

Section 1.1 Whole Numbers

Objectives

In this section, you will learn to:

- Identify the base-10 numbers.
- Find the place value of a digit in a whole number.
- Write a whole number in expanded form and in words.
- Round whole

INTRODUCTION

Numbers have been around since the beginning of language. People first used numbers to count things, especially when items were being traded, such as five sheep for three pigs. As time progressed, numbers were used to measure things, such as the length of an ark, the distance between two cities, the amount of daylight in a day, and so on.

Eventually people started doing calculations with numbers, and it was called “arithmetic.” They started drawing circles and triangles, and it was called “geometry.” They started using letters for numbers, and it was called “algebra.” They gave all of it a name; they called it “mathematics.”

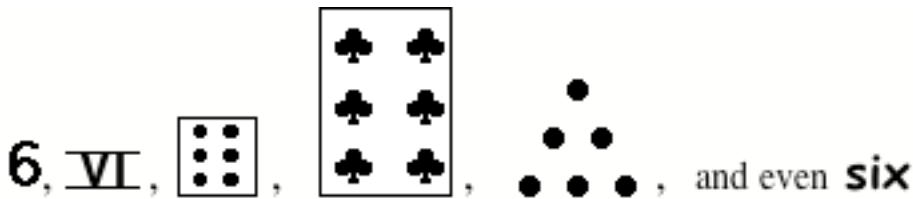
Mathematics is at the heart of every computer, every television set, and every cell phone. In short, mathematics is a hidden part of your everyday life. As complex as our world has become, even the simplest mathematics plays a role.

Most students of arithmetic—basic math—will continue their studies through algebra; some of you may even go on to study higher levels of math, such a statistics or calculus. Whatever your goal, learning basic math—the first building block—is an important step.

NUMBERS AND NUMERALS

In the world of language, a number is an adjective; it describes something. Just like the color red is an adjective, as in “the *red* balloon,” a **number** describes *how many* items there might be, as in “Tina has *six* balloons.”

A **numeral** is a symbol that represents a number. For example, the number *six* can be expressed by any of these symbols (and there are others):



In this text we often use the word “number” in place of the word “numeral” to make the reading easier.

Our way of counting is built on a **base-ten numbering system** that uses ten **digits**, or numerals:

0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

These digits are the first of the **whole numbers**. When we get to 9, we have exhausted all of the single digit whole numbers, so in order to represent the next whole number, ten, we must use two digits, 10.

Here is a diagram of the place values (a place-value chart). Notice the groups of threes, called **periods**, and the pattern of repetition (hundreds, tens, ones) within each period.

Trillions period			Billions period			Millions period			Thousands period			Ones period		
h _{und}	t _{ens}	o _{nes}	h _{und}	t _{ens}	o _{nes}	h _{und}	t _{ens}	o _{nes}	h _{und}	t _{ens}	o _{nes}	h _{und}	t _{ens}	o _{nes}

ten billions place
hundred millions place
one thousands place

To make large numbers easier to read, we typically use a comma to separate numbers every three digits—reading from right to left—into their periods. You will probably agree that the number 28930564 is too hard to read without commas. Written as 28,930,564 it can be more easily read. Let’s place the number, one digit for each place, in the place-value chart:

Trillions period			Billions period			Millions period			Thousands period			Ones period		
h _{und}	t _{ens}	o _{nes}	h _{und}	t _{ens}	o _{nes}	h _{und}	t _{ens}	o _{nes}	h _{und}	t _{ens}	o _{nes}	h _{und}	t _{ens}	o _{nes}
							2	8	9	3	0	5	6	4

This large number can be read (and written) as:

twenty-eight *million*, nine hundred thirty *thousand*, five hundred sixty-four.

Notice that:

- (1) We use commas to separate the number even when it is written out in words;
- (2) The numerical period is written before each comma. (The word *one*, for the ones period, is never written.)
- (3) When spelled out, two-digit numbers, such as 28, are hyphenated: *twenty-eight*.

Example 1: Identify in which period and place you find the digit 4, then write the value the 4 represents.

- a) 62,439 b) 94,023,186

Procedure: Let's use a place-value chart to help us read the numbers more easily.

Millions period			Thousands period			Ones period		
hund	ten _s	one _s	hund	ten _s	one _s	hund	ten _s	one _s
(a)				6	2	4	3	9
(b)	5	9	4	0	2	3	1	8

Answer: a) In 62,439 the 4 is in the ones period and in the *hundreds place*, so the 4 represents four hundred (400).
 b) In 94,023,186 the 4 is in the millions period and in the *ones place*, so the 4 represents four million (4,000,000).

