Section 1.2 Definitions and Properties

Objectives
In this section you will learn to:

- Identify and apply the four basic operations.
- Evaluate a mathematical expression.
- Identify and apply the Commutative Properties.
- Identify and apply the Associative Properties.
- Recognize the identities for addition and multiplication.
- Add by grouping to 10.

INTRODUCTION

There are a variety of occupations that have their own vocabulary and expressions. For example, nursing students need to learn relevant medical terminology and procedures; this is how doctors and nurses communicate with each other. A nurse must be able to understand what a doctor is saying and the doctor must understand the nurse.

The same is true of math students. As a math student, you must learn the definitions and properties of math so that you can understand your instructor—or this textbook—when those terms are discussed.

Likewise, if you have a question or a problem with math, you need to be able to effectively communicate to the instructor or a tutor. If you are unclear in your question, then the instructor may not answer it properly.

BASIC MATH TERMINOLOGY

There are four basic **operations** in mathematics, addition (+), subtraction (–), multiplication (x), and division (÷).

The *written form* of an operation, such as **5 + 4**, is called an **expression**.

We also have words for the <u>results</u> of the operations. To get the result of an operation we must *apply* the operation.

The chart below shows:

(1) the name of each operation;

(2) the operation as an expression (in written form);

(3) the given names of the parts of the operation; and

(4) the result of applying the operation to the numbers given.

Operation	As an expression (written form)	Result
Addition (**plus**)	**Addend** + **addend** 3 + 5	= **sum** = 8
Subtraction (**minus**)	**minuend** – **subtrahend** 9 – 5	= **difference** = 4
Multiplication (**times**)	**multiplier** x **multiplicand** 2 x 3 Also, **factor** x **factor**	= **product** = 6 = **product**
Division (**divided by**)	**dividend** ÷ **divisor** 6 ÷ 3	= **quotient** = 2

For multiplication, the numbers in a product are called **factors**. For example, 2 and 3 are factors of 6 because 2 x 3 = 6. 1 and 6 are also factors of 6 because 1 x 6 = 6.

For division, the quotient can also be expressed using the long division symbol

$$\begin{array}{r} \textbf{quotient} \\ \textbf{divisor}\,\overline{\big)\,\textbf{dividend}} \end{array} \text{, as in } \begin{array}{r} 2 \\ 3\,\overline{\big)\,6} \end{array}.$$

Example 1: Write both the expression and the result of each of the following.

	Answer:	**As an expression**		**As a result**
a)	the sum of 5 and 1	5 + 1	=	6
b)	the difference between 10 and 6	10 – 6	=	4
c)	the product of 4 and 7	4 x 7	=	28
d)	the quotient of 20 and 4	20 ÷ 4	=	5

YTI #1 Write both the expression and the result of each of the following. Use Example 1 as a guide.

		As an expression		**As a result**
a)	the sum of 9 and 3	________________	=	____________
b)	the difference between 12 and 4	________________	=	____________
c)	the product of 5 and 8	________________	=	____________
d)	the quotient of 30 and 5	________________	=	____________

YTI #2 Determine whether the expression shown is a sum, difference, product, or quotient.

Expression	Operation	Expression	Operation
a) 10 x 6	_______________	b) 15 – 8	_______________
c) 27 ÷ 9	_______________	d) 12 + 4	_______________

> To **evaluate** means to "find the value of."
>
> When we find the sum of the expression 3 + 5, we are simply finding a different way to express the *value* of 3 + 5; in other words, we are *evaluating* 3 + 5.

YTI #3 Evaluate each expression.

a) 5 + 3 = _____ b) 9 – 2 = _____ c) 4 x 5 = _____ d) 12 ÷ 3 = _____

THE COMMUTATIVE PROPERTY OF ADDITION

Evaluate 2 + 4 = _____ and 4 + 2 = _____ . For each of these, the resulting sum is 6. These two sums illustrate a simple, yet important, property of mathematics, the *Commutative Property of Addition*.

