2.2 Solving Linear Equations With More Than Two Operations

Objectives
In this section, you will learn to:

- Solve equations involving more than one variable term.
- Simplify expressions before solving.

To successfully complete this section, you need to understand:

- Operations with real numbers (Chapter 1)
- Combining like terms (1.9)
- The Distributive Property (1.10)
- Solving standard form linear equations (2.1)

INTRODUCTION

In Section 2.1, we became familiar with solving equations in standard form, $ax + b = c$, which involves two operations, addition (or subtraction) and multiplication, such as $6x + 9 = -15$. To solve, we reduce the equation by first clearing the constant term, 9.

Sometimes, though, an equation has more than two operations. Examples of such equations are

$$4x + 17 = 6x + 5, \qquad 2x + 8 + 4x = -4 + 3, \qquad \text{and} \qquad 12 - 2y = 3(y - 1).$$

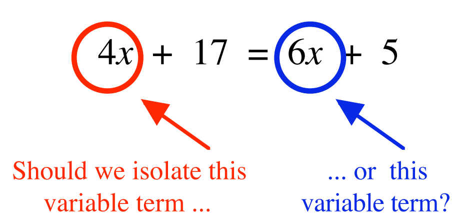
As explained throughout this section, our initial goal in solving these more complex linear equations will be to reduce each equation to standard form. This may take a few steps, but once an equation is in standard form, we can apply what we learned in Section 2.1 to finish solving.

SOLVING LINEAR EQUATIONS CONTAINING MORE THAN ONE VARIABLE TERM

Remember, the ultimate goal in solving any linear equation is to isolate the variable, and this means first isolating the variable term. However, some equations have more than one variable term, one on each side, such as $4x + 17 = 6x + 5$.

Question: To solve this equation, which variable term should we isolate?

Answer: Remember that any solution we find becomes a replacement value. This solution replaces the variable in *both* variable terms.



In the example $4x + 17 = 6x + 5$, the x's are the same variable and must be combined as a single term before we can truly isolate the variable. However, because the variables are in different expressions—on different sides of the equal sign—the only way we can combine them is to clear one of the variable terms by adding its opposite to each side.

Does it really matter which variable term we choose to clear? No. Because the equation has only one solution, whichever variable term we choose to clear will lead us to the solution.

Example 1 and You Try It 1 solve the same equation using two different approaches.

Example 1: Solve $4x + 17 = 6x + 5$

Procedure: First, combine the variable terms by adding the opposite of one them to each side.

Answer:

$4x + 17 = 6x + 5$ — Prepare the equation for solving by clearing $+4x$: add $-4x$ to each side of the equation.

$4x + (-4x) + 17 = 6x + (-4x) + 5$

$0 + 17 = 2x + 5$ — $4x + (-4x) = 0x$, or just 0.

$17 = 2x + 5$ — Reduce this equation by clearing the constant: add -5 to each side.

$17 + (-5) = 2x + 5 + (-5)$

$12 = 2x + 0$

$12 = 2x$ — Divide each side by 2.

$\frac{12}{2} = \frac{2x}{2}$

$6 = x$ — Write the variable on the left side.

$x = 6$

Verify the solution, 6. Replace each variable in the *original* equation:

$4(6) + 17 \overset{?}{=} 6(6) + 5$

$24 + 17 \overset{?}{=} 36 + 5$

$41 = 41$ ✓

You Try It 1 Solve this equation by clearing the $+6x$ term from each side. The first step is shown. Complete the solving process. Use Example 1 as a guide.

$4x + 17 = 6x + 5$

$4x + (-6x) + 17 = 6x + (-6x) + 5$

Verify the solution:

Think about it #1: Comparing the work shown in Example 1 and your own work in You Try It 1, which variable term, $4x$ or $6x$, would you likely clear if you had your choice? Why? Share your answer with a classmate.

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You Try It 2 Solve each equation. Check each answer to show that it is a solution. Use Example 1 as a guide.

a) $4w - 5 = 4 + 7w$

b) $3y + 9 = -5y + 41$

c) $x + 10 = 2 - 3x$

d) $2p - 4 = -p - 25$

SIMPLIFYING EXPRESSIONS BEFORE SOLVING

Sometimes, in reducing an equation to standard form, we must simplify one (or both) of the sides; after all, each side is an expression, and it's common for us to simplify expressions.

For example,

(1) we may need to first distribute a number to a quantity, as in the expression $3(y - 1)$

$$3(y - 1)$$
$$= 3y - 3$$

(2) we may need to combine like terms, as in the expression $2x + 8 + 4x$.

$$2x + 8 + 4x.$$
$$= 6x + 8$$

Example 2: Solve $12 - 2y = 3(y - 1)$

Procedure: First simplify the right side by distributing 3 to the quantity $(y - 1)$. Then reduce the equation to standard form by clearing one of the variable terms.

Answer:

$12 - 2y = 3(y - 1)$ — First distribute on the right side. Do not try to clear any terms just yet.

$12 - 2y = 3y - 3$ — Reduce this equation to standard form. Let's clear $-2y$ by adding its opposite, $+2y$, to each side.

$12 - 2y + \mathbf{2y} = 3y + \mathbf{2y} - 3$

$12 + 0 = 5y - 3$

$12 = 5y - 3$ — Now isolate the variable term by clearing the constant: add +3 to each side.

$12 + \mathbf{3} = 5y - 3 + \mathbf{3}$

$15 = 5y + 0$

$15 = 5y$ — Divide each side by 5.

