Section 1.7 Solving Applications Using Equations

Objectives
In this section, you will learn to:

- Solve applications involving addition and subtraction.
- Solve applications involving multiplication and division.
- Solve applications involving averages.
- Solve applications involving perimeter and area.

To successfully complete this section, you need to understand:

- Rounding Whole Numbers (1.1)
- Adding Whole Numbers (1.3)
- Subtracting Whole Numbers (1.3)
- Multiplying Whole Numbers (1.4)
- Dividing Whole Numbers (1.5)
- Solving Equations (1.6)

INTRODUCTION

Now we are ready to apply our understanding of solving equations to applications. Here is a situation we might find in the workplace:

> Tony needs to make sure his truck isn't carrying too much weight for the road conditions ahead. His truck weighs 3,125 pounds and it is carrying a 586-pound load. What is the total weight of the truck and the load?

As you can see, an application is simply an equation written in sentence form. Approach the problem as if it were an assignment given to you by a boss or supervisor. Impressing a supervisor with your math skills makes you more valuable to the company.

One formula that we use very often is:

The sum of all of the parts equals the whole.

As you might imagine, the "whole" is always larger than any individual "part."

Some words that might help you identify the whole are "total," "combined" (as in *combined* weight) and "altogether." Mostly, though, you should rely less on words and more on *thinking* about the situation to identify the whole and the parts.

If an application has only two parts, say Part 1 and Part 2, and if the unknown value is the whole, then the rule could be written as:

Part 1 + Part 2 = Whole (n)

If, instead, the unknown value is one of the parts, then this formula could be written as

Part 1 + Unknown Part (n) = Whole

Key steps for Solving Application Problems

1. Think about the application

a) by putting yourself in the situation,

b) by drawing a diagram,

c) by making estimates about the answer, and

d) by using smaller numbers and making a model of the situation.

2. Decide what is known and what is unknown.

a) Each number is either a part or the whole. Usually, the last sentence in the problem indicates what is unknown; this is *n*.

b) Write a **legend** identifying what n represents.

The **legend**, in this regard, is like that found on a map. A legend lists symbols and explains what each symbol represents.

3. Write an equation, based on a formula.

4. Solve the equation by isolating the variable.

5. Write a complete sentence to answer the question in the application problem. Usually, it's possible to just reword the question and put it in the form of an answer.

In the problem-solving process, it's a good idea to read the problem through once, just to get an idea of the situation. Then read it a second time and look for the important information, thinking about which might be the whole and which might be the parts.

As you read it the second time, either underline or put a box around important information. Also, in the last sentence underline the unknown value and write *n* under it; you will use this for your legend.

Tony needs to make sure his truck isn't carrying too much weight for the road conditions ahead. His truck weighs 3,125 pounds and it is carrying a 586-pound load. What is the total weight of the truck and the load?

n

APPLICATIONS INVOLVING ADDITION AND SUBTRACTION

Let's use the key steps for solving applications to answer the questions in Examples 1 and 2.

Example 1: Tony needs to make sure his truck isn't carrying too much weight for the road conditions ahead. His truck weighs 3,125 pounds and it is carrying a 586-pound load. What is the total weight of the truck and the load?

Procedure: First, reread the problem and put a box around important information. Put yourself in Tony's spot. Picture the truck and picture the load going into the truck. As you might imagine, the truck weighs more *with* the load than it does when empty.

In this case, the parts are

(1) The weight of the truck when there is no load (3,125 pounds), and

(2) the weight of the load that he is carrying (586 pounds).

From the last sentence we know the unknown value, the total weight, is the whole.

Answer: We start with the **legend**: Let n = the total weight

Part 1 + Part 2 = Whole

Write the equation: 3,125 + 586 = n

Apply addition: 3,711 = n

$$\begin{array}{r} 3125 \\ +\ 586 \\ \hline 3711 \end{array}$$

Since the legend says that n is the total weight, we can write a sentence based on the question, "What is the total weight of the truck and the load?"

Sentence: The total weight is 3,711 pounds.

