{-x} \quad \leftrightarrow \quad \mathbf{-x}}$$

Add $-x$ to each side to get the variable terms together.

$$2x - 5 < 3$$
$$\underline{\mathbf{+5} \quad \leftrightarrow \quad \mathbf{+5}}$$

Isolate the variable term by adding 5 to each side.

$$2x < 8$$

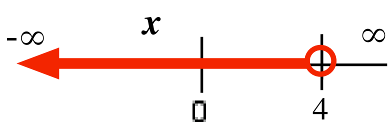
The coefficient is positive. Divide each side by 2.

$$\frac{2x}{\mathbf{2}} < \frac{8}{\mathbf{2}}$$

Simplify.

$$x < 4$$

Draw the graph of the solution set.



b)

$$17 \leq 5w + 2$$

There is only one variable term and it is on the right side. Let's switch sides to put it on the left.

$$5w + 2 \geq 17$$
$$\underline{\mathbf{-2} \quad \leftrightarrow \quad \mathbf{-2}}$$

Isolate the variable term by adding -2 to each side.

$$5w \geq 15$$

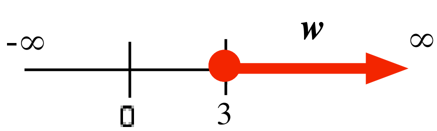
The coefficient is positive. Divide each side by 5.

$$\frac{5w}{\mathbf{5}} \geq \frac{15}{\mathbf{5}}$$

Simplify.

$$w \geq 3$$

Draw the graph of the solution set.



You Try It 7 Solve each inequality. Draw the graph of the solution set on a number line. Use Example 7 as a guide.

a) $3x + 5 > 11$

b) $7 \geq 5v - 3$

c) $5 > 4 - w$

d) $2n - 3 \leq 7 + 6n$

Example 8: Solve each inequality. Draw the graph of the solution set on a number line.

a) $2(y - 6) - 6y > 16$

b) $-3v - 9 \leq 2v - \frac{3}{2}$

Procedure: Use only those guidelines that are necessary.

Answer: a) $2(y - 6) - 6y > 16$ First, clear the parentheses.

$2y - 12 - 6y > 16$ Simplify the left side. Combine like terms

$-4y - 12 > 16$ Multiply each side by -1 and change the direction of the inequality sign.

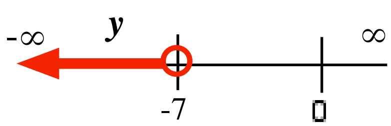
$4y + 12 < -16$ Isolate the variable term by adding -12 to each side.

$\underline{\quad\ \ \mathbf{-12} \leftrightarrow \mathbf{-12}}$

$4y < -28$ Divide each side by 4.

$\dfrac{4y}{\mathbf{4}} < \dfrac{-28}{\mathbf{4}}$ Simplify.

$y < -7$ Draw the graph of the solution set.



b) $-3v - 9 \le 2v - \dfrac{3}{2}$ The LCD is 2. Prepare the inequality by placing parentheses around each side. Multiply each side by 2.

$\mathbf{2} \cdot (-3v - 9) \le \mathbf{2} \cdot \left(2v - \dfrac{3}{2}\right)$ Distribute 2, on each side, to each term.

$-6v - 18 \le 4v - 3$ There is a variable term on each side. Let's add $6v$ to each side.

$\underline{\mathbf{+\,6v} \quad \leftrightarrow \quad \mathbf{+\,6v}}$

$-18 \le 10v - 3$ Isolate the variable term by adding 3 to each side.

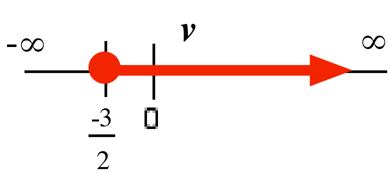
$\underline{\mathbf{+\,3} \leftrightarrow \quad \mathbf{+\,3}}$

$-15 \le 10v$ Divide each side by 10.

$\dfrac{-15}{\mathbf{10}} \le \dfrac{10v}{\mathbf{10}}$ Simplify.

$\dfrac{-3}{2} \le v$ Switch sides and switch the direction of the inequality sign.

$v \ge \dfrac{-3}{2}$ Draw the graph of the solution set.



