Section 2.1 Solving Linear Equations Involving Two Operations

Objectives
In this section, you will learn to:

- Define the term *equation*.
- Apply the Addition Property of Equality.
- Apply the Multiplication Property of Equality.
- Isolate variables.
- Solve equations involving one operation.
- Solve equations involving two operations.

To successfully complete this section, you need to understand:

- Additive identity (1.1)
- Multiplicative identity (1.1)
- Adding and subtracting real numbers (1.3 and 1.4)
- Multiplying and dividing real numbers (1.5)
- The main operation (1.7)
- The reciprocal (R.3)

INTRODUCTION

The equal sign, =, is used primarily two different ways in mathematics. In Chapter 1, we used *continued equal signs* when simplifying an expression to indicate that one step is equivalent the next step.

This is illustrated in Example 1 from Section 1.7, simplifying an expression using the order of operations:

$(4 - 16) \div 2^2$	Subtract first.
$= -12 \div 2^2$	Apply the exponent
$= -12 \div 4$	Divide.
$= -3$	

We also sometimes see continued equal signs when simplifying a fraction, such as $\frac{48}{72}$. We might need to divide out some smaller common factors before it is completely simplified:

$$\frac{48}{72} = \frac{24}{36} = \frac{4}{6} = \frac{2}{3}$$

$\frac{48}{72}$	$\frac{24}{36}$	$\frac{4}{6}$
↑ Simplify by a factor of 2	↑ Simplify by a factor of 6	↑ Simplify by a factor of 2

An equal sign is also used to indicate the equivalence of two expressions that may or may not contain variables. This equivalence is called an *equation*. The equations that we see in this text include numbers and variables and require us to find the value of the variable that makes the equation true. These equations are often true for only one or two numbers.

For example, the equation $5 + x = 7$ is true only when $x = 2$:

$$5 + (2) = 7$$
$$7 = 7 \quad \textit{True!}$$

and the equation $x + 7 = 2x + 1$ is true only when $x = 6$:

$$(6) + 7 = 2(6) + 1$$
$$13 = 12 + 1$$
$$13 = 13 \quad \textit{True!}$$

Some equations, called **identities**, are true for all numbers.

For example, the equation $x + 3 = x + 3$ is true when

$x = 2$:	$x = 10$:	$x = -7$:
$(2) + 3 = (2) + 3$	$(10) + 3 = (10) + 3$	$(-7) + 3 = (-7) + 3$
$5 = 5$ *True!*	$13 = 13$ *True!*	$-4 = -4$ *True!*

Really, the equation $x + 3 = x + 3$ is an identity and is true for all values of x.

Equations that have only one or two solutions are called **conditional equations**. The equations above, $5 + x = 7$ and $x + 7 = 2x + 1$ each have only one solution, so they are conditional equations.

The discussion in this section is about *solving* conditional equations, finding the value of the variable that makes an equation true.

PREPARING TO SOLVE LINEAR EQUATIONS

There is a class of conditional equations called *linear equations*. In a linear equation, the highest power of any variable is 1. Here are some examples of linear equations:

(a) $3x + 5 = 23$ (b) $-9 = 15 - 2y$ (c) $6w + 11 = -4w - 9$

To be well prepared to solve linear equations, we must be familiar with the following ideas from Chapter 1.

