Section 1.5 Exponents, Square Roots, and the Order of Operations

Objectives
In this section, you will learn to:

- Identify perfect squares.
- Use exponents to abbreviate repeated multiplication.
- Evaluate powers of 10.
- Find square roots of perfect squares.
- Evaluate expressions by applying the order of operations.

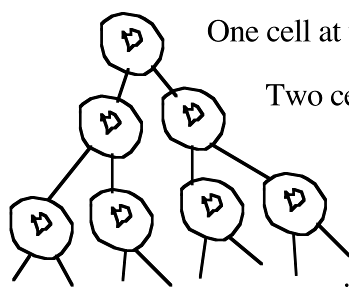
To successfully complete this section, you need to understand:

- Evaluating quantities (1.1)
- The multiplication table (1.1)
- The definition of *factors* (1.1)

INTRODUCTION

Exponents and square roots are found in many formulas in math, chemistry, physics, economics, and statistics. The ability to perform calculations with exponents and square roots, for instance, helps astronomers provide a safe flight for the space shuttle and for the many satellites orbiting the earth.

Research biologists often use exponents to explain the increase or decrease in the number of cells they are observing. When cells divide, split in two, the researcher can count twice as many cells as before.



For example, Sheila is watching over the growth of a bacteria used in making yogurt, and she is taking notes on what she sees. Sheila notices that the cells double in number every minute. Here is a summary of her notes:

Number of cells at start	at 1 minute	at 2 minutes	at 3 minutes	at 4 minutes	at 5 minutes
1	$1 \cdot 2$	$2 \cdot 2$	$4 \cdot 2$	$8 \cdot 2$	$16 \cdot 2$
1 cell	= 2 cells	= 4 cells	= 8 cells	= 16 cells	= 32 cells

As part of her research, Sheila must make a prediction about the number of cells present at 8 minutes. Assuming the pattern will continue, Sheila notices that the number of cells can be predicted each minute by multiplying by another factor of 2:

Number of cells at start	At 1 minute	At 2 minutes	At 3 minutes	At 4 minutes	At 5 minutes
1	2	$2 \cdot 2$	$2 \cdot 2 \cdot 2$	$2 \cdot 2 \cdot 2 \cdot 2$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

She realizes that at 3 minutes, there are three factors of 2, at 4 minutes there are four factors of 2, and so on. With this in mind, Sheila predicts that at 8 minutes there will be eight factors of 2 bacteria:

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 256.$$

As you might imagine, representing the number of bacteria by this repeated multiplication can become a bit tedious. Instead, there is a way that we can abbreviate repeated multiplication, and that is with an exponent.

EXPONENTS

Eight factors of 2 can be abbreviated as 2^8. The repeated factor, 2, is called the **base**, and the small, raised ^{8} is called the **exponent** or **power**. 2^8 is read as "2 to the eighth power."

For values raised to the second power, such as 6^2, it's most common to say that the base is "squared," such as "6 squared." The phrase 6 squared comes from the area of a square in which each side has a length of 6 units, as you will see later in this section.

2^8 is called the exponential form and $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ is called the expanded form.

An exponent indicates the number of factors of the base.

Example 1: Write each in exponential form.

a) $7 \cdot 7 \cdot 7$ b) $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ c) $5 \cdot 5$

Procedure: To identify the exponent, count the number of factors of the base.

Answer:

a) 7^3 There are three factors of 7.

b) 10^6 There are six factors of 10.

c) 5^2 There are two factors of 5.

YTI #1 Write each in exponential form.. Use Example 1 as a guide.

a) $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ b) $3 \cdot 3 \cdot 3 \cdot 3$ c) $1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$

Example 2: Expand each and find its value.

a) 2^3 b) 3^4 c) 7^2

Answer:

a) $2^3 = 2 \cdot 2 \cdot 2 = 8$ — 2^3 means three factors of 2.

b) $3^4 = 3 \cdot 3 \cdot 3 \cdot 3$

$= (3 \cdot 3) \cdot 3 \cdot 3)$

$= 9 \cdot 9 = 81$

If you use the Associative Property to group two factors at a time and then multiply them, you'll get $9 \cdot 9$ which is 81.

c) $7^2 = 7 \cdot 7 = 49$ — 7^2 means two factors of 7.