You Try It 8 Solve each inequality. Draw the graph of the solution set on a number line. Use Example 8 as a guide.

a) $-7 - 3y \geq -2(y - 4)$

b) $-(2x - 3) > 4(x - 3)$

c) $\frac{3}{4}w + 1 > 1 - w$

d) $\frac{1}{3}m + \frac{1}{2} \geq 1 - \frac{2}{3}m$

INEQUALITIES INVOLVING THE "NOT EQUAL" SIGN

There are circumstances in algebra where we must solve identities involving the "not equal" sign, ≠. All of the rules for solving inequalities apply to this type as well, except that there is never a need to switch the direction of ≠. In other words, dividing each side by a negative number is no different than dividing each side by a positive number.

Example 9: Solve the inequality. Draw the graph of the solution set on a number line.

$1 - 3p \neq 4(p + 2)$

Procedure: Use only those guidelines that are necessary.

Answer: a) $1 - 3p \neq 4(p + 2)$ First, clear the parentheses on the right side. Also, write the left side with the variable term first.

$-3p + 1 \neq 4p + 8$ Add -4p to each side to get the variable terms together.

$\underline{\mathbf{-4}\boldsymbol{p} \quad \leftrightarrow \quad \mathbf{-4}\boldsymbol{p}}$

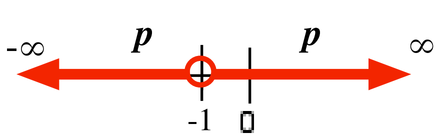
$-7p + 1 \neq 8$ Isolate the variable term by adding -1 to each side.

$\underline{\mathbf{-1} \quad \leftrightarrow \quad \mathbf{-1}}$

$-7p \neq 7$ Divide each side by -7.

$\dfrac{-7p}{\mathbf{-7}} \neq \dfrac{7}{\mathbf{-7}}$ Simplify.

$p \neq -1$ Draw the graph of the solution set.



You Try It 9 Solve each inequality. Draw the graph of the solution set on a number line. Use Example 9 as a guide.