		Section	Property	Example
1.	The additive identity is 0:	1.1	$a + 0 = a$ and $0 + a = a$	$9 + 0 = 9$
2.	The multiplicative identity is 1:	1.1	$b \cdot 1 = b$ and $1 \cdot b = b$	$1 \cdot 15 = 15$
3.	For addition, the sum of a number and its opposite is 0:	1.3	$a + (-a) = 0$	$4 + (-4) = 0$
4.	For multiplication, the product of a number and its reciprocal is 1: If a number is negative, its reciprocal is also negative:	1.5	$\frac{a}{b} \cdot \frac{b}{a} = 1$	$\frac{3}{4} \cdot \frac{4}{3} = 1$ $\frac{-2}{5} \cdot \frac{-5}{2} = 1$
5.	The main operation.	1.9	The main operation of an expression is the last operation to be applied, according to the order of operations.	In the expression $6y + 5$, the main operation is *addition*.
6.	The sign of a term.	1.9	The sign in front of a term belongs to that term.	In the expression $3w - 4$, the second term is -4.
7.	If a variable is in the numerator of a fraction, the coefficient of the variable is the whole fraction, not just the numerator.	1.8	$\frac{ax}{c} = \frac{a}{c}x$	$\frac{3x}{4} = \frac{3}{4}x$ The coefficient of x is $\frac{3}{4}$.

THE VOCABULARY OF EQUATIONS

An **equation** is a mathematical sentence in which

one expression = another expression

Each expression has one or more terms, and each term is either a constant or contains a variable. Any term that contains a variable is called a **variable term**.

If the highest exponent of the variable is 1, such as x^1 or x, then the equation is a **linear equation**. A linear equation typically has only one solution.

Note:

Linear is pronounced lih´-nee-er.

Here are some examples of linear equations. Notice that the variable terms can be on the left side, on the right side, or on both sides:

$$n + 3 = -8$$
$$18 = 6v$$
$$6x + 5 = x$$
$$64 + y = 178 - 2y$$

Each of these equations is a conditional equation and is true for only one value. To **solve** an equation is to find the one number that makes the equation true; this number is called the **solution**.

Throughout this section, and the next two, we look at a variety of strategies for solving linear equations. If a mistake is made during the solving process, then we can still get an answer, but it won't be the correct answer, it won't be the *solution*.

After we solve an equation and arrive at an answer, we can verify if the answer is, indeed, the solution by placing the answer into the equation, as demonstrated in Example 1.

Note:
In the process of determining if a replacement value is a solution, we use the symbol $\stackrel{?}{=}$ until we can verify whether the last statement is true or not.

Example 1: For each equation, a student solved the equation and arrived at the answer shown. Verify whether the answer is a solution or not.

a) $12 = 8 - 4x$; $\boldsymbol{x = 5}$ b) $3w - 2 = 2w - 5$; $\boldsymbol{w = -3}$

Procedure: For each equation, replace the variable with the given answer and evaluate each side. If the two sides are equal, then the answer makes the equation true, and it is the solution. If they are not equal, then the answer is not the solution.

Answer: a)

$$12 = 8 - 4x \qquad \text{Replace } x \text{ with 5.}$$
$$12 \stackrel{?}{=} 8 - 4(5)$$
$$12 \stackrel{?}{=} 8 - 20$$
$$12 = -12 \quad \text{False}$$

No, 5 is not the solution.

b) $3w - 2 = 2w - 5$ Replace each w with -3.

$3(-3) - 2 \stackrel{?}{=} 2(-3) - 5$

$-9 - 2 \stackrel{?}{=} -6 - 5$

$-11 = -11$ True **Yes, -3 is the solution.**

You Try It 1 For each equation, a student solved the equation and arrived at the answer shown. Verify whether the answer is a solution or not. Use Example 1 as a guide.

a) $15 - 5m = 30$; $\boldsymbol{m = 3}$ b) $18 = 5p - 2$; $\boldsymbol{p = 4}$ c) $6v + 3 = 4v - 1$; $\boldsymbol{v = -2}$

BALANCING EQUATIONS

The strategy for solving a linear equation is to *isolate the variable*. To **isolate the variable** means to get the variable alone on one side of the equation and a single number on the other, such as $x = -5$ or $3 = y$.

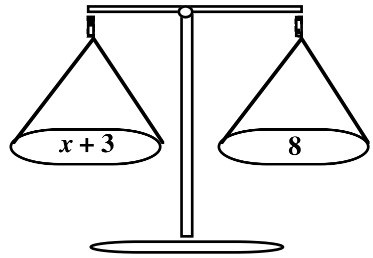
For example, to isolate x in the equation $x + 3 = 8$, we must remove the constant 3 from the left side. This will leave x alone (isolated) on that side.