YTI #2 Expand each and find its value. Use Example 2 as a guide.

a) 6^2 b) 2^4 c) 9^3

____________________ ____________________ ____________________

How should we interpret 5^1? 5^1 means "one factor of 5." In this case, there are no repeated factors of 5, there is only 5 itself. In other words, $5^1 = 5$.

This principle is true for any base: $\boxed{\text{If } b \text{ represents any base, then } b^1 = b.}$

YTI #3 Rewrite each without an exponent.

a) 2^1 b) 9^1 c) 17^1 d) 1^1

THE POWERS OF 10

Numbers such as 100, 1,000 and 10,000 are called **powers of 10** because

$$10^2 = 10 \cdot 10 = 100$$
$$10^3 = 10 \cdot 10 \cdot 10 = 1{,}000$$
$$10^4 = 10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$$

Based on the results above, notice that ...

The exponent of 10 indicates the number of zeros that follow the 1.

So, 10^6 is a 1 followed by six zeros: 1,000,000; in other words, 1,000,000 is the 6th power of 10. Likewise, 10,000,000 is a 1 followed by seven zeros and is abbreviated as 10^7.

Example 3: For each power of 10, write its value or abbreviate it with an exponent, whichever is missing.

a) 1,000 b) 100,000 c) 10^4 d) 10^8

Procedure: For a) and b), count the number of zeros; for c) and d), the exponent indicates the number of zeros following a 1.

Answer:
a) $1{,}000 = 10^3$ b) $100{,}000 = 10^5$

c) $10^4 = 10{,}000$ c) $10^8 = 100{,}000{,}000$

YTI #4 For each power of 10, write its value or abbreviate it with an exponent, whichever is missing. Use Example 3 as a guide.

a) 10^5 b) 10^7 c) 10^1

d) 100 e) 10,000 f) 1,000,000

Three hundred can be written as 300 and as $3 \cdot 100$. This can be further abbreviated as $3 \cdot 10^2$. In other words, $300 = 3 \cdot 10^2$.

When a number ends in one or more zeros, it can be written as a product of a whole number and a power of 10. For example, 45,000,000 has six zeros following the 45, so the *sixth* power of 10, or 10^6, is a factor: $45{,}000{,}000 = 45 \cdot 10^6$.

Example 4: Identify the power of 10, and rewrite the number as a product of a whole number and a power of 10.

a) 6,000 b) 290,000 c) 506,000,000 d) 80

Procedure: Count the number of zeros following the non-zero digit(s); that is the power of 10.

Answer:
a) $6{,}000 = 6 \cdot 10^3$ 6 followed by three zeros.

b) $290{,}000 = 29 \cdot 10^4$ 29 followed by four zeros.

c) $506{,}000{,}000 = 506 \cdot 10^6$ 506 followed by six zeros.

d) $80 = 8 \cdot 10^1$ 8 followed by one zero.

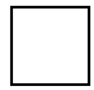
YTI #5 As outlined in Example 5, rewrite the number as a product of a whole number and a power of 10.

a) 240 = ____________________ b) 5,600 = ____________________

c) 308,000,000 = ____________________ d) 7,260,000 = ____________________

PERFECT SQUARES

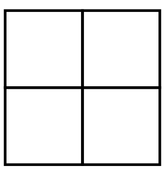
This is a diagram of a *unit* square, just as we saw in Section 1.4.



1 unit

1 unit

In the squares below, notice that there is a number associated with each—the number of unit squares within. Each number is called a **perfect square** because the unit squares form a square.

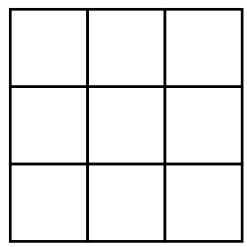


2 units

2 units

2 units x 2 units = 4 square units

4 is a perfect square.



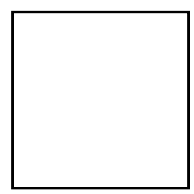
3 units

3 units

3 units x 3 units = 9 square units

9 is a perfect square.

