Section 5.4 Multiplying Decimals

Objectives
In this section, you will learn to:

- Multiply a decimal by a decimal.
- Multiply a decimal by a whole number.
- Multiply a decimal by a power of 10.

To successfully complete this section, you need to understand:

- Multiplying whole numbers (1.3)
- Multiplying fractions (4.4)
- Writing decimals as fractions (5.1)

INTRODUCTION

True story, though the names have been changed to protect the innocent.

> Lana went to a small specialty shop to buy a music CD for a friend. While there, the power went out, and the cash register would not work. Neither the employee nor the manager knew how to calculate sales tax, so they didn't want to sell the CD to Lana. Lana, the owner of her own retail shop, showed them how to find sales tax by multiplying the price of the CD, $15.95, by the sales tax rate, 0.08.

The moral of this story is, don't be totally dependent on machines to do all of the work for you. Learn to know what the machine is doing so that you can do it on your own should you ever need to.

MULTIPLYING DECIMAL NUMBERS

We know that a decimal can always be written as a decimal fraction. For example, 0.3 can be written as $\frac{3}{10}$ and 0.07 can be written as $\frac{7}{100}$.

This concept is helpful when we multiply two decimal numbers. The advantage of writing decimals as fractions is that the numerators are whole numbers, and the denominators are powers of 10. We will use this concept to develop a consistent procedure for multiplying decimals.

Consider the product 0.3×0.07. Because each decimal can be written as a fraction, we can multiply using the product rule for fractions:

$$0.3 \times 0.07 = \frac{3}{10} \times \frac{7}{100} = \frac{21}{1{,}000} = 0.021$$

Example 1:	Multiply 0.06×0.8 by first rewriting each decimal as a fraction.
Answer:	$0.06 \times 0.8 = \frac{6}{100} \times \frac{8}{10} = \frac{48}{1{,}000} = 0.048$

YTI #1 Multiply by first rewriting each decimal as a fraction. Multiply the fractions, then convert the answer back into a decimal. Use Example 1 as a guide.

a) $0.5 \times 0.7 =$ _____

b) $0.1 \times 0.8 =$ _____

c) $0.04 \times 0.6 =$ _____

d) $0.11 \times 0.09 =$ _____

Consider 0.8×0.21. Let's think about what we see here.

$$0.8 \times 0.21 = \frac{8}{10} \times \frac{21}{100} = \frac{168}{1{,}000} = 0.168$$

In this product, whether we use fractions or decimals, we are multiplying tenths by hundredths and getting thousandths.

In each factor, if we count the number of zeros in each denominator, we will know the number of denominator zeros in the end result:

Fractions:

Tenths x *hundredths* = *thousandths*

$\frac{8}{10} \times \frac{21}{100} = \frac{168}{1{,}000}$

One zero; *Two* zeros; *Three* zeros

Number of zeros: *one + two = three*

In each factor, if we count the number of decimal places in each number, we will know the number of decimal places in the end result:

Decimals:

Tenths x *hundredths* = *thousandths*

0.8 x 0.21 = 0.168

One decimal place; *Two* decimal places; *Three* decimal places

Number of decimal places: *one + two = three*

The fractions show us we are simply multiplying whole numbers 8 x 21; the decimals show us that the answer is a decimal number. The question is, after we multiply 8 x 21 = 168, where do we place the decimal point?

The diagrams above show that we can count the number of decimal places in the factors, and the result will have that total number of decimal places.

Multiplying Two Decimal Numbers

Multiplying decimals is exactly the same as multiplying whole numbers, but we must:

(1) Temporarily ignore the decimal points and multiply the numbers (factors) as if they were both whole numbers; and

(2) count up the total number of decimal places in the factors; this total is the number of decimal places in the product (before any simplifying).

Let's practice identifying the number of decimals, as in part (2) of the procedure.

Example 2: Given the following multiplication, decide how many decimal places the product (the result) will have. *Do not multiply.*

a) 0.3 x 5 b) 1.2 x 0.4 c) 0.16 x 5.3 d) 1.25 x 0.12

Procedure: Count the total number of decimal places in both numbers. This is the number of decimal places that the answer will have.