The **You Try It** exercises—**YTI**—are presented within the section because you are expected to do these exercises at this point, while the material is fresh in your mind. These exercises will sometimes lead to other ideas that rely on what you do here. The answer to each YTI can be found at the end of the section.

YTI #1

Identify in which period and place you find the digit 6, then write the value the 6 represents.

Use Example 1 as a guide.

Millions period			Thousands period			Ones period		
hund	ten _s	one _s	hund	ten _s	one _s	hund	ten _s	one _s

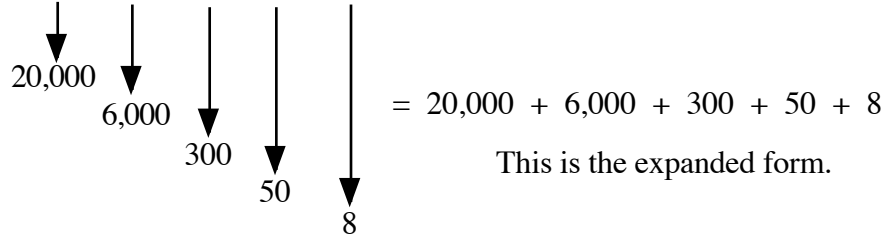
- a) 629,418 _____ _____ _____
- b) 65,023,189 _____ _____ _____

WRITING A NUMBER IN EXPANDED FORM

We can write a number in *expanded form* when we consider (1) the value of each digit, and (2) the place of each digit. For example, consider the digit 6 in the number 26,358. It is in the thousands place so it represents 6,000 (six thousand).

Likewise, the digit 3 is in the hundreds place and represents 300 (three hundred). In fact, we can break down the entire number into its individual digits, each according to its place, in the following manner:

Millions period			Thousands period			Ones period		
hund	ten	ones	hund	ten	ones	hund	ten	ones
				2	6	3	5	8



YTI #2

Write each number in expanded form.

a) 3,075 = _____

b) 503,142 = _____

WRITING WHOLE NUMBERS IN WORDS

You saw the number 28,930,564 written in words as

twenty-eight *million*, nine hundred thirty *thousand*, five hundred sixty-four.

Again, notice that commas are used to separate each period and that the period (million and thousand) is written before each comma.

A simpler example is 307. We write this as three hundred seven.

CAUTION:

You might be tempted to include the word *and* in three hundred seven, but this is not appropriate when writing whole numbers. We reserve the word *and* for mixed numbers that include both whole numbers along with either fractions or decimals. In this sense, the word *and* means *plus*.

For example, compare two hundred six million, which is 206,000,000
to two hundred and six million, which is $200 + 6,000,000 = 6,000,200$.

To eliminate this confusion, we do not use the word *and* when writing out whole numbers.

Example 2: Write each number in words.

- a) 239 b) 8,104 c) 403,005 d) 49,023,000

Procedure: Write the number, reading left to right, using a comma to indicate the conclusion of one period and the beginning of a new period.

Answer:

- a) 239 is two hundred thirty-nine. Notice that *thirty-nine* is spelled with a hyphen between 'thirty' and 'nine'.
- b) 8,104 is eight thousand, one hundred four. Once we write the word *thousand*, we place a comma after it and finish writing the number.
- c) 403,005 is four hundred three thousand, five. After the thousands, the ones period contains only 5; it is the comma that shows us the separation.
- d) 49,023,000 is forty-nine million, twenty-three thousand. Because there are no non-zero digits in the ones period, we stop at the thousands.

YTI #3

Write the number in words. Use Example 2 as a guide.

- a) 863 _____
- b) 3,075 _____
- c) 62,009 _____
- d) 5,003,102 _____

Example 3: Write the whole number in numeral form.

- a) Seven thousand, five hundred nine. b) Two hundred four thousand, fifty-three.
- c) Six million, eighteen

Procedure: Look for the numerical period and the comma.

Answer:

- a) Seven thousand, five hundred nine is 7,509. There are no tens between the five hundred and the nine. No tens is represented by 0 in the tens place.
- b) Two hundred four thousand, fifty-three is 204,053. This time there are no ten thousands and no hundreds, each of which is represented by a 0.
- c) Six million, eighteen is 6,000,018. Since we never see the word *thousand*, it must be that there are no thousands at all. We can't skip them, but represent that fact with three 0's in the thousands place.

YTI #4

Write the whole number in numeral form. Use Example 3 as a guide.