Whenever we create a square with a number, n, as the length of one side, that same number, n, must appear as the length of the other side. The product of these two numbers, $n \cdot n$, is called a perfect square number, or just a perfect square.



n

n

In other words, because

$1 \cdot 1 = 1^2 = 1$, 1 is a perfect square

$2 \cdot 2 = 2^2 = 4$, 4 is a perfect square

$3 \cdot 3 = 3^2 = 9$, 9 is a perfect square

$4 \cdot 4 = 4^2 = 16$, 16 is a perfect square

$5 \cdot 5 = 5^2 = 25$, 25 is a perfect square

The list of perfect squares goes on and on. Any time you multiply a whole number by itself you get a result that is (automatically) a perfect square.

Notice, also, that there are a lot of numbers that are *not* perfect squares: 2, 3, 5, 6, 7, ... and the list goes on and on. 2 is not a perfect square because no whole number multiplied by itself equals 2. The same is true for all the numbers in this list.

Example 5:	Is it possible to draw a square that has:	
	a) 81 unit squares within it?	b) 24 unit squares within it?
Answer:	a) Yes, a square with 9 units on each side will have 81 square units within it because $9 \cdot 9 = 81$.	
	b) No, there is no whole number that, when multiplied by itself, will give 24.	

YTI #6 Use Example 5 as a guide to answer the following questions. Is it possible to draw a square that has:

a) 36 unit squares within it? ______________________________

b) 12 unit squares within it? ______________________________

c) 49 unit squares within it? ______________________________

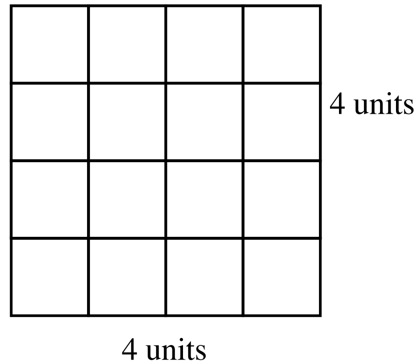
Example 6:	Find the perfect square number associated with each product.			
a) 7^2	b) 11^2	c) 13^2	d) 20^2	
Procedure:	Expand each and evaluate.			
Answer:	a) $7 \cdot 7 = \mathbf{49}$	c) $13 \cdot 13 = \mathbf{169}$		
	b) $11 \cdot 11 = \mathbf{121}$	d) $20 \cdot 20 = \mathbf{400}$		

YTI #7 Find the perfect square number associated with each product. Use Example 6 as a guide.

a) 6^2 b) 10^2 c) 18^2

SQUARE ROOTS

Consider this perfect square of 16. We know from our discussion of perfect squares, the number 4 makes the perfect square 16. For this reason, we call 4 a **square root** of 16.



The Square Root Property

If **p** = $\mathbf{r}^2$ (**p** is a **p**erfect square using **r** as one side), then **r** is a square **r**oot of **p**.

Example 7: What is a square root of the following numbers?

a) 49 b) 36 c) 1 d) 25

Procedure: What number, multiplied by itself, gives each of those above?

Answer: a) A square root of 49 is **7**. b) A square root of 36 is **6**.

c) A square root of 1 is **1**. d) A square root of 25 is **5**.

YTI #8 What is a square root of the following numbers? Use Example 7 as a guide.

a) A square root of 4 is _______ . b) A square root of 9 is _______ .

c) A square root of 64 is_______ . d) A square root of 100 is _______ .

THE SQUARE ROOT SYMBOL

The square root symbol, $\sqrt{\ }$, called a **radical**, makes it easier to write square roots.

So, instead of writing "A square root of 16 is 4," we can simply write $\sqrt{16} = 4$. The number within the radical, in this case 16, is called the **radicand**.

When we use the radical to find a square root of a number, like 16, the radical becomes an operation. Just like the plus sign tells us to apply addition to two numbers, the radical tells us to take the square root of a number.