In the solving process, as we work our way toward isolating the variable, we must often write several equations, each one a bit simpler than the previous one. This is called *reducing the equation.* Each equation in this series of reduced equations is equivalent to the others, and these **equivalent equations** all have the same solution.

To create simpler, reduced, equations we make changes to the expressions in the equation. To maintain the solution, though, we must keep the equations balanced by performing the same operation—addition, subtraction, multiplication, or division—on each side of the equation.

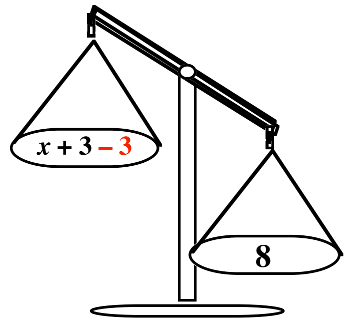
This idea of keeping an equation balanced can be visualized using a scale.

Here we see the equation $x + 3 = 8$. The left side, $x + 3$, and the right side, 8, are equal to each other, so they are in balance.

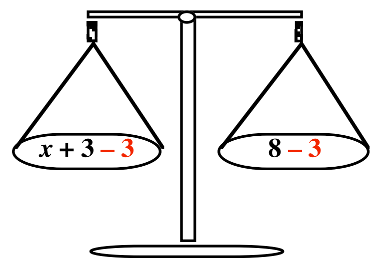


To isolate the variable, we might want to remove the 3 from the left side by subtracting it away.

However, if we subtract 3 from only one side, then that side becomes lighter and the scale (and the equation) is out of balance.



If we subtract 3 from each side of the scale at the same time, then the scale remains in balance and we maintain a balanced equation.



In the example above, subtracting 3 from each side creates a constant term of 0 on the left side, thereby isolating the variable:

$x + 3 = 8$	Subtract 3 from each side.
$x + 3 - 3 = 8 - 3$	Simplify each side
$x + 0 = 5$	Simplify the left side.
$x = 5$	The solution is **5**.

Although the solution, itself, is 5, it's common to state it as $x = 5$.

In general, to isolate the variable or the variable term, we create 0 by adding the opposite of any term we want to remove, or *clear*. The property that allows us do this is the Addition Property of Equality:

The Addition Property of Equality

We may add any number or term, c, to *each side* of an equation.

If $a = b$,

then $a + c = b + c$

When clearing the constant, it is important to keep in mind the notion that *the sign in front of the term belongs to that term.*

This helps us to identify the sign of the constant term, and we can then add its opposite to each side of the equation.

Note:
As we see in Example 2b), it is common to write the final equation with the variable on the left and the constant on the right.

This will be most beneficial in Section 2.7, *Solving Inequalities,* so we'll get in the habit now of placing the variable on the left side in the answer.

Example 2: Solve each equation by clearing the constant. Also, verify the solution.

a) $x - 6 = 4$ b) $-9 = 4 + y$ c) $w - \frac{2}{5} = \frac{8}{5}$

Procedure: Apply the Addition Property of Equality. Recognize the constant that must be cleared, and add its opposite to isolate the variable. Verify the solution by using it as a replacement value for the variable in the original equation.

Answer:

a) $x - 6 = 4$ — Clear the constant -6 by adding +6 to each side.

$x - 6 + 6 = 4 + 6$ — Simplify each side; $-6 + 6 = 0$.

$x + 0 = 10$ — $x + 0$ is just x.