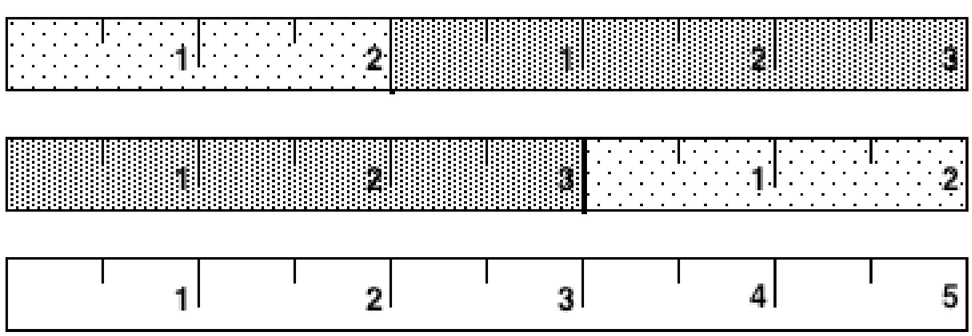
The **Commutative Property of Addition** states: When adding two numbers, it doesn't matter which number is written first, the resulting sum will be the same.

To understand this property visually, let's use pieces of two separate rulers, one that is 2 inches long and another that is 3 inches long:

First, as you know, 2 inches + 3 inches = 5 inches,

and it's also true that 3 inches + 2 inches = 5 inches.

In other words, it doesn't matter which measure is written first, the sum of 2 inches and 3 inches will always be 5 inches.



The Commutative Property of Addition is more formally written using letters, *a* and *b*, to represent any two numbers:

The Commutative Property of Addition

If a and b are any two numbers, then $a + b = b + a$.

In other words, the order in which we add two numbers doesn't affect the resulting sum.

Example 2: Write each expression two different ways using the Commutative Property.

Answer:

a) The sum of 7 and 3 can be written 7 + 3 or 3 + 7 .

b) The sum of 5 and 4 can be written 5 + 4 or 4 + 5 .

YTI #4 Write each expression two different ways using the Commutative Property. Use Example 2 as a guide.

a) The sum of 2 and 8 can be written ________________ or ________________ .

b) The sum of 6 and 1 can be written ________________ or ________________ .

THE COMMUTATIVE PROPERTY OF MULTIPLICATION

There is also a *Commutative Property of Multiplication.* We can use the multiplication table to see many examples of this property.

x	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	**24**	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	**24**	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

In the multiplication table, we can see that **3** x **8** = **24** and that **8** x **3** = **24**, and we can write this as 8 x 3 = 3 x 8.

In other words, when multiplying two numbers, it doesn't matter which number is written first, the resulting product will be the same. This is an example of the **Commutative Property of Multiplication**:

The Commutative Property of Multiplication

If ***a*** and ***b*** are any numbers, then $a \times b = b \times a$.

In other words, the order in which we multiply two numbers doesn't affect the resulting product.

Example 3: Write each expression two different ways using the Commutative Property.

Answer:

a) The product of 9 and 2 can be written 9 x 2 or 2 x 9 .

b) The product of 4 and 6 can be written 4 x 6 or 6 x 4 .

YTI #5 Write each expression two different ways using the Commutative Property. Use Example 3 as a guide.

a) The product of 5 and 6 can be written ______________ or ______________ .

b) The product of 4 and 9 can be written ______________ or ______________ .

THINK ABOUT IT:

What could you tell a classmate to help him or her remember why the two properties above are called *commutative* properties?

____

____

Caution: Division is *not* commutative. If we switch the order of the numbers in a division, then we don't get the same result. For example, 10 ÷ 5 is *not* the same as
5 ÷ 10. Here's an illustration:

At a youth car wash, if 5 youths wash a truck for $10, they get $2 each (10÷ 5 = 2); however, if 10 youths wash a car for $5, they get only $0.50 each (5 ÷ 10 = 0.50).

Similarly, subtraction is *not* commutative.

THE ASSOCIATIVE PROPERTIES

Another important property of mathematics is the *Associative Property*. There is an Associative Property for addition and one for multiplication. Before discussing these properties, we'll look at the role parentheses can play in an expression.