Example 2: Julia is delivering old newspapers to the recycler. The scale indicates the total weight to be 3,928 pounds. After the papers have been emptied, the scale indicates the weight to be 3,195 pounds. What was the weight of the newspaper she delivered for recycling?

Procedure: First, reread the problem and put a box around important information. Use the outline shown below. These steps aren't necessary but are a useful aid in thinking about the situation.

a) Write down any known information.

- The total weight (whole) is 3,928 pounds.
- The empty weight is 3,195 pounds.

Also write what we're trying to find.

- Find the weight of the newspapers.

c) Write the legend: Let n = the weight of the newspapers. We know the whole, so the unknown value is one of the parts.

Answer:

d) Write the formula: Part 1 + Unknown Part = Whole

Write the equation: 3,195 + n = 3,928

Subtract 3,195 from each side: 3,195 – 3,195 + n = 3,928 – 3,195

n = 733

$$\begin{array}{r} 3928 \\ -\ 3195 \\ \hline 733 \end{array}$$

e) **Sentence:** The weight of the newspapers is 733 pounds.

Caution:	Although it's true that the whole will be larger than any of the individual parts, just because one number is larger than another doesn't make it the whole.

The You Try It exercises in this section show how you might organize the written information given in a problem. Use these exercises as guides to help you in solving the Focus Exercises at the end of the section.

YTI #1 Mark owns a courier service and uses his small plane to make deliveries. His plane can carry cargo that has a combined weight of 1,280 pounds. One customer has asked him to deliver several large packages that total 891 pounds. How much more cargo weight can his plane carry? (Hint: His plane is not full, so there is space available.)

a) Write down any known information.

Whole: _____ Parts: _____

b) Legend: _____

c) Write the equation and solve by isolating the variable.

d) Sentence: _____

YTI #2 One of Kami's customers places an order for 113 shirts. Kami already has 22 of that style in stock. How many more shirts of that style does Kami need?

a) Write down any known information. Also write what we're trying to find.

Whole: _____ Parts: _____

b) Legend: _____

c) Write the equation and solve by isolating the variable.

d) Sentence: _____

APPLICATIONS INVOLVING MORE THAN TWO PARTS

There are many applications that require adding more than two numbers together. Also, some applications involve both addition and subtraction.

Example 3: Mark owns a courier service and uses his small plane to make deliveries. One customer has asked him to deliver cargo that weighs 289 pounds. Another needs to ship 481 pounds and a third customer wishes him to deliver 340 pounds. What is the total weight of the cargo he will be carrying?

Procedure: First, reread the problem and put a box around important information. This problem has three parts, a cargo weight from each customer. The unknown value is the total weight, the whole.

Answer: Legend: Let n = the total weight of the cargo.

$$\text{Part 1} + \text{Part 2} + \text{Part 3} = \text{Whole}$$

$$289 + 481 + 340 = n$$

$$\begin{array}{r} 289 \\ 481 \\ +\ 340 \\ \hline 1110 \end{array}$$

$$1{,}110 = n$$

$$n = 1{,}110$$

Sentence: Mark will be carrying 1,110 pounds of cargo.

Example 4: [This is the same situation as Example 3 with a different question to answer.]

If Mark's plane can carry a maximum cargo weight of 1,280 pounds, how much more cargo weight can his plane carry?

Procedure: First, reread the problem and put a box around important information. This problem has four parts, the original three cargo weight and how much more weight his plane can carry.. This time the whole is the total weight that he is allowed to carry, the 1,280 pounds.

Answer: Legend: Let n = how much more weight his plane can carry.

Equation: $\text{Part 1} + \text{Part 2} + \text{Part 3} + \text{Part 4} = \text{Whole}$

$$289 + 481 + 340 + n = 1{,}280$$

$$\begin{array}{r} 289 \\ 481 \\ +\ 340 \\ \hline 1{,}110 \end{array}$$

$$1{,}110 + n = 1{,}280$$

$$n + 1{,}110 - 1{,}110 = 1{,}280 - 1{,}110$$

$$\begin{array}{r} 1{,}280 \\ -\ 1{,}110 \\ \hline 170 \end{array}$$

$$n = 170$$

Sentence: Mark's plane can carry 170 more pounds of cargo.