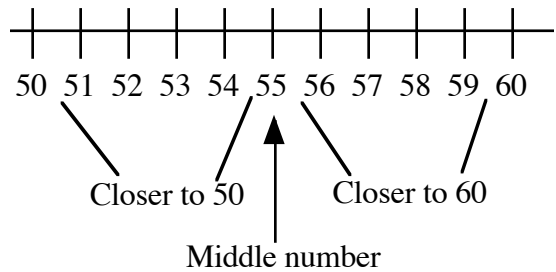
- a) Two thousand, forty-eight _____
- b) Seventy-five thousand, four _____
- c) One hundred four thousand, two hundred _____

ROUNDING VALUES

Sometimes, when working with a large number, it is easier to think of the number in *rounded* terms. For example, if the price of a lawn mower is \$287.99, it is easier to think of it as, maybe, \$290, or maybe as \$300. These rounded numbers, \$290 and \$300, are called *approximations*, or *estimates*.

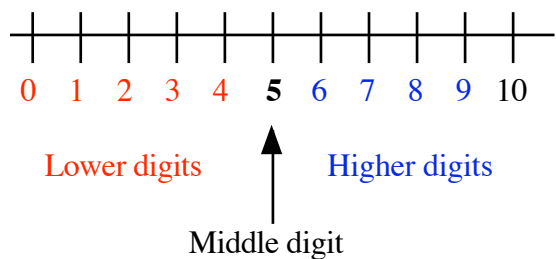
In a moment, you will be introduced to the rules for rounding numbers. First, though, let’s take a look at how to visualize numbers in a “higher and lower” sense.

The diagram on the right shows the whole numbers 0 through 10. The digit 5 is in the middle; the digits 6, 7, 8, and 9 are **higher** digits (to the right of the middle); and the digits 0, 1, 2, 3 and 4 are considered **lower** digits.



When rounding a number to the nearest *ten*, we rewrite the number with a zero in the *ones* place.

For example, to round 57 to the nearest ten, we must either round it *down* to 50 or round it *up* to 60. In other words, it’s like asking the question, “Is 57 closer to 50 or to 60?”



It’s the ones digit that indicates whether we should round up or round down. In 57, the ones digit is 7, a higher digit; this means that we should round *up* to 60.

When rounding to the nearest ten we use the digit in the ones place—called the *rounding digit*—to determine whether we should round the number up or down. The diagram indicates that

- When the rounding digit is either 0, 1, 2, 3 or 4, we round *down*.
- When the rounding digit is either 6, 7, 8 or 9, we round *up*.

What happens when the rounding digit is 5? Should we round up or round down? Since there are five digits (0, 1, 2, 3 and 4) that cause a number to be rounded down, there should also be five digits that cause a number to be rounded up (5, 6, 7, 8 and 9). So,

- When the rounding digit is 5, we round *up*.

Below are the rules for rounding. We use two examples—one that rounds up and one that rounds down—to show the application of the various steps.

Rules for Rounding:

1. Identify the place digit that is to be rounded.

a) “Round 28,714 to the nearest thousand” means that the 8, in the thousands place, is the place digit.

↓
28, 7 14

b) “Round 28,714 to the nearest hundred” means that the 7, in the hundreds place, is the place digit.

↓
28, 7 14

2. Identify the number to its *immediate right*, called the rounding digit.

a) Rounding 28,714 to the nearest thousand means the rounding digit is the 7.

↓
28, 7 14

b) Rounding 28,714 to the nearest hundred means the rounding digit is the 1.

↓
28, 7 1 4

3. i) If the rounding digit is 5 or higher, round *up* (add 1 to the place digit).

ii) If the rounding digit is 4 or lower, round *down* (add 0 to the place digit).

4. All digits to the right of the place digit are written as zeros. Rewrite the number showing the appropriate approximation.

a) In rounding 28,714 to the nearest thousand, the rounding digit, 7, is more than 5, so we need to add 1 to the place digit 8, making a 9 in the thousands place, and all of the digits following it are zeros:

↓
28, 7 14
+ 1
↓
29, 0 00

b) In rounding 28,714 to the nearest hundred, the rounding digit, 1, is less than 5, so we need to add 0 to the place digit, meaning the hundreds place will remain a 7, and all of the digits following it are zeros:

↓
28, 7 1 4
+ 0
↓
28, 7 00

So, 28,714 can be approximated as 29,000 when rounded to the nearest *thousand*

and can be approximated as 28,700 when rounded to the nearest *hundred*.

Notice that the digits to the *left* of the place digit remain unchanged. This is true for most rounding. As you will see in a little bit, when the place digit is 9, rounding up will change more than just the place digit.

Example 4: For each of the following, identify the place digit and the rounding digit, decide whether the number will round up or down, and round the number.