$x = 10$

Verify the solution, 10:

$x - 6 = 4$

$10 - 6 \stackrel{?}{=} 4$

$4 = 4$ ✓

b) $-9 = 4 + y$	Clear the constant 4 by adding -4 to each side.	Verify the solution, -13:
$-9 + (-4) = 4 + (-4) + y$	Simplify each side; $4 + (-4) = 0$.	$-9 = 4 + y$
$-13 = 0 + y$	$0 + y$ is just y.	$-9 \stackrel{?}{=} 4 + (-13)$
$-13 = y$	Write the equation with the variable on the left side.	$-9 = -9$ ✓
$y = -13$		
c) $w - \frac{2}{5} = \frac{8}{5}$	Clear the constant $-\frac{2}{5}$ by adding $\frac{2}{5}$ to each side.	Verify the solution, 2 or $\frac{10}{5}$:
$w - \frac{2}{5} + \frac{2}{5} = \frac{8}{5} + \frac{2}{5}$	Simplify each side; $-\frac{2}{5} + \frac{2}{5} = 0$.	$w - \frac{2}{5} = \frac{8}{5}$
$w + 0 = \frac{10}{5}$		$\frac{10}{5} - \frac{2}{5} \stackrel{?}{=} \frac{8}{5}$
$w = 2$	To verify the solution, use $w = \frac{10}{5}$.	$\frac{8}{5} = \frac{8}{5}$ ✓

Look back at Example 2 and notice these three things:

1. Each step in the solving process is directly below the preceding step.
2. The equal signs are lined up, one below the other.
3. We isolated the variable by creating 0, the additive identity, on the side of the variable term.

You Try It 2 Solve each equation by clearing the constant. Also, verify the solution. Use Example 2 as a guide.

a) $x + 7 = 20$

b) $-19 = b - 12$

c) $w - 9 = -9$

d) $\frac{8}{3} = m + \frac{7}{3}$

SOLVING EQUATIONS INVOLVING MULTIPLICATION

Some linear equations involve multiplication but no addition or subtraction, such as

$$\frac{3}{2}y = 18 \qquad \text{and} \qquad 5x = -35.$$

In each of these equations, the coefficient of the variable is not 1, so to isolate the variable, we must create a coefficient of 1. We do so by multiplying each side of the equation by the reciprocal of the coefficient.

For example, in the equation $\frac{3}{2}y = 18$, we can create a coefficient of 1 by multiplying each side by $\frac{2}{3}$ (the reciprocal of the coefficient, $\frac{3}{2}$):

$\frac{3}{2}y = 18$ — Multiply each side by the reciprocal of $\frac{3}{2}$, which is $\frac{2}{3}$.

$\frac{2}{3} \cdot \frac{3}{2}y = \frac{2}{3} \cdot 18$ — Simplify. $\frac{2}{3} \cdot \frac{3}{2} = 1$; $\frac{2}{3} \cdot \frac{18}{1} = \frac{2}{1} \cdot \frac{6}{1} = \frac{12}{1} = 12$.

$1 \cdot y = 12$ — $1 \cdot y = y$

$y = 12$ — To verify the answer, use $y = \frac{12}{1}$.

Verify:

$$\frac{3}{2}y = 18$$

$$\frac{3}{2} \cdot \frac{12}{1} \stackrel{?}{=} 18$$

$$\frac{3}{1} \cdot \frac{6}{1} \stackrel{?}{=} 18$$

$$18 = 18 \checkmark$$

The property that allows us to multiply each side of the equation by the same number is the Multiplication Property of Equality.

The Multiplication Property of Equality

Each side of an equation may be multiplied by any non-zero number, c, and the equality remains:

$$\text{If} \quad a = b,$$

$$\text{then} \quad c \cdot a = c \cdot b, \quad c \neq 0$$

If the coefficient is an integer, as in $5x = -35$, then we have two options:

1. We can multiply each side of the equation by the reciprocal of 5, which is $\frac{1}{5}$.

$5x = -35$ — Multiply each side by $\frac{1}{5}$.

