

## Section 1.6 Factors

### Objectives

In this section, you will learn to:

- Identify factors and factor pairs.
- Identify prime numbers and composite numbers.
- Use the divisibility tests to determine if one number is a factor of another number.
- Find the prime factorization of composite numbers.

To successfully complete this section, you need to understand:

- The multiplication table (1.1)
- Multiples of a number (1.1)
- Square roots (1.5)

### INTRODUCTION

Chemists want to know which atoms make up a particular molecule. They know, for example, that

- ▶ Water is two parts hydrogen (*H*) and one part oxygen (*O*), written  $\text{H}_2\text{O}$ .
- ▶ Ammonia is three parts hydrogen (*H*) and one part nitrogen (*N*), written  $\text{NH}_3$ .
- ▶ Methane is four parts hydrogen (*H*) and one part carbon (*C*), written  $\text{CH}_4$ .

In math, we want to understand the factors that make up a number. For example,  $15 = 3 \cdot 5$ . Two of the factors of 15 are 3 and 5. But what about a number such as 105? Does it have a factor of 3? Does it have a factor of 5? What other factors does 105 have? Before the end of this section you will be able to answer each of these questions.

To prepare us to answer those questions let's first recall some definitions about multiplication and division.

From Section 1.1 recall that

1. The product of any two numbers is the result when those two numbers are multiplied together, and the numbers in a product are called *factors*.

For example, the product of 3 and 5 is 15,  $3 \cdot 5 = 15$ . This means that 3 and 5 are factors of 15.

2. A *multiple* of any number, *a*, is a product involving *a* and some other whole number.

For example, 60 is a multiple of 3 because  $60 = 20 \cdot 3$ .

3. If a number, *a*, divides evenly into a number, *b*, then we say that *b* is *divisible* by *a*.

For example, 7 divides evenly into 35, so 35 is *divisible* by 7.

Combining all of these definitions, if two whole numbers,  $m$  and  $n$ , multiply to get a product,  $p$ , that is,  $m \cdot n = p$ , then

1.  $p$  is a *multiple* of both  $m$  and  $n$ .
2.  $p$  is *divisible* by both  $m$  and  $n$ .
3. both  $m$  and  $n$  *divide evenly* into  $p$ .
4. both  $m$  and  $n$  are *factors* of  $p$ .

**Example 1:**      Because  $6 \cdot 7 = 42$ ,

- 42 is a multiple of both 6 and 7.
- 42 is divisible by both 6 and 7.
- both 6 and 7 divide evenly into 42.
- both 6 and 7 are factors of 42.

**YTI #1**      Use Example 1 as a guide to complete each of the following.

a) Because  $4 \cdot 5 = 20$ ,

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

b) Because  $9 \cdot 8 = 72$ ,

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

## FACTORS

Any whole number can be written as the product of two numbers, called a **factor pair**. For example,

- › 12 can be written as  $2 \cdot 6$ , so  $2 \cdot 6$  is a factor pair of 12.
- › 9 can be written as  $3 \cdot 3$ , so  $3 \cdot 3$  is a factor pair of 9.
- › 7 can be written as  $1 \cdot 7$ , so  $1 \cdot 7$  is a factor pair of 7.

Many numbers have more than one factor pair, and we can use a *factor pair table* to help us find all of the factors of a particular number.

Consider this factor pair table of 24. Notice how it is organized to find all of the factor pairs of 24.

	24	
1	24	Start with 1 on the left and write the other factor of the pair, 24, on the
2	12	right. Do the same for 2, 3, and so on. Do not include 5, because 5
3	8	isn't a factor of 24.
4	6	

Now that the factor pair table is complete, we can list all of the factors of 24:

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

Consider this complete list of the factor pairs of 24 (at right). Notice that every factor pair is written twice; this is not necessary. In other words, if we include the factor pair 3 and 8, then we don't need to also include the factor pair 8 and 3.