YTI #3 At her bookstore this month, Gena sold 167 books the first week, 228 books the second week, and 174 books the third week. How many books must she sell during the fourth week to reach her monthly goal of 700?

a) Write down any known information.

Whole: _____ Parts: _____

b) Write what we're trying to find in the legend.: _____

c) Write the equation and solve by isolating the variable.

d) Sentence: _____

APPLICATIONS INVOLVING MULTIPLICATION

In this section we will solve applications that involve multiplication. Just as in situations that involve addition, there are some values that are known and some that are unknown. As with addition, we use an equation to solve for the unknown value.

The formula for addition is: the sum of all of the parts equals the whole.

For multiplication, we use the same formula; the difference is that the parts are *repeated* parts. The formula can be shown as:

Part + Same Part + Same Part + ... + Same Part = Whole.

This formula can also be rewritten as a product:

(Number of Same Parts) x Part = Whole.

For example, Kahlil earns $87.00 per day working as a plumber's assistant. How much does he earn in a 5-day week?

His earnings can be looked at as $\underbrace{\$87}_{\text{Monday}} + \underbrace{\$87}_{\text{Tuesday}} + \underbrace{\$87}_{\text{Wednesday}} + \underbrace{\$87}_{\text{Thursday}} + \underbrace{\$87}_{\text{Friday}}$

This is the same part added repeatedly, and there are 5 of them, so the equation is $5 \times \$87 = \text{whole}$

$5 \times \$87 = \$435$

In the application where a number is added repeatedly, the *unknown value* will be either:

(1) the part,

(2) the number of parts (number of repeats), or

(3) the whole.

Identifying the unknown value is relatively simple because it is usually mentioned in the question. However, determining whether the unknown is the whole or not can be a little more challenging.

Guidelines for Recognizing the Whole, the Part, and the Number of Parts

1. The whole and the part always have the same unit measure. They might both be inches, or cookies, or dollars, or miles, or many other units of measure, but they must always be the same.
2. The whole is always larger than the part.
3. The number of parts will almost always be of a different unit of measure, such as the number of days, the number of boxes, or the number of months.

Yet, with all of the helpful information you might find on these pages, it's still best to approach the problem by *thinking*, by putting yourself in the situation and by looking for key words.

The examples presented in this section will give you some insight into the best approach; however, it is still up to you to think about the situation, beyond simply reading the words.

Example 5: Union dues are $432 a year and are split into 12 equal monthly payments. How much must Gloria pay each month for union dues?

Procedure: First, reread the problem and put a box around important information. Use the guidelines above to think about the problem and to set up the equation.

The unknown value is: n = how much (dollars) Gloria pays each month.

The whole is the total dues (also dollars) she pays for the year: $432

The part is how much she pays each month. The part occurs 12 times.

Notice that the whole and the part are both measured in dollars; the other measure is number of months.

Answer: Legend: The amount Gloria pays each month for union dues.

The equation is this: $12 \cdot n = 432$

Solve it:

$12 \div 12 \cdot n = 432 \div 12$

$n = 36$

$$\begin{array}{r} 36 \\ 12\overline{)\,432} \\ -\,36 \\ \hline 72 \\ -\,72 \\ \hline 0 \end{array}$$

Sentence: Gloria must pay $36 each month for union dues.

YTI #4 Keith works in a factory that produces 600 nails per hour, and 40 nails fit into each box. How many boxes are needed for those 600 nails?

a) What is the unknown value? Legend: n = _____

b) What value (known or unknown) represents the whole? _____

c) What value (known or unknown) represents the part? _____

d) What value (known or unknown) represents the number of times the part occurs? _____

e) Write the equation and solve it.

f) Sentence: _____

Example 6: It cost Adrian $29 per day to rent a car. He rented the car for 7 days. How much total rent did Adrian pay for the car?