a) $3v + 8 \neq 0$

b) $5w + 8 \neq 3w - 2$

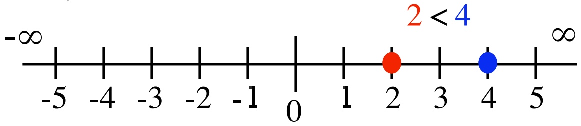
c) $5 - \frac{1}{2}x \neq x - 4$

d) $3y - 5 \neq 5(2y - 1)$

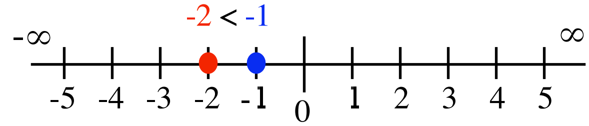
You Try It Answers

You Try It 1

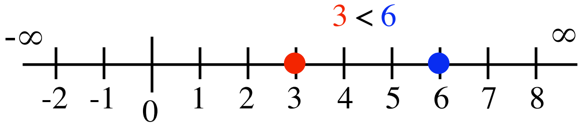
a) $2 < 4$



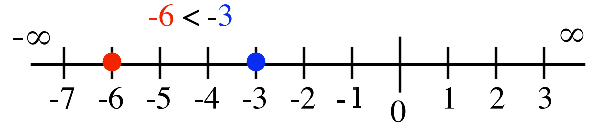
b) $-2 < -1$



c) $3 < 6$

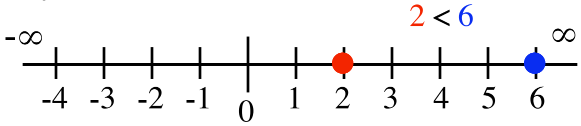


d) $-6 < -3$

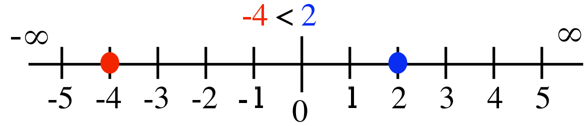


You Try It 2

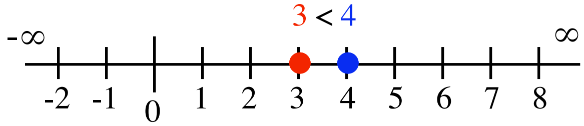
a) $2 < 6$



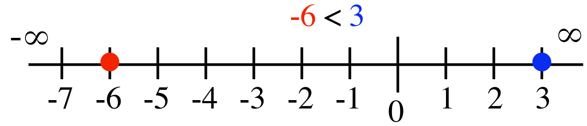
b) $-4 < 2$



c) $3 < 4$

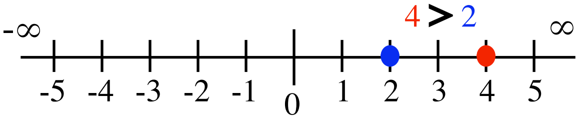


d) $-6 < 3$

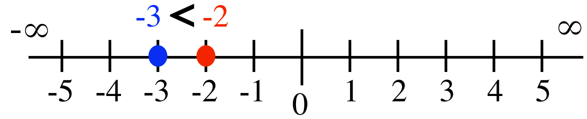


You Try It 3

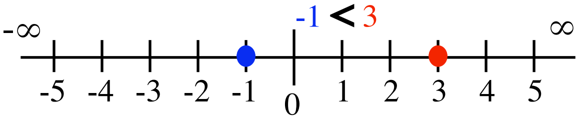
a) $4 > 2$



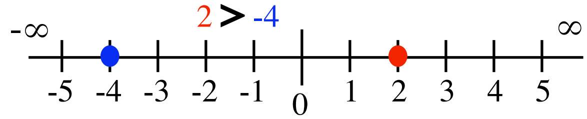
b) $-3 < -2$



c) $-1 < 3$



d) $2 > -4$



You Try It 4

a) $p \le -6$ b) $w > 1$ c) $-10 < 5x$

d) $8 \ge 4m$ e) $x - 3 < -9$ f) $7 + y \le 3y - 4$

You Try It 5

a) $p \le -2$ b) $w < 3$ c) $y > -5$ d) $x \ge 4$

You Try It 6

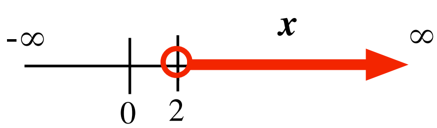
a) $x \le 7$ b) $w > 6$ c) $y < -8$

d) $x + 5 < 9$ e) $d - 8 \ge -4$ f) $5x - 2 \le 3x$

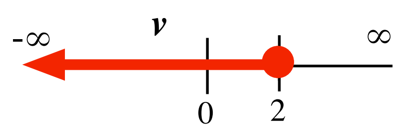
g) $4x + 8 > 1 - 3x$ h) $6 - 4x < 2x + 3$ i) $2p - 10 \ge 5 - 3p$