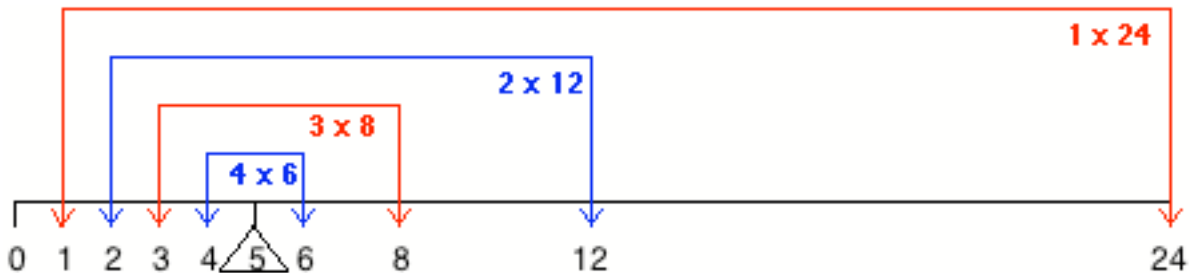
	24
1	24
2	12
3	8
4	6
6	4
8	3
12	2
24	1

It's a good idea, when creating a factor pair table, to write the left side in numerical order, starting with 1 so that we don't accidentally skip any factor pairs. But how do we know when to stop searching for factor pairs? Is there a number that tells us when to stop searching for factors?

Yes. This number is *the square root of the first perfect square larger than the given number*.

For example, if we're trying to find all of the factor pairs of 24, the first perfect square after 24 is **25**. Because  $\sqrt{25} = 5$ , we don't need to go beyond **5** in our search for factor pairs.

It's true that there are factors of 24 beyond 5, such as 8, but this factor is paired with a number less than 5, namely 3.



**Example 2:** Use a factor pair table to find all of the factor pairs of 30. From the table, write a list of the factors of 30.

**Procedure:** Think of 30 as a product of two numbers. Start with 1 on the left side, then 2, then 3, and so on, and decide whether those numbers are factors of 30.

**Answer:**

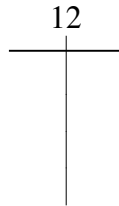
	<u>30</u>		
1	30	$1 \cdot 30 = 30$	Notice that 4 is not on this list because 4 doesn't divide evenly into 30.
2	15	$2 \cdot 15 = 30$	
3	10	$3 \cdot 10 = 30$	
5	6	$5 \cdot 6 = 30$	

So, the factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.

**YTI #2**

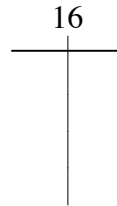
Use a factor pair table to find all of the factor pairs of each number. From the factor pair table, write the list of factors of that number. Use Example 2 as a guide.

a)



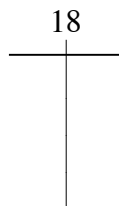
The factors of 12 are \_\_\_\_\_

b)



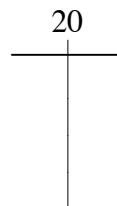
The factors of 16 are \_\_\_\_\_

c)



The factors of 18 are \_\_\_\_\_

d)



The factors of 20 are \_\_\_\_\_

**PRIME AND COMPOSITE NUMBERS**

Whole numbers greater than 1 fall into one of two categories: they are either *prime* or *composite*.

A whole number is a **prime number** if it has exactly two distinct, whole number factors: 1 and itself.

A whole number that has *more* than two distinct factors is a **composite number**. A composite number is any whole number (greater than 1) that is not prime.

**Caution:** *1 is not a prime number* because it has only one factor.

**Example 3:** For each number, determine if it is prime or composite.

- a) 7                      b) 12                      c) 9

- Answer:**
- a) 7 is a prime number because the only factors of 7 are 1 and 7; or we might say, 7 is prime because the only whole numbers that divide evenly into 7 are 1 and 7.
  - b) 12 is a composite number because it has more than two factors. The factors of 12 are 1, 2, 3, 4, 6, and 12.
  - c) 9 is a composite number because it has more than two factors. The factors of 9 are 1, 3, and 9.

**YTI #3** For each number, determine if it is prime or composite. Use Example 3 as a guide.

- a) 15                      b) 13                      c) 1                      d) 4

**YTI #4** Here is the list of prime numbers less than 100.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

a) What do you notice about the first prime number? \_\_\_\_\_

b) What do you notice about all of the other prime numbers? \_\_\_\_\_

c) Are all odd numbers prime numbers? Support your answer with some examples.