Answer:

a) 0.3 x 5 — 0.3 has one decimal place and 5 has none, so their product will have a total of one decimal place.

b) 1.2 x 0.4 — 1.2 has one decimal place and 0.4 has one, so their product will have a total of two decimal places.

c) 0.16 x 5.3 — 0.16 has two decimal places and 5.3 has one, so their product will have a total of three decimal places.

d) 1.25 x 0.12 — 1.25 has two decimal places and 0.12 has two, so their product will have a total of four decimal places.

YTI #2 Given the following multiplication, decide how many decimal places the product (the result) will have. *Do not multiply.* Use Example 2 as a guide.

a) 0.45 x 1.2 b) 6 x 4.3 c) 1.08 x 3.04 d) 4.3 x 2.6

Example 3: Multiply. a) 13 x 32 b) 1.3 x 3.2

Answer:

a)
$$\begin{array}{r} 13 \\ \times\ 32 \\ \hline 26 \\ +\ 390 \\ \hline 416 \end{array}$$

b)
$$\begin{array}{r} 1.3 \\ \times\ 3.2 \\ \hline 26 \\ +\ 390 \\ \hline 4.16 \end{array}$$

Temporarily ignore the decimal points and multiply as if the decimals were whole numbers. The two decimal places appear in the end result only.

Example 4: Multiply these decimal numbers. (In these, when the whole number is 0, we usually don't multiply it.)

a) 0.3 x 5 b) 1.2 x 0.4 c) 0.17 x 5.1 d) 0.63 x 2.08

Procedure: Write the numbers with decimals in place. Temporarily ignore the decimal points and multiply as you would with whole numbers. When complete, count up the number of decimals in each product.

Answer:

a)
$$\begin{array}{r} 0.3 \\ \times\ 5 \\ \hline 1.5 \end{array}$$
One decimal place
No decimal places
One decimal place

b)
$$\begin{array}{r} 1.2 \\ \times\ 0.4 \\ \hline 0.48 \end{array}$$
One decimal place
One decimal place
Two decimal places

c)
$$\begin{array}{r} 0.17 \\ \times\ 5.1 \\ \hline 17 \\ +\ 850 \\ \hline 0.867 \end{array}$$
Two decimal place
One decimal place
↓
↓
Three decimal places

d)
$$\begin{array}{r} 0.63 \\ \times\ 2.08 \\ \hline 504 \\ +\ 12600 \\ \hline 1.3104 \end{array}$$
Two decimal places
Two decimal place
↓
↓
Four decimal places

YTI #3 Follow the procedure for multiplying decimals to find the product. Use Examples 3 and 4 as guides.

a) 0.9 x 7

b) 0.8 x 0.4

c) 0.17 x 0.6

d) 1.8 x 0.12

e) 0.58 x 0.71

Think about it #1: When adding and subtracting decimals, it is important to line up the decimal points, but it is not necessary to do that when multiplying decimals. Why not?

_____

_____

Caution: There are two problems in the next example that involve working with 0 in one way or another. Be careful!

Example 5: Multiply.

a) 0.16 x 0.3 b) 0.25 x 0.72

Procedure: Follow the procedure in Example 4, but notice the following:

The first, a), will require three decimal places, but there will be only two digits in the product. A third digit is required before we can place the decimal point in the answer. That third digit is a 0 in front.

The second, b), will have enough decimal places, but there will be a few extra zeros at the end of the number that we can eventually eliminate.

Answer:

a)
$$\begin{array}{rl} 0.16 & \text{Two decimal places} \\ \underline{\times\ 0.3} & \underline{\text{One decimal place}} \\ 0.048 & \text{Three decimal places} \end{array}$$

0.16 x□□0.3 = 0.048

b)
$$\begin{array}{rl} 0.25 & \text{Two decimal places} \\ \underline{\times\ 0.72} & \underline{\text{Two decimal places}} \\ 50 & \downarrow \\ \underline{+\ 1750} & \downarrow \\ 0.1800 & \text{Four decimal places} \end{array}$$

We can simplify 0.1800 to 0.18.