Example 8:	Evaluate each square root.			
	a) $\sqrt{1}$	b) $\sqrt{25}$	c) $\sqrt{144}$	
Procedure:	Because each radicand, here, is a perfect square, applying the radical leaves a number with no radical symbol.			
Answer:	a) $\sqrt{1} = 1$	b) $\sqrt{25} = 5$	c) $\sqrt{144} = 12$	

YTI #9 Evaluate each square root. Use Example 8 as a guide.

a) $\sqrt{4} =$ b) $\sqrt{9} =$ c) $\sqrt{121} =$

d) $\sqrt{36} =$ e) $\sqrt{100} =$ f) $\sqrt{81} =$

THE ORDER OF OPERATIONS

Let's review the operations we've worked with so far:

Operation	**Example**	**Name**
Multiplication	$3 \cdot 5 = 15$	**product**
Division	$14 \div 2 = 7$	**quotient**
Subtraction	$12 - 9 = 3$	**difference**
Addition	$2 + 8 = 10$	**sum**
Exponent	$2^3 = 8$	**power**
Radical	$\sqrt{25} = 5$	**square root**

As we learned earlier, to evaluate a mathematical expression means to find the value of the expression.

Consider this expression: $3 + 4 \cdot 5$. How could it be evaluated?

If we apply the operation of addition first, and then multiplication, we get

$$\begin{aligned} & 3 + 4 \cdot 5 \\ =\ & \ \ 7 \ \cdot 5 \\ =\ & \mathbf{35} \end{aligned}$$

If, however, we evaluate using multiplication first and then addition, we get

$$\begin{aligned} & 3 + 4 \cdot 5 \\ =\ & 3 + \ 20 \\ =\ & \mathbf{23} \end{aligned}$$

Notice that, depending on which operation is applied first, we get two different results. Math, however, is an exact science and doesn't allow for two different values of the same expression.

Because of this, a set of guidelines, called the **order of operations**, was established. The order of operations tells us which operation should be applied first in an expression. The order of operations was developed with these thoughts in mind:

1. Any quantity within grouping symbols should be applied first. Grouping symbols includes parentheses ( ), brackets [ ], braces { }, and the radical, or the square root symbol, $\sqrt{\ }$.

2. Because an exponent is an abbreviation for repeated multiplication, exponents should rank higher than multiplication.

3. Because multiplication and division are inverse operations they should be ranked together. Also, because multiplication is an abbreviation for repeated addition (and division is an abbreviation for repeated subtraction), multiplication (and division) should rank higher than addition (and subtraction).

4. Because addition and subtraction are inverse operations they should be ranked together.

There are basically two types of grouping symbols:
• Those that form a quantity, like ( ), [ ], and { }, and
• those that are actual operations, like $\sqrt{\ }$.
The radical is *both* a grouping symbol *and* an operation!

In summary,

The Order Of Operations

1. Evaluate within all grouping symbols (one at a time), if there are any.
2. Apply any exponents.
3. Apply multiplication and division *reading from left to right*.
4. Apply addition and subtraction *reading from left to right*.

We sometimes refer to the order of the operations by their rank. For example, we might say that an exponent has a higher rank than multiplication.

Because multiplication and division have the same rank, we must apply them (carefully) from *left to right*. You'll see how we do this in the following examples.

Think about it: When evaluating an expression, is it ever possible to apply addition before multiplication? Explain.

____

____

The best way to understand these guidelines is to put them to work. We'll find that there is only one way to evaluate an expression using the rules, but we'll also find that some steps can be combined in certain situations. For now, though, let's evaluate each expression one step at a time.

Example 9: Evaluate each according to the order of operations.

a) $14 - 6 \div 2$ b) $(14 - 6) \div 2$ c) $7 + 3^2$

d) $(7 + 3)^2$ e) $24 \div 4 \cdot 2$ f) $24 \div (4 \cdot 2)$

Answer: Each of these has two operations; some have grouping symbols that will affect the order in which the operations are applied.

a) $14 - 6 \div 2$ Two operations, subtraction and division: divide first.

$= 14 - 3$ Notice that the minus sign appears in the second step. That's because it hasn't been applied yet.

$= \boxed{11}$

b) $(14 - 6) \div 2$ — Here are the same two operations as above, this time with grouping symbols. Evaluate the expression inside the parentheses first.

$= 8 \div 2$ — Because we've already evaluated within the grouping symbols, we don't need the parentheses any more.

$= \boxed{4}$

c) $7 + 3^2$ — Two operations, addition and exponent: apply the exponent first, then add.