\_\_\_\_\_

Refer to the list of prime numbers less than 100 in YTI #4. Notice that the only even prime number is 2 and that no *two-digit* prime number ends in 5. Also notice that there really is no pattern to the list of primes; in other words, you can't predict what the next prime number is going to be just by looking at the list.

**DIVISIBILITY TESTS: 2, 3, 5, 9, AND 10**

Sometimes it is not easy to know what factors a composite number may have, especially if the number has three or more digits. To assist in identifying some factors, there are some tests for divisibility. What follows are divisibility tests for 2, 3, 5, 9, and 10. We can use these tests to find some factors of some composite numbers.

**Divisibility Test for 2**

2 is a factor of a whole number if and only if\* the number is even (has either 0, 2, 4, 6, or 8 in the ones place).

\*This means that if the number is odd, *not* even, then 2 is *not* a factor. (An odd number has a 1, 3, 5, 7, or 9 in the ones place.)

**YTI #5**

Without trying to find any other factors or factor pairs, determine if 2 is a factor of each. Explain why or why not.

a) 52

b) 61

c) 70

\_\_\_\_\_  
\_\_\_\_\_

\_\_\_\_\_  
\_\_\_\_\_

\_\_\_\_\_  
\_\_\_\_\_

**Divisibility Test for 5**

5 is a factor of a whole number if and only if the number has either 5 or 0 in the ones place.

**YTI #6**

Without trying to find any other factors or factor pairs, determine if 5 is a factor of each. Explain why or why not.

a) 90

b) 175

c) 608

\_\_\_\_\_  
\_\_\_\_\_

\_\_\_\_\_  
\_\_\_\_\_

\_\_\_\_\_  
\_\_\_\_\_

## Divisibility Test for 10:

10 is a factor of a whole number if and only if the number has 0 in the ones place.

10 is a factor of each of these numbers: 30, 160, 1,420 and 700 because each has 0 in the ones place.

## Divisibility Test for 3:

3 is a factor of a whole number if and only if the number's digits add to a multiple of 3.

**Example 4:** Determine if 3 is a factor of the number. Verify each answer by dividing the number by 3.

a) 285

b) 473

**Procedure:** a) Add the digits:  $2 + 8 + 5 = 15$ ;  
3 is a factor of 15.

b) Add the digits:  $4 + 7 + 3 = 14$ ;  
3 is not a factor of 14.

**Answer:** *Yes*, 3 is a factor of 285.

*No*, 3 is *not* a factor of 473.

**Check:**

$$\begin{array}{r} 95 \\ 3 \overline{)285} \\ \underline{-27} \\ 15 \\ \underline{-15} \\ 0 \end{array}$$

So, 285 is divisible by 3, and 3 is a **factor** of 285.

$$\begin{array}{r} 157 \text{ r } 2 \\ 3 \overline{)473} \\ \underline{-3} \\ 17 \\ \underline{-15} \\ 23 \\ \underline{-21} \\ 2 \end{array}$$

Because the remainder is not 0, 473 is not divisible by 3. Therefore 3 is *not* a factor of 473.

### YTI #7

Determine if 3 is a factor of the number. Verify each answer by dividing the number by 3. Use Example 4 as a guide.

a) 87: \_\_\_\_\_

b) 671: \_\_\_\_\_

c) 8,395: \_\_\_\_\_

d) 25,074: \_\_\_\_\_

### Divisibility Test for 9:

9 is a factor of a whole number if and only if the number's digits add to a multiple of 9.

**Example 5:** Determine if 9 is a factor of the number. Verify each answer by dividing the number by 9.

a) 675

b) 1,983

**Procedure:** a) Add the digits:  $6 + 7 + 5 = 18$ ; 18 is a multiple of 9.

b) Add the digits:  $1 + 9 + 8 + 3 = 21$ ; 21 is *not* a multiple of 9.

**Answer:** *Yes*, 9 is a factor of 675.

*No*, 9 is *not* a factor of 1,983.