Procedure: First, reread the problem and put a box around important information. The unknown value is the total paid (in dollars), the whole:

The part is $29 per day (notice that it's also in dollars, same as the whole.)

The part is repeated 7 times (7 days).

Answer: Legend: n = the total rent Adrian paid for the car.

$$7 \cdot 29 = n$$

$$203 = n$$

$$n = 203$$

$$\begin{array}{r} 29 \\ \times\ 7 \\ \hline 203 \end{array}$$

Sentence: Adrian paid $203 to rent the car.

YTI #5 Mary contributes $15 from her paycheck each month to the United Way. How much does she contribute in a year (in 12 months)?

a) What is the unknown value? Legend: n = _____

b) What value (known or unknown) represents the whole? _____

c) What value (known or unknown) represents the part? _____

d) What value (known or unknown) represents the number of times the part occurs? _____

e) Write the equation and solve it.

f) Sentence: _____

APPLICATIONS INVOLVING AVERAGES

For this next situation, you see the key words "on average." An average suggests that the part was divided *evenly* among the whole. We usually also find the word "each" with the words "on average;" this, again, indicates a part.

Example 7: Sally, a saleswoman at an electronic appliance store, worked 20 days in February. She sold a total of 140 televisions during that month. How many televisions did she sell, on average, each day?

Procedure: First, reread the problem and put a box around important information. The key words, appearing in the question, are on average. This means the unknown value is the part.

The measure that appears twice in the problem is televisions. The unknown value is the average number of televisions sold (the part).

The whole is 140 televisions.

The part is repeated 20 times (20 days).

Answer: Legend: n = the average number of televisions sold per day.

$$20 \cdot n = 140$$

$$20 \div 20 \cdot n = 140 \div 20$$

$$n = 7$$

Sentence: Sally sold, on average, 7 television sets each day.

Does this mean that Sally actually sold 7 TV sets each day? Probably not.

If she sold 7 sets every day, a chart of her TV sales might look like this:

Day	Feb. 1	Feb. 2	Feb. 3	Feb. 4	Feb. 5	Feb. 8	Feb. 9	**etc.**
# TV sets	7	7	7	7	7	7	7	.....

Most likely, though, Sally had some good selling days and some poor selling days. One day she may have sold as many as 15 TV sets and another day she may have sold as few as 2. But, over the course of one full month, she was able to sell an average of 7 sets per day.

YTI #6 At her bookstore, Gena sold 165 books in one five-day period. How many books did Gena sell, on average, each day?

a) What is the unknown value? Legend: n = _____

b) What value (known or unknown) represents the whole? _____

c) What value (known or unknown) represents the part? _____

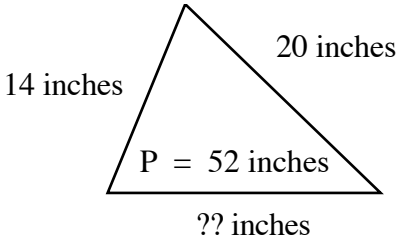
d) What value (known or unknown) represents the number of times the part occurs? _____

e) Write the equation and solve it.

f) Sentence: _____

APPLICATIONS INVOLVING GEOMETRY

Recall from Section 1.3 that the perimeter of a figure is the sum of the lengths of its sides. If we know the perimeter (P) of a triangle, but only two of the side measures, then we can use an equation to find the length of the third side.



We can let *n* be the length of the unknown side and solve for it using *the sum of the parts equals the whole*.

Example 8: The perimeter of a triangle is 52 inches. The longest side measures 20 inches and the shortest side measures 14 inches. What is the length of the third side?

Procedure: First, reread the problem and put a box around important information. If a triangle is not provided, you can draw one of your own and label the sides. The triangle above can be used for this problem.

Answer: Legend: n = the measure of the third side

$20 + 14 + n = 52$	Simplify the left side by combining the numbers.
$34 + n = 52$	Subtract 34 from each side.
$34 - 34 + n = 52 - 34$	Simplify.
$n = 18$	

Sentence: The third side is 18 inches long.