$\frac{1}{5} \cdot \frac{5}{1} x = \frac{1}{5} \cdot \frac{-35}{1}$ — Simplify. $\frac{1}{5} \cdot \frac{-35}{1} = -7$

$1 \cdot x = -7$

$x = -7$

2. We can divide each side by the coefficient, 5:

$5x = -35$ — Divide each side by 5.

$\frac{5x}{5} = \frac{-35}{5}$ — Simplify each fraction.

$1x = -7$ — x has a coefficient of 1.

$x = -7$

← We get the same result using either technique. →

This example shows us that we can also divide each side by the same number.

The Division Property of Equality

We may divide each side of an equation by any non-zero number, c:

If $a = b$,

then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$

This is especially helpful when the coefficient is an integer. In Example 8, there are a variety of coefficients, so read each step carefully to see what the best strategy is.

Example 3: Solve each equation by clearing the coefficient. Also, verify the solution.

a) $-4x = 32$ b) $15 = \frac{-5}{8} y$ c) $\frac{5w}{2} = -20$ d) $18 = 12p$

Procedure: Apply the Multiplication Property of Equality. Recognize the coefficient that must be cleared, and either multiply each side by its reciprocal or divide by the coefficient directly. Verify the solution by using it as a replacement value for the variable in the original equation.

Answer:

a) $-4x = 32$

The coefficient is an integer, -4, so we divide each side by -4.

$$\frac{-4x}{\mathbf{-4}} = \frac{32}{\mathbf{-4}}$$

Simplify each fraction.

$$x = -8$$

Verify the solution, -8:

$$-4x = 32$$

$$-4(\mathbf{-8}) \stackrel{?}{=} 32$$

$$32 = 32 \checkmark$$

b) $15 = \frac{-5}{8}\, y$

The coefficient is a fraction, $\frac{-5}{8}$, so we multiply each side by its reciprocal, $\frac{-8}{5}$.

$$\frac{\mathbf{-8}}{\mathbf{5}} \cdot \frac{15}{1} = \frac{\mathbf{-8}}{\mathbf{5}} \cdot \frac{-5}{8}\, y$$

Simplify each side.

$$\frac{-8}{5} \cdot \frac{15}{1} = \frac{-8}{1} \cdot \frac{3}{1} = -24.$$

$$-24 = 1y$$

Write the equation with the variable on the left.

$$y = -24$$

Verify the solution, -24:

$$15 = \frac{-5}{8}\, y$$

$$15 \stackrel{?}{=} \frac{-5}{8} \cdot (-24)$$

$$15 = \frac{-5}{8} \cdot \frac{-24}{1}$$

$$15 = \frac{-5}{1} \cdot \frac{-3}{1}$$

$$15 = 15 \checkmark$$

c) $\frac{5w}{2} = -20$

First write the left side as $\frac{5}{2}\, w$.

$$\frac{5}{2}\, w = -20$$

The coefficient is a fraction, $\frac{5}{2}$, so we multiply each side by its reciprocal, $\frac{2}{5}$.

$$\frac{\mathbf{2}}{\mathbf{5}} \cdot \frac{5}{2}\, w = \frac{\mathbf{2}}{\mathbf{5}} \cdot \frac{-20}{1}$$

Simplify each side.

$$\frac{2}{5} \cdot \frac{-20}{1} = \frac{2}{1} \cdot \frac{-4}{1} = -8.$$

$$1w = -8$$

$$w = -8$$

Verify the solution, -8:

$$\frac{5w}{2} = -20$$

$$\frac{5(-8)}{2} \stackrel{?}{=} -20$$

$$\frac{-40}{2} \stackrel{?}{=} -20$$

$$-20 = -20 \checkmark$$

d) $18 = 12p$

The coefficient is an integer, 12, so we divide each side by 12.