**Check:**

$$\begin{array}{r} 75 \\ 9 \overline{)675} \\ \underline{-63} \\ 45 \\ \underline{-45} \\ 0 \end{array}$$

So, 675 is divisible by 9, and 9 is a **factor** of 675.

$$\begin{array}{r} 220 \text{ r } 3 \\ 9 \overline{)1983} \\ \underline{-18} \\ 18 \\ \underline{-18} \\ 03 \\ \underline{-00} \\ 3 \end{array}$$

Because the remainder is not 0, 1,983 is not divisible by 9, and 9 is *not* a factor of 1,983.

### YTI #8

Determine if 9 is a factor of the number. Verify each answer by dividing the number by 9. Use Example 5 as a guide.

a) 548: \_\_\_\_\_

b) 3,582: \_\_\_\_\_

c) 8,511: \_\_\_\_\_

d) 20,142: \_\_\_\_\_

**Think about it:**

If we add the digits of 25, we get  $2 + 5 = 7$ . Does this mean that 7 is a factor of 25?

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Let's put the divisibility tests for 2, 3, and 5 together in the following example.

**Example 6:** Which of the first three prime numbers—2, 3 and 5—are factors of the following?

- |                         |                       |                        |                         |
|-------------------------|-----------------------|------------------------|-------------------------|
| a) 42                   | b) 135                | c) 570                 | d) 91                   |
| • 42 is even (2)        | • 135 is not even     | • 570 is even (2)      | • 91 is not even        |
| • $4 + 2 = 6$ (3)       | • $1 + 3 + 5 = 9$ (3) | • $5 + 7 + 0 = 12$ (3) | • $9 + 1 = 10$          |
| • doesn't end in 0 or 5 | • ends in 5 (5)       | • ends in 0 (5)        | • doesn't end in 0 or 5 |

**Answer:**

- a) 2 and 3      b) 3 and 5      c) 2 and 3 and 5      d) none of these

**YTI #9**

Which of the first three prime numbers—2, 3 and 5—are factors of the following? Use Example 6 as a guide.

- a) 213      b) 390      c) 419      d) 2,835
- 

**Think about it:**

If a number, such as 169, doesn't have 2, 3, or 5 as one or more of its factors, is it a prime number? Explain your answer.

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**Think about it:**

On the first page of this section, the following questions were asked about 105. Can you answer them now?

Does 105 have a factor of 3? \_\_\_\_\_

Does 105 have a factor of 5? \_\_\_\_\_

What other factors does 105 have? \_\_\_\_\_

**PRIME FACTORIZATION**

Just as a chemist breaks up an element into its most basic atoms, we will break up a composite number into its prime factors. When we write a composite number as a product of its prime factors we call it **prime factorization**.

To help us understand prime factorization, we look at this analogy about primes and composites.

In paints we have three primary colors, red, blue and yellow. For the purposes of this analogy, we can think of these as representing the prime numbers.

We can mix any two of these primary colors together to get other colors, called secondary colors. In particular,

mixing equal amounts of red and yellow makes orange;

mixing equal amounts of red and blue makes purple;

mixing equal amounts of blue and yellow makes green.

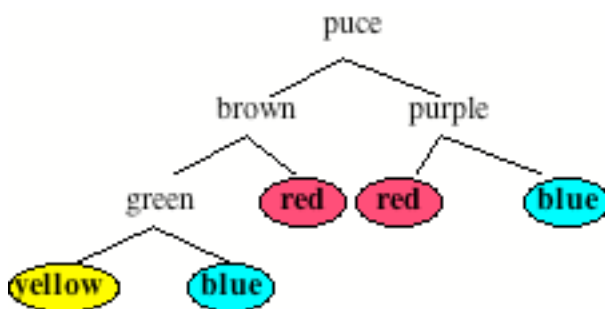
These secondary colors are like composite numbers. You can't get green without both blue and yellow, just as a whole number can't have 6 as a factor without also having both 2 and 3 as factors.

We can even mix all three primary colors together to form brown, so we can say that brown is composed of red, blue and yellow. Brown is like 30; 30 is a composite number. It is composed of the prime factors 2, 3 and 5:  $30 = 2 \cdot 3 \cdot 5$ . (This is the prime factorization of 30.)