Parentheses, (and), are considered **grouping symbols**. Parentheses group different values together so that they can be treated as one *quantity*.

A **quantity** is an expression that is considered to be *one value*. For example, (6 + 2) is a quantity; it has just one value: 8.

Whenever we need to add three numbers together, such as 3 + 2 + 4, we must always choose to add two of them first. We can use parentheses to group two of them to create a quantity, then evaluate the quantity first.

> Generally, in any expression that has parentheses, the quantity within those parentheses should be evaluated *first*.

For example, with the sum 3 + 2 + 4, we can either group the first two numbers or the last two numbers:

a) (3 + 2) + 4

Add 3 and 2 first.

= 5 + 4

= 9

or

b) 3 + (2 + 4)

Add 2 and 4 first.

= 3 + 6

= 9

Notice that the resulting sum, 9, is the same no matter which grouping we choose.

The same is true of multiplying three numbers together, such as 3 x 2 x 4:

c) (3 x 2) x 4

Multiply 3 and 2 first.

= 6 x 4

= 24

or

d) 3 x (2 x 4)

Multiply 2 and 4 first.

= 3 x 8

= 24

Notice that the resulting product, 24, is the same no matter which grouping we choose.

The Associative Property uses letters *a*, *b,* and *c* to represent any numbers we wish to put in their place.

The Associative Properties of Addition and Multiplication

If ***a***, ***b***, and ***c*** are any three numbers, then

Addition	Multiplication
$(a + b) + c = a + (b + c)$	$(a \times b) \times c = a \times (b \times c)$
If the only operation is addition, we can change the grouping of the numbers without affecting the resulting sum.	If the only operation is multiplication, we can change the grouping of the numbers without affecting the resulting product.

Example 4: Write each expression two different ways using parentheses, then evaluate it.

a) 1 + 6 + 3 b) 6 x 2 x 3

Procedure: Use the Associative Property; notice that the order in which the numbers are written doesn't change.

Answer:

a) 1 + 6 + 3 = (1 + 6) + 3 = 1 + (6 + 3) = **10**
   (7 + 3) (1 + 9)

b) 6 x 2 x 3 = (6 x 2) x 3 = 6 x (2 x 3) = **36**
   (12 x 3) (6 x 6)

YTI #6 Write each expression two different ways using parentheses, then evaluate it. Use Example 4 as a guide.

a) 2 + 8 + 6 = __________ = __________ = ______

b) 4 + 6 + 9 = __________ = __________ = ______

c) 3 x 2 x 4 = __________ = __________ = ______

d) 6 x 5 x 2 = __________ = __________ = ______

THINK ABOUT IT:

What could you tell a classmate to help him or her remember why the two properties above are called *associative* properties?

____

____

THE IDENTITIES

The notion of *identity* is another important property of addition and multiplication. An **identity** is a number that, when applied, won't change the value of another number or quantity. In other words, it keeps the original number's *identity*, its value.

For addition, the identity is 0 (zero), because

a) 6 + **0** = 6 b) **0** + 3 = 3 0 is called the **additive identity**.

Notice that adding 0 (zero) to a number doesn't change the value of the number.

For multiplication, the identity is 1 (one), because

a) 5 x **1** = 5 b) **1** x 8 = 8 1 is called the **multiplicative identity**.

Notice that multiplying a number by 1 doesn't change the value of the number.

YTI #7 Apply the idea of identity by filling in the blank.

a) 4 + 0 = ______ b) 9 + ______ = 9 c) 0 + ______ = 12

d) 7 x 1 = ______ e) 23 x ______ = 23 f) 1 x ______ = 15

THINK ABOUT IT:

What could you tell a classmate to help him or her remember why the two numbers above are called *identities*?

____

____

FAST ADDITION, GROUPING TO 10

The Associative and Commutative Properties allow us to rearrange the numbers in a sum (or product) without changing the value of the end result.

For example, the sum 7 + 9 + 3 could be rearranged as and then we could group the first two numbers:

  7 + 3 + 9,
  (7 + 3) + 9
= 10 + 9
= 19

The advantage of rearranging the numbers this way is to add 7 and 3 first because they add to 10, and 10 is a good number to work with in addition (and in multiplication).