Verify the solution, $\frac{3}{2}$:

$$18 = 12p$$

$\frac{18}{12} = \frac{12p}{12}$	Simplify each fraction.	$18 \stackrel{?}{=} 12 \cdot \frac{3}{2}$
	$\frac{18}{12}$ simplifies by a factor of 6 to $\frac{3}{2}$.	$18 \stackrel{?}{=} \frac{12}{1} \cdot \frac{3}{2}$
$\frac{3}{2} = p$	Write the equation with the variable on the left.	$18 \stackrel{?}{=} \frac{6}{1} \cdot \frac{3}{1}$
$p = \frac{3}{2}$		$18 = 18$ ✓

Look back at Example 3 and notice three things:

1. Each step in the solving process is directly below the preceding step.
2. The equal signs are lined up, one below the other.
3. We created a coefficient of 1 (the multiplicative identity) to isolate the variable.

When the coefficient is -1, we can clear it by either dividing or multiplying each side by -1.

Example 4: Solve $-x = 7$ by (a) dividing by -1 (b) multiplying by -1.

Procedure: The coefficient is -1.

Answer:

a) $-x = 7$ ← Divide each side by -1.

$\frac{-1x}{-1} = \frac{7}{-1}$

$x = -7$ ← We get the same solution. →

b) $-x = 7$ ← Multiply each side by -1.

$-1 \cdot (-1x) = -1 \cdot 7$

$x = -7$

You Try It 3 Solve each of these equations by clearing the coefficient. Verify the solution. Use Examples 3 and 4 as guides.

a) $\frac{5}{6} v = -30$

b) $-20 = -5x$

c) $12 = \frac{-3y}{4}$

d) $-w = -14$

e) $-21p = 6$

b) $8 = -x$

Remember:	Whatever we do to modify one side of an equation—by either adding, subtracting, multiplying, or dividing a number or term—we must do likewise to the other side.

SOLVING LINEAR EQUATIONS IN STANDARD FORM

In the linear equations we have solved so far in this section, to isolate the variable, we had to clear either a constant or a coefficient. We now turn our attention to equations that require us to clear both a constant and a coefficient, such as $3x + 5 = -7$. Equations of this type are considered to be in *standard form*:

The **standard form** of a linear equation is

$$ax + b = c \quad \text{or} \quad c = ax + b,$$

where x is a variable and a, b, and c are constants, $a \neq 0$.

In the standard form of a linear equation, a cannot be 0, but either b or c (or both) may be 0.

Think about it 1 In the standard form of a linear equation, why is it important that $a \neq 0$?

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Because an equation in standard form involves two operations, the question becomes, “To isolate the variable, which number should we clear first, the constant or the coefficient?”

For example, in the equation $3x + 5 = -7$, should we first clear the constant, 5, by adding -5 to each side, or should we first clear the *coefficient*, 3, by dividing each side by 3?

There are two reasons we must clear the constant first:

1. Because the goal is to isolate the variable, we must prepare this equation—and others like it—by first *isolating the variable term*. This means that we must clear the constant first and reduce the equation.

2. In any equation involving more than one operation, we must first clear the *main operation*. For example, because $3x + 5$ is a sum (the main operation is addition), we must clear the sum by adding the opposite of the constant.

In either case, once the constant has been cleared, we have an equation that can be solved by clearing the coefficient.

Example 5: Solve each equation, and verify the solution.

a) $3x + 5 = -7$ b) $-15 = 6 - 7y$

Procedure: First isolate the variable term by clearing the constant.

Answer:

a)

$3x + 5 = -7$	The constant term is 5, so add -5 to each side.
$3x + 5 + (-5) = -7 + (-5)$	Simplify each side be combining like terms.
$3x + 0 = -12$	Simplify the left side.
$3x = -12$	Clear the coefficient by dividing each side by 3.
$\frac{3x}{3} = \frac{-12}{3}$	Simplify each side. $\frac{3x}{3}$ simplifies to $1x$, or just x.
$x = -4$ →	Verify the solution. Replace x with -4 in the original equation.

$$3(-4) + 5 \stackrel{?}{=} -7$$

$$-12 + 5 \stackrel{?}{=} -7$$

$$-7 = -7 \checkmark$$

b)

$-15 = 6 - 7y$	The constant term is 6, so add -6 to each side.
$-15 + (-6) = 6 + (-6) - 7y$	Simplify each side be combining like terms.