Also, we can mix yellow and blue to get green, and then we can mix in more yellow for lime green. Lime green is a composite color with two amounts of yellow and one amount of blue. Similarly, 12 is a composite number with two factors of 2 and one factor of 3:  $12 = 2 \cdot 2 \cdot 3$ . (This is the prime factorization of 12.)

To complete this analogy, consider how a paint scientist might break down the color *puce*. She might first discover that puce is a mixture of brown and purple, then recognize that brown is composed of red and green, and so on.

Here is a diagram of what this breakdown into primary colors might look like. The primary colors are circled to indicate that they can't be broken down further.



We see that puce is composed of one amount of yellow, two amounts of red, and two amounts of blue.

<b>Example 7:</b>	Find the prime factorization of each composite number.		
	a) 6	b) 14	c) 15
<b>Procedure:</b>	Think of the factor pairs that make up the composite number. Because 1 is not a prime number, it should not be included.		
<b>Answer:</b>	a) $6 = 2 \cdot 3$ .	b) $14 = 2 \cdot 7$	c) $15 = 3 \cdot 5$

**YTI #10** Find the prime factorization of each composite number. Use Example 7 as a guide.

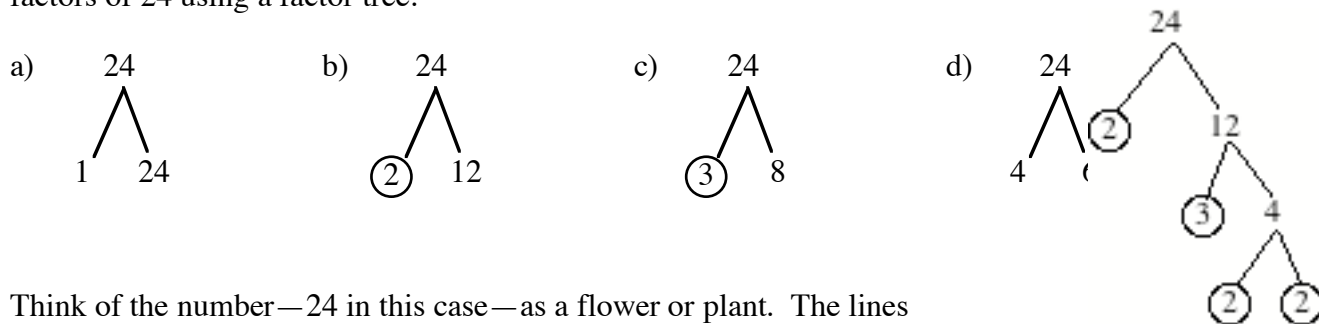
- a) 21                      b) 22                      c) 35                      d) 77

**Think about it:**

Though it's true that  $7 = 7 \cdot 1$ , we can't use this as the prime factorization. Why not?

**FACTOR TREES**

A **factor tree** is a visual method used to look at the factors of a number. First let's look at the variety of factors of 24 using a factor tree:

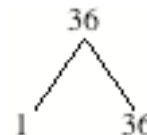


Think of the number—24 in this case—as a flower or plant. The lines leading to the factors are like *branches*. The circled numbers are prime numbers, like the fruit of the tree, and this is the purpose of a factor tree: to identify prime factors of the original number.

Each branch that leads to a prime number bears fruit, and each branch that leads to a composite number must branch again until it bears the fruit of a prime number. At right is the completed factor tree for 24. This factor tree indicates that  $24 = 2 \cdot 2 \cdot 2 \cdot 3$ .

**Think about it:**

Is this factor tree a good start for the prime factorization for 36. Why or why not?



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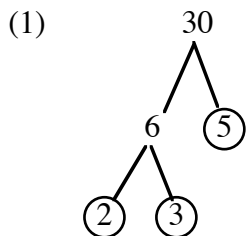
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**Example 8:** Find the prime factorization of 30.

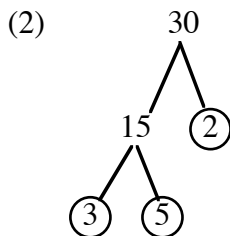
**Procedure:** We'll use a factor tree to first find any two factors of 30. We'll circle any primes that appear to indicate that the branch can't be factored further.

If we arrive at any composite numbers, we must factor them further to continue our search for prime factors.