	Round	To the nearest	Place digit	Rounding digit	Round up/down	Rounded Number
a)	726	ten	<u>2</u>	<u>6</u>	<u>up</u>	<u>730</u>
b)	726	hundred	<u>7</u>	<u>2</u>	<u>down</u>	<u>700</u>
c)	1,082	thousand	<u>1</u>	<u>0</u>	<u>down</u>	<u>1,000</u>
d)	235,471	ten-thousand	<u>3</u>	<u>5</u>	<u>up</u>	<u>240,000</u>

YTI #5

For each of the following, identify the place digit and the rounding digit, decide whether the number will be rounded up or down, and round the number. Use Example 4 as a guide.

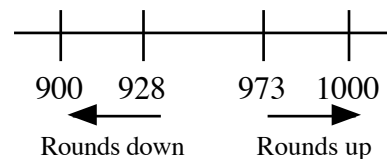
	Round	to the nearest	Place digit	Rounding digit	Round up/down	Rounded number
a)	528	ten	_____	_____	_____	_____
b)	4,609	hundred	_____	_____	_____	_____
c)	4,609	thousand	_____	_____	_____	_____
d)	75,406	ten-thousand	_____	_____	_____	_____
e)	1,925,046	hundred-thousand	_____	_____	_____	_____

ROUNDING WHEN THE PLACE DIGIT IS 9

What happens when the place digit is 9? If the number needs to be rounded up, then it rounds up to 10. This means that more than just the place digit is affected; the digit to the left will also increase by 1. (If the number is to be rounded down, the 9 will stay a 9.)

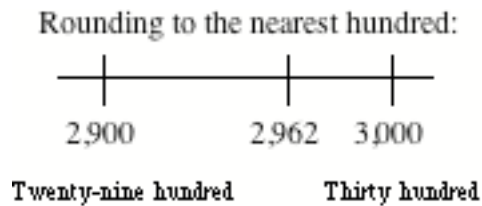
Rounding 928 to the nearest hundred means rounding down to 900.

Rounding to the nearest hundred:



Rounding 973 to the nearest hundred means rounding it up to “one more than 9” hundred. This means rounding it to ten hundred (1000), or 1,000.

Likewise, rounding 2,962 to the nearest hundred means rounding it up to “one more than 29” hundred. This means rounding it to *thirty* hundred (3000), or 3,000.



So, when the place digit is 9, rounding up affects more than just the place digit; it affects the digit to its left, as well.

Example 5: For each of these the place digit is 9. Think about them carefully.

	Round	To the nearest	Round up/down	Rounded Number
	↓			
a)	97	ten	<u>up</u>	100
	↓			
b)	928	hundred	<u>down</u>	900
	↓			
c)	4,951	hundred	<u>up</u>	5,000
	↓			
d)	29,950	hundred	<u>up</u>	30,000
	↓			
e)	9,850	thousand	<u>up</u>	10,000

YTI #6

Round each of these numbers to the place shown. Use Example 5 as a guide.

	Round	To the nearest	Round up/down	Rounded number
a)	597	ten	_____	_____
b)	987	hundred	_____	_____
c)	3,948	hundred	_____	_____
d)	691,956	ten thousand	_____	_____
e)	399,604	thousand	_____	_____

APPLICATIONS

Sometimes it's helpful when working with large numbers to round them first. What follows are some statistical data involving large numbers. Your job is to round them and write a sentence that shows the approximation.

Example 6:	In April 2005, the U.S. population was believed to be 295,981,000. Round this number to the nearest million, and write a sentence indicating the approximation. <i>Source: www.census.gov/main/www/popclock.html</i>
Procedure:	First round the number to the nearest million. It rounds to 296,000,000. Then write a sentence for the approximation. Use the word "about" in the sentence.
Answer:	In April 2005, the U.S. population was about 296,000,000.

YTI #7 In 2003, the median (average) income of a four-person California family was \$63,761. Round this number to the nearest thousand, and write a sentence indicating the approximation. Use Example 6 as a guide. *Source: www.census.gov*

Sentence: _____

YTI #8 In 2000, the United States Census Bureau counted 105,522,964 registered women voters. Round this number to the nearest **million**, and write a sentence indicating the approximation. *Source: www.census.gov*

Sentence: _____

Think about it:	Do you ever round numbers in any of your daily activities at work, while shopping, in your hobbies, or other daily routines? Write a few examples.
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You Try It Answers