$-21 = \mathbf{0} - 7y$	$0 - 7y$ is $-7y$.
$-21 = -7y$	Clear the coefficient by dividing each side by -7.
$\frac{-21}{-7} = \frac{-7y}{-7}$	Simplify each side. $\frac{-7y}{-7}$ simplifies to $1y$, or just y.
$3 = y$	Verify the solution. Replace y with 3 in the original equation. $-15 \stackrel{?}{=} 6 - 7(3)$
$y = 3$ ↗	$-15 \stackrel{?}{=} 6 - 21$
	$-15 = -15$ ✓

You Try It 4 Solve each equation by first isolating the variable term. Verify the solution. Use Example 5 as a guide.

a) $3x - 5 = 19$

b) $\frac{4}{3}y + 15 = 3$

c) $4 = 3m - 6$

c) $-4v + 12 = -16$

e) $-1 = 9 + 2w$

f) $-2 = 18 - 5p$

Answers: You Try It and Think About It

YTI 1: a) 0 = 30; No, 3 is not the solution. b) 18 = 18; Yes, 4 is the solution.
c) -9 = -9; Yes, -2 is the solution.

YTI 2: a) $x = 13$ b) $b = -7$ c) $w = 0$ d) $m = \frac{1}{3}$

YTI 3: a) $v = -36$ b) $x = 4$ c) $y = -16$ d) $w = 14$
e) $p = \frac{-2}{7}$ f) $x = -8$

YTI 4: a) $x = 8$ b) $y = -9$ c) $m = \frac{10}{3}$ d) $v = 7$
e) $w = -5$ f) $p = 4$

Think About It: **1.** If $a = 0$, then the equation would have no variable to solve for.

Section 2.1 Exercises

Think Again.

1. Refer to the definition of *continued equal signs,* at the beginning of this section, and how they are used in simplifying a fraction. Is it possible to solve a linear equation using continued equal signs? Explain your answer. *(Refer to Think About It 1.)*

2. In the standard form of a linear equation, why is it important that $a \neq 0$? *(Refer to Think About It 2.)*

3. To clear a constant, we add its opposite to each side. To clear a coefficient, why don't we divide by its opposite?

4. What extra step(s) would be required to solve this equation: $-1 = 3y - 5y + 9$?

Focus Exercises.

For each, replace the variable with 16 and decide whether 16 is the solution.

5. $29 + m = 55$ **6.** $114 = k + 98$ **7.** $5 \cdot d = 80$ **8.** $210 = x \cdot 15$

For each, replace the variable with $\frac{2}{3}$ and decide whether $\frac{2}{3}$ is the solution.

9. $-5 = 6k - 9$ **10.** $7 + 3m = 11$ **11.** $-12x + 7 = 1$ **12.** $-4 = 6 - 15y$

Solve each equation. Verify the solution.

13. $x - 12 = 6$ **14.** $p - 2 = 8$ **15.** $-7 = w - 4$

16. $-10 = m - 3$ **17.** $y - 9 = -8$ **18.** $u - 12 = -6$

19. $2 = p + 8$ **20.** $6 = a + 9$ **21.** $m + 6 = -4$

22. $x + 9 = -8$ **23.** $0 = x + 6$ **24.** $0 = w + 8$

25. $p - 5 = 0$ **26.** $v - 10 = 0$ **27.** $4 = v + 4$

28. $9 = k + 9$ **29.** $m - 6 = -6$ **30.** $x - 8 = -8$

31. $x + \frac{5}{12} = \frac{7}{12}$ **32.** $y + \frac{4}{18} = \frac{13}{18}$ **33.** $m + \frac{11}{6} = \frac{5}{6}$