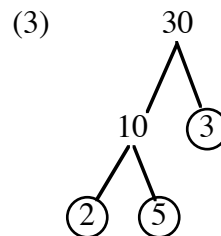
Here are three different paths to finding the prime factors of 30:



**Answer:**  $30 = 2 \cdot 3 \cdot 5$



$30 = 3 \cdot 5 \cdot 2$



$30 = 2 \cdot 5 \cdot 3$

Notice that the result (the prime factorization) is the same no matter which factor path we choose. Generally, though, we write the prime factorization in numerical order, starting with the lowest. So we'd write  $30 = 2 \cdot 3 \cdot 5$ .

**YTI #11**

Find the prime factorization of the following. Create a factor tree and put a circle around any prime factor. Use Example 8 as a guide.

a) **42**

b) **54**

c) **70**

42 =

54 =

70 =

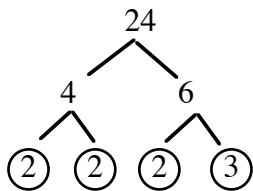
**Example 9:** Find the prime factorization of 24. Write the answer two ways, with and without exponents.

**Procedure:** This time you are given one of three paths. It is up to you to find two more. Any correct path you choose will give the same prime factorization.

(1) Our path:

(2) Your first path

(3) Your second path



Notice this time we have repeated prime factors, and we must list them all in writing the prime factorization. We can write the answer:

**Answer:**  $24 = 2 \cdot 2 \cdot 2 \cdot 3$  or  $2^3 \cdot 3$ .

**YTI #12**

Find the prime factorization of the following using a factor tree. Write the answer two ways: with and without exponents, as shown in Example 9. Be sure to show the factor tree and circle the prime factors as they appear.

a) 12

b) 50

12 = \_\_\_\_\_ or \_\_\_\_\_  
(without exponents) OR (with exponents)

50 = \_\_\_\_\_ or \_\_\_\_\_

c) 36

d) 27

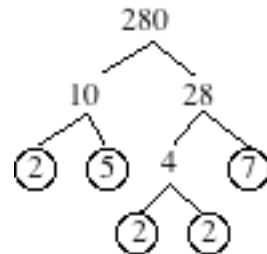
36 = \_\_\_\_\_ or \_\_\_\_\_

27 = \_\_\_\_\_ or \_\_\_\_\_

**Example 10:** Find the prime factorization of 280.

**Procedure:** When a number is rather large, don't be intimidated by it. Identify at least one number (prime *or* composite) that is a factor and begin the process. The other factors will quickly become smaller and easier to work with.

- (1) Is 2 is a factor of 280?      10 is a good factor to start with.
- (2) Is 3 is a factor of 280?
- (3) Is 5 is a factor of 280?
- (4) Is 9 is a factor of 280?
- (5) Is 10 is a factor of 280?



**Answer:**  $280 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 7$  or  $2^3 \cdot 5 \cdot 7$ .

**YTI #13** Find the prime factorization of the following. Write it both with and without exponents. Use Example 10 as a guide.

a) 100

b) 260

c) 1,540

100 = \_\_\_\_\_  
without exponents

260 = \_\_\_\_\_  
without exponents

1,540 = \_\_\_\_\_  
without exponents

100 = \_\_\_\_\_  
with exponents

260 = \_\_\_\_\_  
with exponents

1,540 = \_\_\_\_\_  
with exponents



c) 240 is even, so the first prime divisor is 2:  $240 \div 2 = 120$ .

$$2 \overline{) 240}$$

120 is also even, so we'll divide by 2 again:  $120 \div 2 = 60$ .

$$2 \overline{) 120}$$

60 is even, so we divide by 2 again:  $60 \div 2 = 30$ .

$$2 \overline{) 60}$$

30 is also even, so we divide by 2 again:  $30 \div 2 = 15$ .

$$2 \overline{) 30}$$

Finally, no more even quotients; we know that 3 is a factor of 15, giving a quotient of 5, which is prime.

$$3 \overline{) 15} \\ 5$$

The prime factorization of 240 is the product of all of the prime divisors and the last prime (5).

So, the prime factorization of 240 is  $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$  or  $2^4 \cdot 3 \cdot 5$ .