$= 7 + 9$

$= 16$

d) $(7 + 3)^2$ — The same two operations as in (c); work within the grouping symbols first.

$= (10)^2$ — $(10)^2$ is also 10^2; at this point, the parentheses are no longer necessary.

$= \boxed{100}$

e) $24 \div 4 \cdot 2$ — Two operations: division and multiplication. Because they have the same rank, and there are no grouping symbols to tell us which to apply first, we apply them in order from *left to right*. That means that division is applied first.

$= 6 \cdot 2$

$= \boxed{12}$

f) $24 \div (4 \cdot 2)$ — This time we do have grouping symbols, so we begin by evaluating the expression within the parentheses.

$= 24 \div 8$

$= \boxed{3}$

YTI #10 Evaluate each according to the order of operations. First identify the two operations, then identify which is to be applied first. Show all steps! Use Example 9 as a guide.

a) $24 \div 6 + 2$

b) $24 \div (6 + 2)$

c) $10 - 3 \cdot 2$

d) $(10 - 3) \cdot 2$

e) $12 \div 2^2$

f) $(12 \div 2)^2$

Some expressions contain more than two operations. In those situations, we need to be even more careful when we apply the order of operations.

Example 10: Evaluate each according to the order of operations.

a) $36 \div 3 \cdot 6 - 2$ b) $36 \div (3 \cdot 6) - 2$ c) $36 \div [3 \cdot (6 - 2)]$

Answer: Each of these has three operations. In part c) there is a smaller quantity $(6 - 2)$ within the larger quantity of the brackets, [ ].

a) $36 \div 3 \cdot 6 - 2$ Because multiplication and division have the same rank, we apply them from left to right. We divide first.

$= 12 \cdot 6 - 2$

Notice that we are applying only one operation at a time and rewriting everything else

$= 72 - 2$

$= 70$

b) $36 \div (3 \cdot 6) - 2$ Evaluate the expression within the grouping symbols first.

$= 36 \div 18 - 2$ Divide.

$= 2 - 2$ Subtract.

$= 0$

c) $36 \div [3 \cdot (6 - 2)]$ Start with what is inside the large brackets. Inside those grouping symbols is another quantity, and we must evaluate it first: $6 - 2 = 4$.

$= 36 \div [3 \cdot 4]$ Evaluate within the brackets: $3 \cdot 4 = 12$.

$= 36 \div 12$ Divide.

$= 3$

Example 10(c) illustrates that when one quantity is within another one, the inner quantity is to be evaluated first.

Sometimes an expression will have two sets of grouping symbols that are unrelated to each other, meaning that evaluating one does not affect the evaluation of the other. In other words, some quantities can be evaluated at the same time.

For example, in the expression $(8 - 3) \cdot (12 \div 4)$ we can evaluate within each grouping symbol regardless of the operation:

$(8 - 3) \cdot (12 \div 4)$	Here there are three operations: subtraction, multiplication and division.
$= (5) \cdot (3)$	Subtraction and division can be applied at the same time because the order of operations tells us to begin by evaluating what is inside of the grouping symbols.
$= 15$	

YTI #11 Evaluate each according to the order of operations. Identify the order in which the operations should be applied. On each line, write the operation that should be applied and then apply it. Use Example 10 as a guide.

a) $36 \div 3 + 3 \cdot 2$

b) $36 \div (3 + 3) \cdot 2$

c) $36 \div (3 + 3 \cdot 2)$

d) $11 + 4 \cdot 6 - 1$

e) $(11 + 4) \cdot (6 - 1)$

f) $11 + [4 \cdot (6 - 1)]$

g) $2 \cdot 3^2 \div (6 + 3)$

h) $(2 \cdot 3)^2 \div (6 + 3)$

Now let's look at some examples that contain a radical.

Example 11: Evaluate each completely. Remember, the radical is both a grouping symbol and an operation.

a) $\sqrt{5 + 11}$ b) $\sqrt{3^2 + 4^2}$ c) $13 - 2 \cdot \sqrt{9}$

Procedure: Because the radical is a grouping symbol, we must evaluate within it first.

Answer:

a) $\sqrt{5 + 11}$ First apply addition.

$= \sqrt{16}$ Now apply the square root.