34. $v + \frac{17}{24} = \frac{5}{24}$ **35.** $w - \frac{11}{30} = \frac{1}{30}$ **36.** $p - \frac{9}{28} = \frac{15}{28}$

37. $v + \frac{3}{16} = \frac{-5}{16}$ **38.** $u + \frac{14}{45} = \frac{-1}{45}$ **39.** $\frac{-7}{4} = x + \frac{5}{4}$

40. $\frac{-3}{8} = y + \frac{7}{8}$ **41.** $\frac{-5}{3} = m - \frac{1}{3}$ **42.** $\frac{-7}{10} = v - \frac{11}{10}$

Solve each equation. Verify the solution.

43. $6n = 42$ **44.** $5x = 20$ **45.** $7y = -42$

46. $4m = -28$ **47.** $-3k = -36$ **48.** $-2x = -18$

49. $18 = -9p$ **50.** $35 = -7w$ **51.** $-x = -9$

52. $-w = -11$ **53.** $-r = 8$ **54.** $-q = 12$

55. $4w = 18$ **56.** $10y = 25$ **57.** $-9y = 21$

58. $-8y = 20$ **59.** $\frac{2}{3}v = 10$ **60.** $\frac{4}{5}m = 12$

61. $\frac{-2}{9}\ y = 6$

62. $\frac{-9}{5}\ x = 36$

63. $8 = \frac{2x}{7}$

64. $20 = \frac{5y}{4}$

65. $\frac{-3k}{5} = -\frac{12}{25}$

66. $\frac{-2v}{3} = -\frac{14}{9}$

Solve each equation. Verify the solution.

67. $3x + 5 = 17$

68. $22 = 7w + 1$

69. $4x + 9 = 9$

70. $5p - 4 = -4$

71. $4 - 5w = -36$

72. $11 = -7 - 9c$

73. $45 = 4x + 9$

74. $-27 = 8w + 5$

75. $-7v - 7 = 14$

76. $-2y - 9 = -17$

77. $-y + 12 = 10$

78. $-k - 10 = -3$

79. $-8 = -p + 12$

80. $1 = -m - 10$

81. $-8 = 5x - 8$

82. $-5 = -5 - 6m$

83. $2y + 6 = 0$

84. $12 + 3v = 0$

85. $3y - 5 = 0$

86. $4v - 3 = 0$

87. $3v + 1 = 0$

88. $2m + 9 = 0$

89. $0 = -20 - 4x$

90. $0 = -18 - 6v$

91. $0 = 15 - 10x$

92. $0 = 9 - 21y$

93. $2 - 2k = -7$

94. $12 = -18 - 4w$

95. $4m - 9 = -39$

96. $6v - 8 = 20$

97. $10 = -5p + 18$

98. $-7 = 8x - 5$

99. $16 + 12k = 1$

100. $8 - 15y = -2$

101. $\frac{1}{3}\ x - 2 = 3$

102. $\frac{1}{4}\ y - 5 = 2$

103. $\frac{2}{3}\ x + 9 = 7$

104. $\frac{3}{4}\ p + 15 = 3$

105. $\frac{3}{5}\ x - 7 = -11$

106. $\frac{2}{7}\ v - 5 = -8$

107. $6 - \frac{2y}{5} = 10$

108. $3 - \frac{7x}{2} = 10$

Think Outside the Box.

Solve each equation by first combining like terms, as necessary.

109. $8w - 5w + 1 = -8$

110. $-2y - 3 - 4y = -27$

111. $13 = 2x - 7x + 28$

112. $-15 = -7q + 4q + 21$

Each of these contains a variable term on each side of the equation. To solve, first get each to standard form by clearing one of the variable terms (add its opposite to each side of the equation). Then finish solving the equation.

113. $7w + 19 = 4w - 8$

114. $-2y + 1 = 4y + 16$

115. $13 - 5x = -8 - 2x$

116. $-14 - 3p = 7p + 26$