**YTI #14**

Find the prime factorization of the following. Use the division method outlined in Example 11.

a) 24

$$24 = \frac{\quad}{\text{without exponents}}$$

$$24 = \frac{\quad}{\text{with exponents}}$$

b) 60

$$60 = \frac{\quad}{\text{without exponents}}$$

$$60 = \frac{\quad}{\text{with exponents}}$$

c) 175

$$175 = \frac{\quad}{\text{without exponents}}$$

$$175 = \frac{\quad}{\text{with exponents}}$$

d) 252

$$252 = \frac{\quad}{\text{without exponents}}$$

$$252 = \frac{\quad}{\text{with exponents}}$$

e) 405

$$405 = \frac{\quad}{\text{without exponents}}$$

$$405 = \frac{\quad}{\text{with exponents}}$$

f) 660

$$660 = \frac{\quad}{\text{without exponents}}$$

$$660 = \frac{\quad}{\text{with exponents}}$$

For additional practice with the division method, go back to YTI #3 and 4 and use the division method there. You will come up with the same prime factorizations if everything is done correctly.

## You Try It Answers: Section 1.6

- YTI #1:**
- |  |  |
|--|--|
| <p>a) • 20 is a multiple of both 4 and 5.<br/>         • 20 is divisible by both 4 and 5.<br/>         • 4 and 5 divide evenly into 20.<br/>         • 4 and 5 are both factors of 20.</p> | <p>b) • 72 is a multiple of both 9 and 8.<br/>         • 72 is divisible by both 9 and 8.<br/>         • 9 and 8 divide evenly into 72.<br/>         • 9 and 8 are both factors of 72.</p> |
|--|--|

**YTI #2:**

a) 
$$\begin{array}{r|l} 12 & \\ \hline 1 & 12 \\ 2 & 6 \\ 3 & 4 \end{array}$$

Factors of 12: 1, 2, 3, 4, 6, and 12

b) 
$$\begin{array}{r|l} 16 & \\ \hline 1 & 16 \\ 2 & 8 \\ 4 & 4 \end{array}$$

Factors of 16: 1, 2, 4, 8, and 16

c) 
$$\begin{array}{r|l} 18 & \\ \hline 1 & 18 \\ 2 & 9 \\ 3 & 6 \end{array}$$

Factors of 18: 1, 2, 3, 6, 9, and 18

d) 
$$\begin{array}{r|l} 20 & \\ \hline 1 & 20 \\ 2 & 10 \\ 4 & 5 \end{array}$$

Factors of 20: 1, 2, 4, 5, 10, and 20

- YTI #3:**
- |                                |              |
|--------------------------------|--------------|
| a) Composite                   | b) Prime     |
| c) Neither prime nor composite | d) Composite |

- YTI #4:**
- a) The first prime number is even. (It is the only even prime number.)  
 b) All other prime numbers are odd numbers.  
 c) No, there are many odd numbers that are not prime. The number 9 is an example of an odd number that is not prime. Other examples of odd numbers that are not prime are 15, 21, 25, 27 and 33.

- YTI #5:**
- a) 2 is a factor of 52 because 52 is an even number.  
 b) 2 is not a factor of 61 because 61 is an odd number.  
 c) 2 is a factor of 70 because 70 is an even number.

- YTI #6:**
- a) 5 is a factor of 90 because 90 has a 0 in the ones place.  
 b) 5 is a factor of 175 because 175 has a 5 in the ones place.  
 c) 5 is not a factor of 608 because 608 does not have a 0 or 5 in the ones place.

- YTI #7:**
- a) Because  $8 + 7 = 15$ , and because 3 is a factor of 15, 3 is a factor of 87.
  - b) Because  $6 + 7 + 1 = 14$ , and because 3 is not a factor of 14, 3 is not a factor of 671.
  - c) Because  $8 + 3 + 9 + 5 = 25$ , and because 3 is not a factor of 25, 3 is not a factor of 8,395.
  - d) Because  $2 + 5 + 0 + 7 + 4 = 18$ , and because 3 is a factor of 18, 3 is a factor of 25,074.