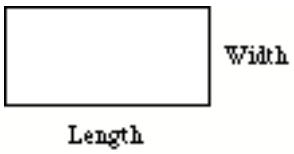
YTI #7 A park is in the shape of a triangle. The perimeter of the sidewalk around it is 168 yards. The shortest side is 42 yards and the longest side is 70 yards. What is the length of the third side? Draw a triangle and place the known measures around it.

Legend: n = _____

Sentence: _____

Recall from Section 1.4 that the area of a rectangle is the product of its length and width:

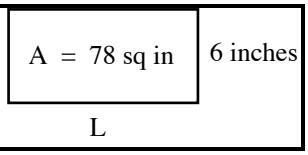
Area of a rectangle = Length x Width



If the area and the measure of one side are known, then we can find the measure of the other side using an equation.

Example 9: The area of a rectangle is 78 square inches. The width is 6 inches. What is the length?

Procedure: We can draw a rectangle and label the width as 6 and the length as L. We can even use L, instead of n, in the equation: $A = L \cdot W$.



Answer: Legend: L = the length of the rectangle

$$78 = L \cdot 6 \quad \leftarrow \text{Divide each side by 6.}$$

$$78 \div 6 = L \cdot 6 \div 6$$

$$13 = L$$

$$\begin{array}{r} 13 \\ 6\overline{)78} \\ -\ 6 \\ \hline 18 \\ -\ 18 \\ \hline 0 \end{array}$$

Sentence: The length of the rectangle is 13 inches.

YTI #8 A rectangular dance floor has an area of 255 square feet. The length is 17 feet. What is the width? Draw a rectangle and place the known measures around it.

Legend: n = _____

Sentence: _____

You Try It Answers

YTI #1:
a) Whole is 1,280 pounds; part is 891 pounds
b) Let n = how much more weight can be carried
c) $891 + n = 1{,}280$
d) Mark's plane can carry 389 pounds more.

YTI #2:
a) Whole is 113 shirts; part is 22 shirts
b) Let n = the # of shirts needed
c) $22 + n = 113$
d) Kami needs to order 91 more shirts.

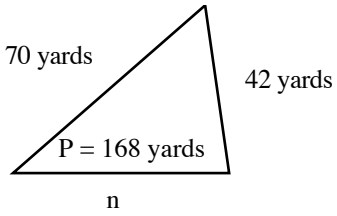
YTI #3:
a) Whole is 700 books; parts are 167 books, 228 books, and 174 books
b) Let n = the # of books Gena must sell during the fourth week.
c) $167 + 228 + 174 + n = 700$
d) Gena must sell 131 books during the fourth week.

YTI #4:
a) Let n = the # of boxes needed.
b) Whole is 600 nails
c) Part is 40 nails
d) The number of times the part occurs is n.
e) $40 \cdot n = 600$
f) Keith needs 15 boxes for the 600 nails.

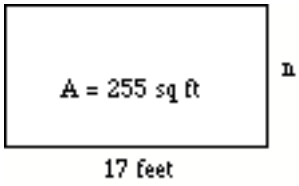
YTI #5:
a) Let n = the amount (in dollars) she will contribute in one year.
b) Whole is n
c) Part is \$15
d) The number of times the part occurs is 12 months.
e) $12 \cdot 15 = n$
f) In one year, Mary will contribute \$180 to the United Way.

YTI #6:
a) Let n = the average number of books sold each day.
b) Whole is 165 books
c) Part is n
d) The number of times the part occurs is 5 days.
e) $5 \cdot n = 165$
f) Gena sold, on average, 33 books each day.

YTI #7:
Let n = the length of the third side.
$42 + 70 + n = 168$
The third side is 56 yards long.



YTI #8:
Let n = the width of the rectangle.
$17 \cdot n = 255$
The width is 15 feet.



Focus Exercises

Work each application. Write a legend and an equation; solve the equation, and answer the question in a complete sentence.