In rewriting 7 + 9 + 3 as 7 + 3 + 9 and then as (7 + 3) + 9 we have used both the Commutative and Associative Properties of Addition.

This idea of combining the Associative and the Commutative Properties of Addition together leads to a quick addition process called *grouping to 10*. This speedy process works well when we need to add a lot of numbers.

The idea of grouping to 10 is to find numbers that easily add to 10, such as 4 and 6. We sometimes use three numbers; for example, 2 and 3 and 5 add up to 10 as well. We can use these facts to quickly add a list of numbers.

Example 5: Given the following list of numbers, find groups that add to 10, then find the sum of the entire list.

a) 6, 4, 8, 2, 1 and 9

b) 9, 3, 8, 1, 5, 7, 2 and 5

c) 1, 6, 3, 2, 4 and 4

d) 9, 6, 1, 2, 5, 4 and 5

Procedure: Look for groups of two or three numbers that add to 10; then add the 10's. Rearrange the list if necessary.

Answer:

a) Sometimes the list is organized nicely:

6, 4, 8, 2, 1, and 9

(6 + 4) + (8 + 2) + (9 + 1)

= 10 + 10 + 10

= **30**

b) Sometimes we need to search for the numbers that add to 10, and rearrange them:

9, 3, 8, 1, 5, 7, 2 and 5

(9 + 1) + (3 + 7) + (8 + 2) + (5 + 5)

= 10 + 10 + 10 + 10

= **40**

c) Sometimes we can get sums of 10 by grouping *three* numbers:

1, 6, 3, 2, 4, and 4

(1 + 6 + 3) + (2 + 4 + 4)

= 10 + 10

= **20**

d) Sometimes there are numbers that can't be grouped to form a sum of 10; these need to be added in at the end:

9, 6, 1, 2, 5, 4 and 5

(9 + 1) + (6 + 4) + (5 + 5) + 2

= 10 + 10 + 10 + 2

= **32**

YTI #8 Given the following list of numbers, find groups that add to 10, then find the sum of the entire list. Use Example 5 as a guide.

a) 6, 4, 9, 1, 8, 2, and 4

b) 9, 2, 5, 8, 5, and 1

c) 5, 2, 4, 3, 1, 3, and 7

GROUPING TO 10 AND THE ASSOCIATIVE PROPERTY

Adding to 10 has its advantages. From Section 1.1 you'll remember that we can exchange ten $1 bills for a single $10 bill. So, we can use 10 as a "target" number when adding two digits whose sum is *greater* than 10. This technique requires the use of the Associative Property, as you'll see in Example 6.

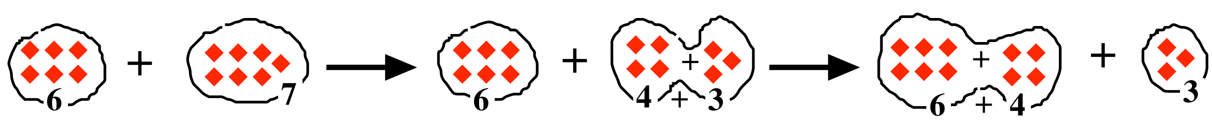
Even though you probably already know that the sum of 6 and 7 is 13 (6 + 7 = 13), the method presented in Example 6 shows *why* the sum is 13.

Example 6: Find the sum of 6 and 7 by breaking up the second number into the sum of two smaller numbers.

Procedure: Keep the number 10 as a target number in the addition process.

Answer:

Start with 6; think of the number that you can add to 6 to make 10: 6 + **4** = 10		6 + 7
Break 7 up into a sum of two numbers, one of which is **4**.	=	6 + (4 + 3)
Use the Associative Property to regroup as shown.	=	(6 + 4) + 3
Now add within the parentheses to get 10.	=	10 + 3
Now complete the addition. (We get 1 ten and 3 ones.)	=	13



Caution: For the sake of understanding this addition better than you already do, please follow the steps outlined in Example 6, using 10 as a target number.