- YTI #1:**
- | | Period | Place | Value |
|----|-----------|---------|----------------------|
| a) | thousands | hundred | six hundred thousand |
| b) | millions | ten | sixty million |
- YTI #2:**
- | | |
|---------------------|-------------------------------------|
| a) $3,000 + 70 + 5$ | b) $500,000 + 3,000 + 100 + 40 + 2$ |
|---------------------|-------------------------------------|
- YTI #3:**
- | | |
|------------------------------|--|
| a) eight hundred sixty-three | b) three thousand, seventy-five |
| c) sixty-two thousand, nine | d) five million, three thousand, one hundred two |
- YTI #4:**
- | | | |
|-----------|-----------|------------|
| a) 2, 048 | b) 75,004 | c) 104,200 |
|-----------|-----------|------------|

- YTI #5:**
- | | Place digit | Rounding digit | Round up/down | Rounded number |
|----|-------------|----------------|---------------|----------------|
| a) | 2 | 8 | up | 530 |
| b) | 6 | 0 | down | 4,600 |
| c) | 4 | 6 | up | 5,000 |
| d) | 7 | 5 | up | 80,000 |
| e) | 9 | 2 | down | 1,900,000 |

- YTI #6:**
- | | Round up/down | Rounded number | | Round up/down | Rounded number |
|----|---------------|----------------|----|---------------|----------------|
| a) | up | 600 | b) | up | 1,000 |
| c) | down | 3,900 | d) | down | 690,000 |
| e) | up | 400,000 | | | |

YTI #7: In 2003, the median income of a four-person California family was approximately \$64,000.

YTI #8: In 2000, the United States Census Bureau counted approximately 106,000,000 registered women voters.

Focus Exercises

Apply what you learned in this section to answer the following.

1. In your own words, what is the difference between whole numbers and counting numbers?
2. Place each number on the number line: 30, 35, 22, 27, 38



Identify in which period and place you find the digit 7, then write the value the 7 represents.

	Period	Place	Value
3. 53,278	_____	_____	_____
4. 352,716	_____	_____	_____
5. 6,703,214	_____	_____	_____
6. 17,480,300	_____	_____	_____

Write each number in expanded form.

7. 486 _____
8. 4,065 _____
9. 203,058 _____
10. 1,500,043 _____

Write each number in words.

11. 498 _____
12. 6,204 _____
13. 507,093 _____
14. 1,013,000 _____

Write the whole number.

15. Five hundred eighteen _____

16. Two thousand, three hundred six _____

17. Two hundred eighty thousand, thirty-four _____

18. Nine hundred five thousand, eight _____

19. One million, four hundred twenty-six _____

20. Three million, two thousand _____

Round each of these numbers to the nearest ten.

21. 67 _____

22. 683 _____

23. 795 _____

24. 1,941 _____

Round each of these numbers to the nearest hundred.

25. 638 _____

26. 2,049 _____

27. 1,708 _____

28. 2,350 _____

29. 6,951 _____

30. 9,962 _____

Round each of these numbers to the nearest thousand.

31. 2,906 _____

32. 8,061 _____

33. 36,407 _____

34. 49,513 _____

35. 699,850 _____

36. 3,580,416 _____

Round each of these numbers to the nearest ten thousand.

37. 25,408 _____

38. 324,612 _____

39. 650,894 _____

40. 3,996,416 _____

Round each of these numbers to the nearest hundred thousand.

41. 624,058 _____

42. 850,043 _____

43. 952,407 _____

44. 9,980,376 _____

Round each of these numbers to the nearest million.

45. 3,580,416 _____ 46. 4,277,095

47. 9,508,416 _____ 48. 46,283,000

Rewrite the underlined sentence or phrase with the requested approximation.

49. In 2005, the Rialto Unified School District's budget revenues was \$192,863,877. Round this number to the nearest million. *Source: www.rialto.k12.ca.us*
50. In 2004, the population of Florida was 17,397,161. Round this number to the nearest hundred thousand. *Source: factfinder.census.gov*
51. In 2002, gas consumption statistics for all international flights showed that the total number of gallons of gas consumed was 4,990,797,640. Round this number to the nearest hundred million. *Source: www.bts.gov*
52. It is estimated that the world's rain forests are being destroyed at a rate of 77,893,900 acres per year. Round this yearly acreage to the nearest million. *Source: www.rain-tree.com*
53. In the 2001 - 2002 school year, 6,248,610 students were enrolled in grades K - 12 in California. Round this number to the nearest hundred thousand. *Source: nces.ed.gov*
54. Statistics compiled by the Center for Disease Control through December, 2003 indicate the total number of reported AIDS cases was 920,566. Round this number to the nearest ten thousand. *Source: www.cdc.gov*