1. Nate is saving his money to buy an Xbox. So far he has $73 saved. The model he wants sells for $162, including tax. How much more money does Nate need to pay for the Xbox?

2. Shika needs 132 yards of material to make enough doll dresses to sell at the craft fair. She already has 58 yards of material. How many more yards of material does Shika need?

3. Adam has saved $900 for his vacation. He has figured that he'll spend $380 on the hotel room, $140 on meals, and $115 on transportation. How much will Adam have left over to spend on other things during his vacation?

4. A cab company spends $18,652 per year in insurance for its drivers. Insuring a new driver costs $1,208. How much total insurance will the company pay if it adds one more driver?

5. Sandra works for the city accounting office. She must prepare a report for the city manager listing a variety of costs that must be paid. For example, it cost the city $21,000 to operate and maintain the public pool. So far, the city has collected $19,629 from user fees. How much more must the city collect to equal the costs of operating the pool?

6. Through a regular deduction from her paycheck, Alexis paid a total of $2,867 last year for federal taxes. In filing her income tax return, she really needed to pay $3,052. How much does Alexis still owe on her federal taxes?

7. Kinko's is putting together 5,000 brochures for a company, due tomorrow at noon. Kelley has assigned the job to three different workers, each using a different photocopier. Heather's copier has produced 1,758 copies; Omar's copier has produced 1,365 copies; and Carla's copier has produced 1,259 copies. How many copies are left to produce?

8. The billing department in a large company had $15,000 in their annual supplies budget. So far this year they have purchased a computer system for $3,193, new furniture for $2,607, and various office supplies for $612. It's Rika's job to update the department's budget. How much is left in the supplies budget after these purchases?

9. Ajay belongs to the local carpenter's union. He pays $252 per year in union dues. If they are spread equally over 12 months, how much does Ajay pay each month in union dues?

10. Kami owns an embroidery business, and she just received a rush order. She has to embroider 168 caps in 7 days. How many caps, on average, must Kami embroider each day?

11. Beth is a nutritionist. She has many overweight clients. Most of her clients are referred to her by doctors who work at a nearby hospital for eating disorders.

a) One of her heaviest clients has been put on a strict diet and exercise program by his doctor. His total weight loss, over a 6 month period, was 138 pounds. How much many pounds did he lose, on average, each month?

b) Beth has another client, Lona, who is on a plan to lose 15 pounds a month for 12 months. Assuming that Lona stays on the plan, what will her total weight loss be?

12. Mansour owns a courier service and uses his small plane to make deliveries.

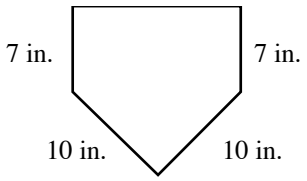
a) One customer has asked him to deliver 25 boxes, each weighing 42 pounds. How much cargo will Mansour's plane be carrying?

b) A customer has 24 boxes weighing a total of 864 pounds. What is the average weight of each box?

13. Aimee pays $375 for rent each month. How much does Aimee pay in rent for a full year?

14. Allison works at a printer. The business often receives large printing orders for pamphlets and flyers. One customer wants 3,200 flyers printed and needs them right away. Allison decides to use 5 printing machines to do the job. On average, how many flyers will each machine print?

15. The home plate in a youth sports league is shown at right. The perimeter of the plate is 48 inches. What is the length of the top side?



16. The Bermuda Triangle, famous for disappearing ships and planes, forms a triangle between three Atlantic locations: Miami, Bermuda, and San Juan. The perimeter of this triangle is 2,992 miles. The distance from Bermuda to Miami is 1,059 miles. The distance from Bermuda to San Juan is 955 miles. What is the distance from San Juan to Miami?

17. A rectangular plot of land is a full acre: 4,840 square yards. If the width is 40 yards, what is the length of the rectangle? Draw a rectangle and place the known measures around it.

18. A hotel pool is in the shape of a rectangle. The rectangle is 15 yards wide and has an area of 540 square yards. What is the length of the pool?