$\frac{15}{\mathbf{5}} = \frac{5y}{\mathbf{5}}$

$3 = y$

$y = 3$ ↗

Verify the solution, 3. Replace each variable in the original equation:

$12 - 2(3) \stackrel{?}{=} 3(3 - 1)$

$12 - 6 \stackrel{?}{=} 3(2)$

$6 = 6$ ✓

Example 3: Solve $2x + 8 + 5x = -4 + 3x$ by first simplifying each side.

Procedure: First simplify the left side by combining like terms. Then reduce the equation to standard form by clearing one of the variable terms.

Answer:

$$2x + 8 + 5x = -4 + 3x$$

Combine like terms on the left side: $2x + 5x = 7x$.

$$7x + 8 = -4 + 3x$$

Reduce this equation to standard form. Let's clear $+3x$ by adding its opposite, $-3x$, to each side.

$$7x + \mathbf{(-3x)} + 8 = -4 + 3x + \mathbf{(-3x)}$$

$$4x + 8 = -4 + 0$$

$$4x + 8 = -4$$

Now isolate the variable term by clearing the constant: add -8 to each side.

$$4x + 8 + \mathbf{(-8)} = -4 + \mathbf{(-8)}$$

$$4x + 0 = -12$$

$$4x = -12$$

Divide each side by 4.

$$\frac{4x}{\mathbf{4}} = \frac{-12}{\mathbf{4}}$$

$$x = -3 \nearrow$$

You finish it:

Verify the solution, 3. Replace each variable in the original equation:

You Try It 3 Solve each equation by first simplifying each side. Use Examples 2 and 3 as guides.

a) $4(y + 1) = 3(2y - 2)$

b) $3m + 6 + 2m = 8 + m - 10$

c) $-2(3w + 5) = 7w - 4 - w$

d) $-x - 9 + 5x = -1 + 2(x - 5)$

Answers: You Try It and Think About It

YTI 1: $x = 6$

YTI 2: a) $w = -3$ b) $y = 4$ c) $x = -2$ d) $p = -7$

YTI 3: a) $y = 5$ b) $m = -2$ c) $w = -\frac{1}{2}$ d) $x = -1$

Think About It:

1. Answers may vary.

Section 2.2 Exercises

Think Again.

1. Why is it important to simplify each side of an equation before applying any of the properties of equality?

2. What would you write to a classmate to explain the importance of doing a check after solving an equation?

Focus Exercises.

Solve each equation. Verify the solution.

3. $3m + 1 = 7m - 15$

4. $4x + 1 = 13 - 2x$

5. $2 + 2x = -10 + 5x$

6. $9 - 2y = 15 - 8y$

7. $3y - 7 = -5y - 39$

8. $2p - 1 = -4p - 19$

9. $5w + 9 = 9w + 33$

10. $3x - 12 = 6 + 5x$

11. $9 - 6x = 14 + 4x$

12. $12 + 6y = 15 - 3y$

13. $2x - 11 = 10x + 13$

14. $20n + 11 = 20 + 8n$

15. $7v - 7 = 5v - 7$

16. $3w - 5 = 7w - 5$

17. $-4c - 4 = -4 - c$

18. $-2h - 3 = -3 - h$

19. $9w + 8 = 7w$

20. $11q + 36 = 7q$

21. $2x - 20 = 7x$

22. $5v - 18 = 8v$

23. $20 - 6x = -4x$

24. $-5w = 3w + 24$

Solve each equation. Verify the solution.

25. $12 - 8a - 6 = -13a + 21$

26. $3 + m + 7 = 19 - 2m$

27. $4x - 2 - 3x = -x + 16$

28. $-y + 18 + 3y = 6 + 5y$

29. $4d + 1 - 6d = 3(5 - 3d)$

30. $2(5 - x) = 3x + 6 - x$

31. $4(3c + 2) = 2c - 2$

32. $3(y - 6) = 4y - 8$

33. $2x + 2 = 6(x + 3)$

34. $4x + 15 = 5(x + 4)$

35. $5p + 7 - 3p = 17 - 9p + 12$

36. $-c + 6 + 3c = -2c + 6$

37. $10 - 3w + 5 = 9w + 3 - 6w$

38. $2y + 5 - 4y = 6y - 11$

39. $15 + 4y - 4 = 12y - 1 - 20y$

40. $10x + 19 - 20x = 4 - 5x$

41. $2v + 3 - 4v = -1(v + 9)$

42. $k - 12 + 5k = 3(k + 2)$

43. $-7(m + 2) = -m - 2 - 5m$

44. $6(r - 1) = r - 10 + 3r$

45. $-3(2q - 5) = 4q + 6 - q$

46. $5(x + 4) = 3x + 5 + 7x$

47. $14 + 8m - 13 = 3(m - 4) + 3$

48. $8c - 2 + 5c = -11 + 2(c - 1)$

49. $7 + 6p = 10p + 7 - 30p$

50. $8x - 2 + 10x = 6 - 7x - 8$

51. $2(3 - v) = v - 9 - 6v$

52. $-1(w - 10) = 3w - 2 - 7w$

53. $n + 10 - 2n = -5(n + 2)$

54. $y + 2 - 6y = -4(y - 3)$

55. $3(x + 20) + (x + 20) + x = 180$

56. $2(w + 9) + 2w = 90$

57. $3(x - 4) + 2x = 7x + 2(-2x - 1)$

58. $5(x + 1) - 3x = -19 + 2(4 - 3x)$

Think Outside the Box.

Solve each equation.

59. $x^2 - 5x + 4 = x^2 + 3x + 52$

60. $2(4x - 3x^2) - 9 = -3(x + 2x^2) + 46$

61. The perimeter of the rectangle is the same as the perimeter of the triangle. What is the length of the rectangle?