- YTI #8:**
- a) Because  $5 + 4 + 8 = 17$ , and because 9 is not a factor of 17, 9 is not a factor of 548.
  - b) Because  $3 + 5 + 8 + 2 = 18$ , and because 9 is a factor of 18, 9 is a factor of 3,582.
  - c) Because  $8 + 5 + 1 + 1 = 15$ , and because 9 is not a factor of 15, 9 is not a factor of 8,511.
  - d) Because  $2 + 0 + 1 + 4 + 2 = 9$ , and because 9 is a factor of 9, 9 is a factor of 20,142.

- YTI #9:** a) 3 only                      b) 2, 3 and 5                      c) none of these                      d) 3 and 5

- YTI #10:** a)  $3 \cdot 7$                       b)  $2 \cdot 11$                       c)  $5 \cdot 7$                       d)  $7 \cdot 11$

- YTI #11:** a)  $2 \cdot 3 \cdot 7$                       b)  $2 \cdot 3 \cdot 3 \cdot 3$                       c)  $2 \cdot 5 \cdot 7$

- YTI #12:** a)  $2 \cdot 2 \cdot 3$  or  $12 = 2^2 \cdot 3$                       b)  $2 \cdot 5 \cdot 5$  or  $50 = 2 \cdot 5^2$   
 c)  $2 \cdot 2 \cdot 3 \cdot 3$  or  $36 = 2^2 \cdot 3^2$                       d)  $3 \cdot 3 \cdot 3$  or  $27 = 3^3$

- YTI #13:** a)  $\frac{2 \cdot 2 \cdot 5 \cdot 5}{2^2 \cdot 5^2}$                       b)  $\frac{2 \cdot 2 \cdot 5 \cdot 13}{2^2 \cdot 5 \cdot 13}$                       c)  $\frac{2 \cdot 2 \cdot 5 \cdot 7 \cdot 11}{2^2 \cdot 5 \cdot 7 \cdot 11}$

- YTI #14:**
- a)  $\frac{2 \cdot 2 \cdot 2 \cdot 3}{2^3 \cdot 3}$                       b)  $\frac{2 \cdot 2 \cdot 3 \cdot 5}{2^2 \cdot 3 \cdot 5}$
  - c)  $\frac{5 \cdot 5 \cdot 7}{5^2 \cdot 7}$                       d)  $\frac{2 \cdot 2 \cdot 3 \cdot 3 \cdot 7}{2^2 \cdot 3^2 \cdot 7}$
  - e)  $\frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 5}{3^4 \cdot 5}$                       d)  $\frac{2 \cdot 2 \cdot 3 \cdot 5 \cdot 11}{2^2 \cdot 3 \cdot 5 \cdot 11}$

## Focus Exercises: Section 1.6

List the first eight multiples of each given number.

1. 4                      2. 5                      3. 7                      4. 9

Use a factor pair table to find all of the factor pairs of:

5. 32                      6. 40                      7. 28                      8. 42

Of the first three prime numbers—2, 3, and 5—which are factors of the following?

9. 32                      10. 80                      11. 127                      12. 414  
13. 76                      14. 57                      15. 125                      16. 390  
17. 315                      18. 860                      19. 156                      20. 4,231  
21. 7,287                      22. 41,592                      23. 322,980                      24. 994,515

Determine if 9 is a factor of the number.

25. 372                      26. 4,797                      27. 7,506                      28. 20,601

Of the following, determine which are prime, which are composite, and which are neither.

29. 0, 7, 9, 23, 8, 40, 15, 33, 32, 12, 41, 51, 50  
30. 6, 17, 2, 27, 31, 38, 1, 29, 41, 49, 55, 57, 61, 71

*Find the prime factorization of the following using a factor tree. Write the answers two ways: with and without exponents.*

**31.**        18

**32.**        48

**33.**        63

**34.**        75

**35.**        105

**36.**        256

**37.**        496

**38.**        588

**39.**        720

**40.**        735

**41.**        945

**42.**        1,050

*Find the prime factorization of the following using the division method. Write the answers two ways: with and without exponents.*

**43.**        20

**44.**        45

**45.**        52

**46.**        72

**47.**        76

**48.**        88

**49.**        98

**50.**        111

**51.**        124

**52.**        135

**53.**        200

**54.**        224