$= 4$

b) $\sqrt{3^2 + 4^2}$ Apply both exponents within the same step.

$= \sqrt{9 + 16}$ Next apply addition.

$= \sqrt{25}$ Apply the square root.

$= 5$

c) $13 - 2 \cdot \sqrt{9}$ The radical is a grouping symbol, so we should apply it—

$= 13 - 2 \cdot 3$ the square root—first. Next, apply multiplication and then

$= 13 - 6$ subtraction.

$= 7$

YTI #12 Evaluate each according to the order of operations. Identify the order in which the operations should be applied. Use Example 11 as a guide.

a) $\sqrt{4 \cdot 9}$

b) $\sqrt{25} - \sqrt{9}$

c) $\sqrt{1 + (12 \cdot 4)}$

d) $\sqrt{(6 - 2) \cdot 5^2}$

Calculator Tip: Scientific Calculator

Is your calculator a non-scientific calculator or a scientific calculator? Input the expression 3 + 4 x 5 to find out.

A non-scientific calculator is programmed to evaluate an expression in the order that the operations are put into it. On this calculator, each time you push an operation key (or the = key) the expression is evaluated, whether you are finished or not.

For example, on a non-scientific calculator, the expression 3 + 4 x 5 would be evaluated in this way:

Key sequence:	$\boxed{3}$	$\boxed{+}$	$\boxed{4}$	$\boxed{\times}$	$\boxed{5}$	$\boxed{=}$
		↓		↓		↓
Display:		$\boxed{3.}$		$\boxed{7.}$		$\boxed{35.}$
		↑		↑		↑
		We have 3 so far.		3 + 4 = 7		7 x 5 = 35

A scientific calculator, though, is programmed to always use the order of operations correctly. In doing so, it is patient and waits see what operation button is push next before deciding what to evaluate.

For example, on a scientific calculator, the expression 3 + 4 x 5 would be evaluated in this way:

Key sequence:	$\boxed{3}$	$\boxed{+}$	$\boxed{4}$	$\boxed{\times}$	$\boxed{5}$	$\boxed{=}$
						↓
Display:						$\boxed{23.}$

To determine if your calculator a scientific calculator or not, input the expression 3 + 4 x 5 and see which answer you get.

You Try It Answers: Section 1.5

YTI #1:	a)	4^7	b)	3^4	c)	1^8		
YTI #2:	a)	$6 \cdot 6 = 36$			b)	$2 \cdot 2 \cdot 2 \cdot 2 = 16$		
	c)	$9 \cdot 9 \cdot 9 = 729$						
YTI #3:	a)	2	b)	9	c)	17	d)	1
YTI #4:	a)	100,000	b)	10,000,000	c)	10		
	d)	10^2	e)	10^4	f)	10^6		
YTI #5:	a)	$24 \cdot 10^1$	b)	$56 \cdot 10^2$	c)	$308 \cdot 10^6$	d)	$726 \cdot 10^4$

YTI #6:
a) Yes, a square with 6 units on each side will have 36 square units.
b) No, there is no whole number that, when multiplied by itself, equals 12.
c) Yes, a square with 7 units on each side will have 49 square units.

YTI #7:	a)	36	b)	100	c)	324		
YTI #8:	a)	2	b)	3	c)	8	d)	10
YTI #9:	a)	2	b)	3	c)	11	d)	6
	e)	10	f)	9				
YTI #10:	a)	6	b)	3	c)	4	d)	14
	e)	3	f)	36				
YTI #11:	a)	18	b)	12	c)	4	d)	34
	e)	75	f)	31	g)	2	h)	4
YTI #12:	a)	6	b)	2	c)	7	d)	10

Focus Exercises: Section 1.5

Expand each and find its value.

1. 6^3 = ______________________________

2. 2^5 = ______________________________

3. 15^2 = ______________________________

4. 3^3 = ______________________________

5. 12^1 = ______________________________

6. 4^3 = ______________________________

7. 5^4 = ______________________________

8. 8^1 = ______________________________

9. 10^3 = ______________________________

10. 10^5 = ______________________________

Express each as a power of 10.

11. 100,000 = ________________

12. 1,000,000,000 = ________________

13. 10,000 = ________________

14. 1,000,000 = ________________

Rewrite each number as a product of a whole number and a power of 10.