0.25 x 0.72 = 0.18

Caution: We cannot eliminate any ending zeros until after the decimal point is placed in the end result.

YTI #4 Follow the procedure for multiplying decimals to find the product. Use Example 5 as a guide.

a) 0.4 x 0.2 b) 0.48 x 0.5 c) 0.06 x 0.75

MULTIPLYING DECIMAL NUMBERS BY POWERS OF 10

Multiplying a decimal by 10, or by a larger power of 10—such as 100 or 1,000—has the same effect as changing the position of the decimal point within the number. In other words, multiplying a decimal number by a power of 10 moves the decimal point a certain number of places to the right. In particular, multiplying a decimal number

- by 10 moves the decimal point one place to the right
- by 100 moves the decimal point two places to the right
- by 1,000 moves the decimal point three places to the right

To understand why, consider that any decimal number, such as 9.413 can be written as a decimal fraction, $\frac{9413}{1000}$. When such a number is multiplied by a power of 10, such as 100, or $\frac{100}{1}$, we can simplify the powers of 10:

$$9.413 \times 100 = \frac{9413}{1000} \times \frac{100}{1} = \frac{9413 \cdot 100}{1000 \cdot 1} = \frac{9413 \cdot 1}{10 \cdot 1} = \frac{9413}{10} = 941.3$$

Divide out a common factor of 100.

In other words, multiplying by 100 has the effect of moving the decimal place two places to the right: **9.413 x 100 = 941.3**

Here are three examples showing the moving of the decimal point directly:

a) $3.6 \times 10 = 36$

$3.6. = 36$

Multiplying by 10, which has only one zero, has the effect of moving the decimal point of 3.6 one place to the right; the product becomes the whole number 36 without the need of a decimal point.

b) $5.24 \times 10 = 52.4$

$5.2.4 = 52.4$

Again, there is one zero in 10, which has the effect of moving the decimal point of 5.24 one place to the right. This time, the decimal point is necessary because 52.4 is still a decimal number.

c) $5.9 \times 1{,}000 = 5{,}900$

$5.900 \times 1{,}000 = 5{,}900$

$5.900. = 5{,}900$

Multiplying by 1,000 suggests that we need to move the decimal point in 5.9 three places to the right.

However, there aren't enough decimal places in 5.9, so we must place some zeros at the end of it so that there is someplace to move the decimal point to.

Here is the procedure that results from all of this:

Multiplying a Decimal Number by a Power of Ten

(1) Count the number of zeros in the power of ten.

(2) Move the decimal point that many places to the right.

YTI #5 Multiply by moving the decimal point the appropriate number of places to the right.

a) 8.1×10

b) 0.764×100

c) 14.6×100

d) $0.0027 \times 1{,}000$

e) $0.09 \times 1{,}000$

Multiplying decimals by powers of 10 will prove useful in understanding dividing decimals (Section 5.5) and working with percents (Section 6.4).

APPLICATIONS

Sales tax is calculated by multiplying the price of an item (usually dollars and cents) by the sales tax rate (a decimal). After the sales tax is calculated, it is rounded to the nearest penny (hundredth) and added to the price of the item.

Example 6 refers to the true story at the beginning of this section.

Example 6: Lana wants to buy a music CD that costs $15.95. If the sales tax rate is 0.08, how much total will she pay for the CD?

Procedure: Calculate the sales tax for the CD and round it to the nearest penny. Then, add the sales tax to the cost of the CD to find the amount Lana must pay.

Answer:

$$\begin{array}{r} 15.95 \\ \times \quad 0.08 \\ \hline 1.2760 \end{array}$$

Round to the nearest penny, $1.2760 \approx 1.28$.

The sales tax is $1.28. Add this to the cost:

$$\begin{array}{r} \$15.95 \\ + \quad 1.28 \\ \hline \$17.23 \end{array}$$

Sentence: Lana will pay a total of $17.23 for the CD.

YTI #6 Tim is buying a baseball glove for his son for little league. The cost of the glove is $34.95, and the tax rate is 0.06. How much in total will Tim pay for the glove?