You Try It 7

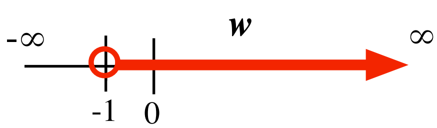
a) $x > 2$



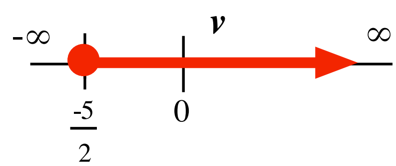
b) $v \le 2$



c) $w > -1$

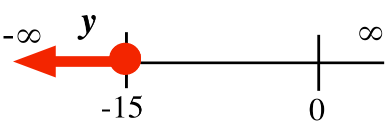


d) $n \ge -\frac{5}{2}$

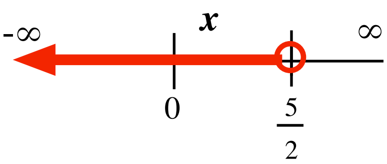


You Try It 8

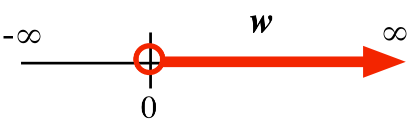
a) $y \leq -15$



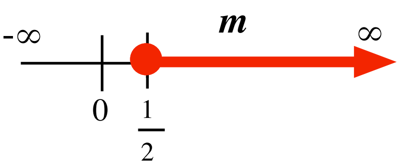
b) $x < \frac{5}{2}$



c) $w > 0$

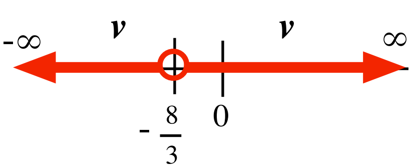


d) $m \geq \frac{1}{2}$

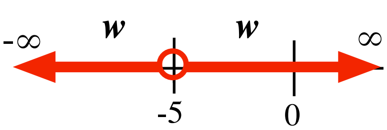


You Try It 9

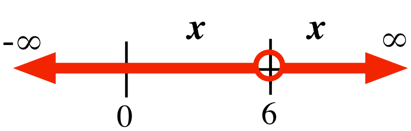
a) $v \neq -\frac{8}{3}$



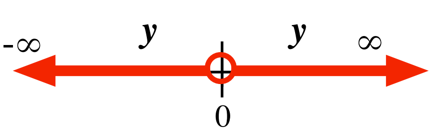
b) $w \neq -5$



c) $x \neq 6$



d) $y \neq 0$



Section 2.7 Exercises

Think Again.

1. To isolate the variable in $4x \geq -20$, must the direction of the inequality sign change, from $\geq$ to $\leq$, when we divide each side by 4? Why or why not? *(Refer to Think About It 2)*

2. Under what circumstances do we change the direction of the inequality sign? *(Refer to Think About It 3)*

Focus Exercises.

Switch sides and change the direction of the inequality sign. Finish solving, if necessary, and draw the graph of the solution set on a number line.

3. $-8 < x$

4. $6 > r$

5. $6 \geq y - 8$

6. $-1 < h - 4$

7. $-5 \geq y - 11$

8. $16 > 7x - 5$

Multiply each side by -1 and change the direction of the inequality sign. Finish solving, if necessary, and draw the graph of the solution set on a number line.

9. $-w < 9$

10. $-3x \geq -3$

11. $21 < -7x$

12. $-3m - 1 \geq -14$

13. $-5 > -2x + 12$

14. $9 - 4k < -1 + k$

Solve each of these inequalities, and draw the graph of the solution set on a number line.

15. $y - 3 \geq 2$

16. $x - 5 < 4$

17. $2 > r - 4$

18. $-6 \leq 2 - x$

19. $10 > -5p$

20. $4x > -20$

21. $-14m \leq -56$

22. $8r + 3 < 51$

23. $-3x + 5 < 14$

24. $-10k + 18 \geq -52$

25. $-15y + 35 > -115$

26. $4x < 3x - 6$

27. $-7x < 2x - 81$

28. $y - 8 \leq -2y + 1$

29. $9s - 13 > -3s + 71$

30. $8 + 2x < 4(x - 3)$

31. $-2(y - 3) \geq 1 + 3y$

32. $2x < 3x - 5$

33. $9(w - 8) > 4(w + 7)$

34. $\frac{3}{4}x + \frac{4}{3} \leq 1 + \frac{5}{6}x$

35. $-\frac{7}{8}y - \frac{3}{4} \geq \frac{1}{4} - y$

36. $-0.3x + 1 \geq 0.2x - 3$

Solve each of these inequalities, and draw the graph of the solution set on a number line.

37. $y - 3 \neq 2$

38. $12 \neq -4x$

39. $-10 \neq c - 6$

40. $2p - 6 \neq 0$

41. $3y + 21 \neq 0$

42. $4r - 2 \neq 0$

43. $8y + 6 \neq 0$

44. $5m - 3 \neq 0$

45. $4x \neq 3x - 6$

46. $5x + 4 \neq 3x$

47. $x - 6 \neq 2x + 5$

48. $11 - 4x \neq 3(x - 1)$

Think Outside the Box:

For each of the following, find the requested set of numbers that makes the statement true. (Hint: The word "is" indicates the beginning of words of comparison, such as "is at least" or "is fewer than.") Draw the graph of the solution set on a number line.

49. The sum of twice a number and 12 is at least 20. What set of numbers makes this true?

50. Three times the difference of a number and 8 is no more than 15. What set of numbers makes this true?

51. The sum of -23 and a number is no more than three times the difference of 7 and the number. What set of numbers makes this true?

52. Nine more than four times a number is not equal to seven less than twice the number.