YTI #9 Find the following sums using the technique outlined in Example 6.

a) 8 + 7

= 8 + (+)

= (8 +) + ____

= 10 + ____

= ____

b) 9 + 5

= 9 + (+)

= (9 +) + ____

= 10 + ____

= ____

c) 6 + 9

= 6 + (+)

= (6 +) + ____

= 10 + ____

= ____

You Try It Answers

YTI #1:	a)	$9 + 3 = 12$	b)	$12 - 4 = 8$	c)	$5 \times 8 = 40$	d)	$30 \div 5 = 6$
YTI #2:	a)	product	b)	difference	c)	quotient	d)	sum
YTI #3:	a)	8	b)	7	c)	20	d)	4

YTI #4: a) $\underline{2 + 8}$ or $\underline{8 + 2}$ b) $\underline{6 + 1}$ or $\underline{1 + 6}$

YTI #5: a) $\underline{5 \times 6}$ or $\underline{6 \times 5}$ b) $\underline{4 \times 9}$ or $\underline{9 \times 4}$

YTI #6:
a) $(2 + 8) + 6 = 2 + (8 + 6) = 16$ b) $(4 + 6) + 9 = 4 + (6 + 9) = 19$
c) $(3 \times 2) \times 4 = 3 \times (2 \times 4) = 24$ d) $(6 \times 5) \times 2 = 6 \times (5 \times 2) = 60$

YTI #7: a) 4 b) 0 c) 12 d) 7 e) 1 f) 15

YTI #8: a) 34 b) 30 c) 25

YTI #9:

a)
$$\begin{aligned} & 8 + 7 \\ = {} & \underline{8 + (2 + 5)} \\ = {} & \underline{(8 + 2) + 5} \\ = {} & \underline{10 + 5} \\ = {} & 15 \end{aligned}$$

b)
$$\begin{aligned} & 9 + 5 \\ = {} & \underline{9 + (1 + 4)} \\ = {} & \underline{(9 + 1) + 4} \\ = {} & \underline{10 + 4} \\ = {} & 14 \end{aligned}$$

c)
$$\begin{aligned} & 6 + 9 \\ = {} & \underline{6 + (4 + 5)} \\ = {} & \underline{(6 + 4) + 5} \\ = {} & \underline{10 + 5} \\ = {} & 15 \end{aligned}$$

Focus Exercises

Which property does each represent?

1. 5 + (8 + 2) = (5 + 8) + 2 ______________________________

2. 12 x 5 = 5 x 12 ______________________________

3. 0 + 11 = 11 ______________________________

4. 8 x (5 x 4) = (8 x 5) x 4 ______________________________

5. 25 + 19 = 19 + 25 ______________________________

6. 13 x 1 = 13 ______________________________

Given the following list of numbers, find groups that add to 10, then find the sum of the entire list.

7. 7, 3, 5, 5, 2, 8, 6

8. 3, 9, 6, 4, 5, 7, 1

9. 1, 6, 3, 4, 1, 5, 8

10. 5, 3, 4, 9, 2, 8, 1, 7, 5

11. 5, 1, 4, 7, 3, 6, 5

12. 5, 3, 2, 1, 6, 3, 2

13. 2, 7, 4, 9, 3, 8, 6

14. 1, 2, 4, 7, 1, 5, 9

Find the following sums using the technique outlined in Example 6.

15. 5 + 7

= 5 + (+)

= ______

= ______

= ______

16. 9 + 8

= 9 + (+)

= ______

= ______

= ______

17. 8 + 6

= 8 + (+)

= ______

= ______

= ______

18. 7 + 6

= 7 + (+)

= ______

= ______

= ______

19. 6 + 9

= 6 + (+)

= ______

= ______

= ______

20. 4 + 8

= 4 + (+)

= ______

= ______

= ______

21. 9 + 9

= 9 + (+)

= ______

= ______

= ______

22. 5 + 8

= 5 + (+)

= ______

= ______

= ______

23. 7 + 9

= 7 + (+)

= ______

= ______

= ______