Show your work:

Sentence: _____

Many more applications involving multiplying decimals appear in Section 5.6.

You Try It Answers: Section 5.4

YTI #1: a) $\frac{5}{10} \times \frac{7}{10} = \frac{35}{100} = 0.35$ b) $\frac{1}{10} \times \frac{8}{10} = \frac{8}{100} = 0.08$

c) $\frac{4}{100} \times \frac{6}{10} = \frac{24}{1{,}000} = 0.024$ d) $\frac{11}{100} \times \frac{9}{100} = \frac{99}{10{,}000} = 0.0099$

YTI #2: a) Three b) One c) Four d) Two

YTI #3: a) 6.3 b) 0.32 c) 0.102 d) 0.216 e) 0.4118

YTI #4: a) 0.08 b) 0.240 = 0.24 c) 0.0450 = 0.045

YTI #5: a) 81 b) 76.4 c) 1,450 d) 2.7 e) 90

YTI #6: Tim must pay a total of $37.05 for the glove.

Focus Exercises: Section 5.4 w/Answers

Think Again.

1. When adding and subtracting decimals, it is important to line up the decimal points, but it is not necessary to do that when multiplying decimals. Why not? *(Think about it #1)*

Answers will vary. One answer is, When multiplying decimals, we ignore the decimal point until the end of the multiplying.

2. Can we multiply a terminating decimal by a repeating decimal? Explain your answer or show an example that supports your answer.

No. (Not in their decimal forms.) We cannot get to the end of the repeating decimal to start the multiplication process.

Multiply by first rewriting each decimal as a fraction. Multiply the fractions, then write the answer as a decimal.

3. 0.7 x 6
4.2

4. 0.3 x 0.8
0.24

5. 1.5 x 0.4
0.6

6. 1.1 x 0.9
0.99

7. 1.3 x 0.05
0.065

8. 0.03 x 6
0.18

Given the following multiplication, decide how many decimal places the product (the result) will have. Do not multiply.

9. 0.9 x 3
2.7

10. 8 x 0.2
1.6

11. 0.7 x 0.6
0.42

12. 0.1 x 0.8
0.08

13. 1.4 x 0.25
0.35

14. 3.9 x 0.86
3.354

15. 0.05 x 0.14
0.007

16. 9.15 x 1.07
9.7905

Multiply by following the procedure for multiplying decimals.

17. 8 x 0.4
3.2

18. 5 x 0.9
4.5

19. 0.3 x 7
2.1

20. 0.2 x 2
0.4

21. 0.9 x 0.7
0.63

22. 0.6 x 0.8
0.48

23. 0.3 x 1.5
0.45

24. 3.5 x 0.4
1.40

25. 9.1 x 0.6
5.46

26. 0.1 x 7.2
0.72

27. 1.4 x 1.1
1.54

28. 5.3 x 0.1
0.53

29. 0.18 x 0.5
0.09

30. 0.15 x 0.9
0.135

31. 0.3 x 0.2
0.06

32. 0.2 x 0.14
0.28

33. 0.03 x 0.5
0.015

34. 0.05 x 0.4
0.02

35. 0.01 x 0.9
0.009

36. 0.01 x 0.7
0.007

37. 0.12 x 0.04
0.0048

38. 0.15 x 0.09
0.0135

39. 0.041 x 0.07
0.00287

40. 0.032 x 0.06
0.00192

41. 0.05 x 0.03
0.00115

42. 0.01 x 0.02
0.0002

43. 9.04 x 0.05
0.452

44. 2.03 x 0.05
0.1015

45. 1.25 x 1.5
1.875

46. 5.25 x 1.9
9.975

47. 4.25 x 5.7
24.225

48. 3.25 x 4.5
14.625

49. 2 x 0.034
0.068

50. 1.6 x 1.2
1.92

51. 2.1 x 4.13
8.673

52. 13.04 x 0.17
2.2168

Use your knowledge of multiplying signed numbers, along with the techniques of multiplying decimals, to find each product.