15. 300 = ____________________

16. 5,000 = ____________________

17. 48,000 = ____________________

18. 710,000 = ____________________

19. 9,500,000 = ____________________

20. 200,000,000 = ____________________

Evaluate the following square roots.

21. $\sqrt{49}$

22. $\sqrt{25}$

23. $\sqrt{36}$

24. $\sqrt{1}$

25. $\sqrt{81}$

26. $\sqrt{16}$

27. $\sqrt{144}$

28. $\sqrt{64}$

Evaluate each according to the order of operations. Simplify just one step, one operation at a time.

29. $30 \div 5 + 1$

30. $30 \div (5 + 1)$

31. $(8 + 5) \cdot 2$

32. $8 + 5 \cdot 2$

33. $5 \cdot 6 \div 3$

34. $5 \cdot (6 \div 3)$

35. $5 \cdot 3^2$

36. $2^3 \cdot 2^2$

37. $2^3 \cdot\cdot 3^2$

38. $28 \div (7 \cdot 2)$

39. $28 \div 7 \cdot 2$

40. $16 \div 4 - 2$

41. $30 \div 2 \cdot 3$

42. $30 \div (2 \cdot 3)$

43. $7^2 + 5 - 3$

44. $5 \cdot 2^2 - 7$

45. $(5 \cdot 2)^2 - 7$

46. $4^2 \div 2 + 2$

47. $(5 + 3) \cdot 9$

48. $(6 + 12) \div (2 \cdot 3)$

49. $6 + \left[12 \div (2 \cdot 3)\right]$

50. $12 + \left[28 \div (7 - 3)\right]$

51. $24 \div (6 - 2) \cdot 3$

52. $6 + 12 \div 2 \cdot 3$

53. $(12 + 28) \div (7 - 3)$

54. $\left[(6 - 2) \cdot 3\right]^2$

55. $(6 - 2) \cdot 3^2$

56. $3 + \sqrt{16}$

57. $9 \cdot \sqrt{25}$

58. $11 - \sqrt{49}$

59. $6^2 - \sqrt{25}$

60. $\sqrt{64} - \sqrt{25}$

61. $9 + \sqrt{2 \cdot 2}$

62. $\sqrt{4 \cdot 9}$

63. $3 \cdot \sqrt{9 + 7}$

64. $\sqrt{8 \cdot 6 + 1}$

65. $24 - 8 \div 2 \cdot 4$

66. $22 - 2^2 \cdot 5$

67. $8 + 2 \cdot 3^2$

68. $6 + 4 \cdot 5^2$

69. $(24 - 8) \div 2 \cdot 4$

70. $(22 - 2)^2 \cdot 5$

71. $(8 + 2) \cdot 3^2$

72. $6 + (4 \cdot 5)^2$

73. $[9 - (3 + 1)]^2$

74. $[9 + (3 - 1)]^2$

75. $5 \cdot 2^2 - 4 \cdot (5 - 2)$

76. $44 - 4 \cdot (5 - 2)^2$

77. A square has a side length of 4 yards.

a) Draw and label all four sides of the square.

b) Find the perimeter of the square.

c) Find the area of the square.

78. A square has a side length of 9 inches.

a) Draw and label all four sides of the square.

b) Find the perimeter of the square.

c) Find the area of the square.

79. The floor of a cottage is in the shape of a square. Each side is 20 feet long.

a) Draw and label all four sides of the square.

b) Find the perimeter of the square.

c) Find the area of the square.

80. A city park is in the shape of a square. Each side is 100 feet long.

a) Draw and label all four sides of the square.

b) Find the perimeter of the square.

c) Find the area of the square.

81. A square has an area of 36 square feet.

a) What is the length of each side of the square?

b) What is the perimeter of the square?

82. A square has an area of 100 square inches.

a) What is the length of each side of the square?

b) What is the perimeter of the square?

83. A table top is in the shape of a square and has an area of 9 square feet.

a) What is the length of each side of the table top?

b) What is the perimeter of the table top?

84. A sand box is in the shape of a square and has an area of 49 square yards.

a) What is the length of each side of the sand box?

b) What is the perimeter of the sand box?