53. **-** 5 x 0.9
-4.5

54. - 6 x 0.2
-1.2

55. - 0.4 x 0.8
-0.32

56. - 0.7 x 0.6
-0.42

57. 2.5 x (- 0.4)
-1

58. 4.5 x (- 0.8)
-3.6

59. 6.5 x (- 0.08)
-0.52

60. 7.5 x (- 0.04)
-0.3

61. - 12.5 x (- 1.6)
20

62. - 6.25 x (- 0.8)
5

63. - 10.4 x (- 0.05)
0.52

64. - 2.25 x (- 0.04)
0.09

Multiply by moving the decimal point the appropriate number of places to the right.

65. 11.9 x 10
119

66. 6.1 x 10
61

67. 0.8 x 10
8

68. 0.1 x 10
1

69. 9.06 x 10
90.6

70. 5.07 x 10
50.7

71. 3.76 x 100
376

72. 9.41 x 100
941

73. 0.08 x 100
8

74. 0.09 x 100
9

75. 1.206 x 100
120.6

76. 8.602 x 100
860.2

77. 4.8 x 100
480

78. 31.5 x 100
3,150

79. 0.401 x 1,000
401

80. 3.159 x 1,000
3,159

81. 8.05 x 1,000
8,050

82. 1.56 x 1,000
1,560

83. 1.3 x 1,000
1,300

84. 6.2 x 1,000
6,200

Solve. Round each dollar answer to the nearest penny. Answer each question with a complete sentence.

85. In Alabama, the sales tax rate is 0.04. How much sales tax is there on a book that costs $19.50?
There is $0.78 in sales tax on the book.

86. In Michigan, the sales tax rate is 0.06. How much sales tax is there on a shirt that costs $22.90?
There is $1.37 in sales tax on the shirt.

87. In Minnesota, the sales tax rate is 0.065. How much sales tax is there on a computer printer that costs $104.00?
There is $6.76 in sales tax on the computer printer.

88. In Oklahoma, the sales tax rate is 0.045. How much sales tax is there on a bicycle that costs $252.00?
There is $11.34 in sales tax on the bicycle.

89. At a hardware store, Arash is buying a pack of batteries priced at $24.95. If the sales tax rate is 0.07, what is the total amount Arash must pay for the batteries?
The total amount for the batteries would be $26.70.

90. At a music and video store, Linda is purchasing a DVD collection priced at $53.90. If the sales tax rate is 0.065, what is the total amount Linda must pay for the DVD collection?
The total amount for the DVD collection would be $57.40.

91. Janelle is buying a used Toyota Camry for $7,600. If the sales tax rate is 0.075, what is the total amount Janelle must pay for the Camry?
The total amount for the Camry is $8,170.00.

92. Mark wants to buy a Tony Gwynn autographed baseball card priced at $120.00 If the sales tax rate is 0.0625, what is the total amount Mark must pay for the card?
The total amount for the card is $127.80.

93. Carlos earns $7.45 per hour. Yesterday he worked 6.5 hours. How much did Carlos earn yesterday?
Carlos earned $48.43 yesterday.

94. Terra earns $8.17 per hour. Yesterday she worked 7.5 hours. How much did Terra earn yesterday?
Terra earned $61.28 yesterday.

95. Oyuki earns $9.60 per hour. Last week she worked 37.25 hours. How much did Oyuki earn last week?
Oyuki earned $357.60 last week.

96. JonRey earns $10.40 per hour. Last week he worked 32.75 hours. How much did JonRey earn last week?
JonRey earned $340.60 last week.

Think Outside the Box.

97. Terrence works for a company that pays each employee a standard hourly wage for the first 40 hours worked each week, and 1.25 times the standard wage for each hour worked beyond 40 hours in one week. Terrence's standard wage is $9.60 per hour. If Terrence worked 48 hours last week, how much did he earn for the week?

Terrence earned $480.00 for the week.

98. Molly is buying a blanket and two pillows for her guest bedroom. The price of the blanket is $22.95, the price of each pillow is $10.95, and the sales tax rate is 0.07. If Molly pays for the whole purchase with two $20 bills and a $10 bill, how much change will she receive back after the purchase?

Molly will receive back $2.01.